Minimum Reinforcement in High-Strength Concrete

By C. Bosco,[1] A. Carpinteri,[2] Member, ASCE, and P. G. Debernardi[3]

Abstract: A dimensional analysis criterion is proposed to compute the minimum amount of reinforcement for high-strength concrete members in flexure. Such an amount is assumed to be provided by the condition of simultaneous first cracking aand steel yielding. The fracture mechanics model utilized defines a brittleness number N_P, which is revealed to be a measure of the experimental brittleness or ductility of the test. N_P is a function of steel-yield strength f_y, concrete fracture toughness K_{IC}, steel percentage A_S/A, and beam depth. The brittleness of the structural member increases by increasing the size and/or decreasing the steel content. On the other hand, a physically similar behavior is revealed in the cases where the brittleness number N_P is the same. The experimental investigation carried out by the writers shows that the failure mechanism changes completely when the beam depth is varied, the steel percentage being the same. Only when the steel percentage is inversely proportional to the square root of the beam depth is the mechanical behavior reproduced.

Introduction

Crack formation in reinforced concrete (R.C.) beams provokes unstable behavior. If the test is performed by controlling deformation or crack opening, the unstability phenomenon can be followed in its actual evolution. Otherwise, a positive jump of deformation or a negative jump of loading capacity occurs, according to the loading control, i.e., dead-load or fixed-grip condition. A qualitative load versus deflection diagram is represented in Fig. 1, where the snap-back and snap-through actual behavior is bypassed along the short cuts AB (load-control) or AC (deflection-control).

The analysis of such experimental behavior is useful in the determination of rational criteria to compute the minimum amount of reinforcement for concrete members in flexure. The minimum amount of reinforcement could be given by the condition for which first concrete cracking and steel yielding are simultaneous. Several national and international standard codes provide empirical formulas for the determination of the minimum amount of reinforcement. At most, two parameters are taken into account in these formulas: the concrete tensile strength and the steel-yield strength. Other important features are neglected, such as the size-scale of the concrete member or its fracture energy. Moreover, current codes establish minimum amounts that sometimes are very different.

In some codes (France, Switzerland, Eurocode EC2) the minimum amount is established on the basis of the ratio between the computed stresses in

[1]Asst. Prof., Dept. of Struct. Engrg., Politecnico di Torino, 24 Corso Duca degli Abruzzi, 10129 Torino, Italy, 39-11-5567832.

[2]Prof., Dept. of Struct. Engrg., Politecnico di Torino, 24 Corso Duca degli Abruzzi, 10129 Torino, Italy.

[3]Assoc. Prof., Dept. of Struct. Engrg., Politecnico di Torino, 24 Corso Duca degli Abruzzi, 10129 Torino, Italy.

Note. Discussion open until July 1, 1990. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on March 20, 1989. This paper is part of the *Journal of Structural Engineering*, Vol. 116, No. 2, February, 1990. ©ASCE, ISSN 0733-9445/90/0002-0427/$1.00 + $.15 per page. Paper No. 24357.

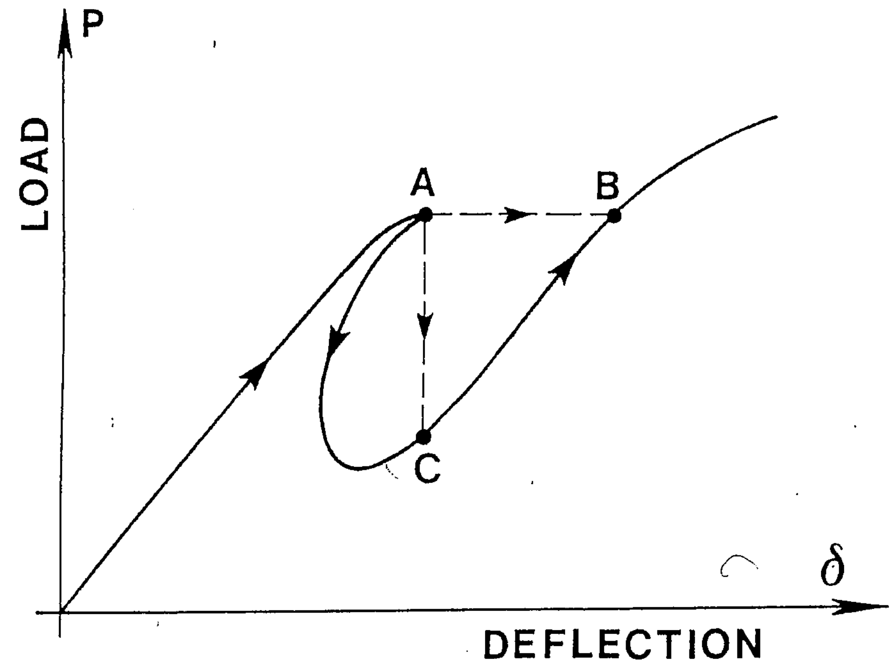

FIG. 1. Snap-Through and Snap-Back Behaviors, Respectively, with Load-Control (AB) and Deflection-Control (AC)

concrete and steel; other codes (Great Britain, American Concrete Institute) take into account only the steel-yield strength, while in Italy and Soviet Union a minimum percentage of steel is fixed independently of any geometrical and mechanical feature (Szalai 1988).

It is possible to take into consideration the beam-size effect on the minimum steel percentage, through the concepts of fracture mechanics. The present theoretical and experimental investigation aims at proposing a dimensional analysis criterion to determine the minimum amount of reinforcement in high-strength concrete beams in flexure. For this purpose a brittleness number is defined (Carpinteri 1981, 1984; Carpinteri and Carpinteri 1984):

$$N_P = \frac{f_y h^{1/2}}{K_{IC}} \frac{A_S}{A} \quad (1)$$

f_y being the steel-yield strength; K_{IC} the concrete fracture toughness; A_S/A the steel percentage; and h the beam depth. The brittleness of the structural member increases by increasing the size or decreasing the steel content. On the other hand, a physically similar behavior is revealed in cases where the brittleness number, N_P, is the same. The experimental investigation carried out by the writers shows that the failure mechanism changes completely when the beam depth is varied, the steel percentage being the same. Only when the steel percentage is inversely proportional to the square root of the beam depth is the mechanical behavior reproduced (Bosco et al. 1988).

Specimen Preparation

The beams used in the present experimental investigation are made of concrete with crushed aggregate of maximum size 12.7 mm. The amount of cement (type 525) is 4.8 kN/m^3, and the water-cement ratio is equal to 0.27. Considerable attention was spent to avoid cracking from hydration and

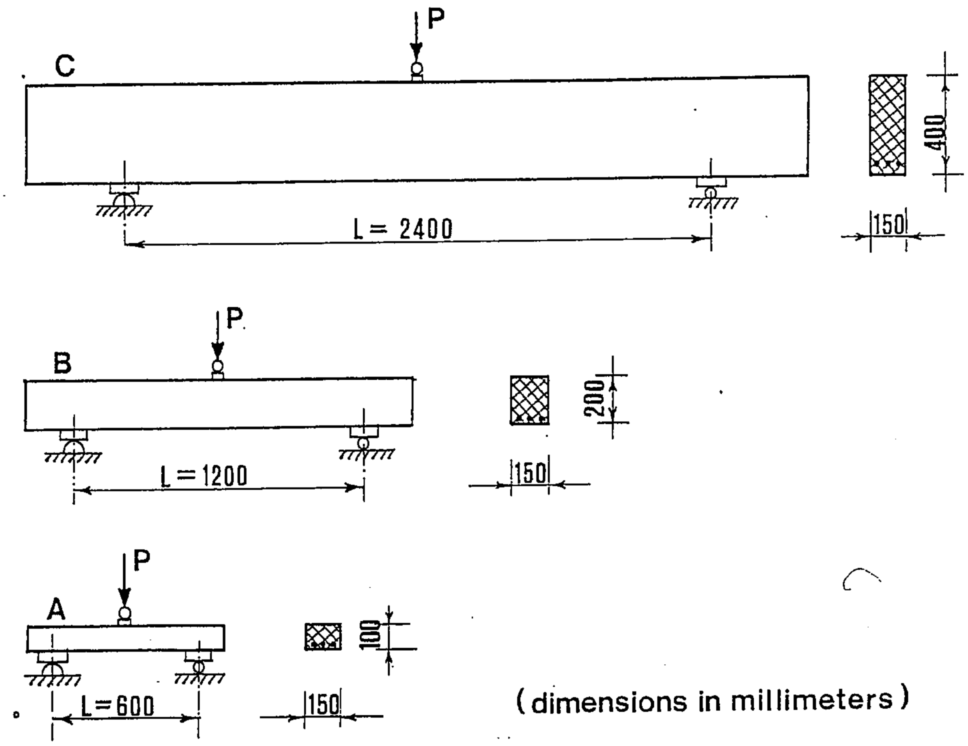

FIG. 2. Three Different Size-Scales Selected for Three-Point-Bending Geometry

shrinkage. The compressive strength (after 28 days) was obtained with 20 cubic specimens measuring 160 mm. The average value was $R_{cm} = 91.2$ N/mm^2, with a standard deviation of 8.8 N/mm^2. The curing time of the beams was three days at 30° C, followed by a second period at 20° C. On average, the tests were carried out 20 days after molding. The tests of elastic modulus were performed on three specimens measuring 150 × 150 × 450 mm, and provided an average value of the secant modulus E (between zero and one-third of the ultimate load) equal to 34,300 N/mm^2.

The fracture energy G_F was determined by three-point-bending tests on three specimens of size $h = 100$ mm, $b = 150$ mm, $l = 750$ mm. The span was equal to $L = 720$ mm and the beams were prenotched on the centerline, the notch depth equaling one half of the beam depth, and the notch width measuring 5 mm. The average value of the fracture energy results was $G_F = 0.090$ N/mm, according to Rilem draft recommendation "Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams," so that the critical value of the stress-intensity factor can be evaluated:

$$K_{IC} = \sqrt{G_F E} = 55.56 \text{ N/mm}^{3/2} \qquad (2)$$

The steel bars had nominal diameters of 4, 5, 8, and 10 mm, respectively. The 4 and 5 mm bars did not exhibit well-defined yield point and conventional limit, obtained from the stress-strain curve at 0.2% permanent deformation, is equal to 637 N/mm^2 and 569 N/mm^2, respectively. The yield strength for the bars of 8 and 10 mm, on the other hand, equaled 441 N/mm^2 and 456 N/mm^2, respectively.

Thirty reinforced concrete beams were tested, with the cross section thickness $b = 150$ mm, and depth $h = 100$, 200, and 400 mm, respectively. The span between the supports was assumed to equal six times the beam depth

TABLE 1. Description of Reinforced Concrete Specimens and Related Loads of First Cracking, Steel Yielding and Final Collapse

Brittleness class (1)	Sizes $b \times h$ (mm) (2)	Nominal content of steel (3)	Actual percentage of steel (4)	Yield limit of steel (N/mm^2) (5)	Actual value of N_P (6)	Cracking load (kN) (7)	Yielding load (kN) (8)	Ultimate load (kN) (9)
			(*a*) Beam size A					
0	150 × 100	0	0.000	0	0	11.38 12.16	0.00 0.00	0.00 0.00
1	150 × 100	1ϕ4	0.085	637	0.097	11.77 11.77	6.43 7.61	6.01 5.55
2	150 × 100	2ϕ5	0.256	569	0.261	12.07 12.95	14.71 15.70	11.77 10.79
3	150 × 100	2ϕ8	0.653	441	0.514	13.73 13.34	26.98 28.90	22.48 21.64
4	150 × 100	2ϕ10	1.003	456	0.847	15.30 14.52	34.14 34.88	48.79 46.83
			(*b*) Beam size B					
0	150 × 200	0	0.000	0	0	23.08 22.03	0.00 0.00	0.00 0.00
1	150 × 200	1ϕ5	0.064	569	0.093	21.40 17.66	10.42 10.18	6.10 5.50
2	150 × 200	3ϕ5	0.190	569	0.275	19.50 22.19	23.04 23.16	17.69 16.59
3	150 × 200	3ϕ8	0.490	441	0.550	21.19 23.54	40.79 42.06	57.53 55.91
4	150 × 200	3ϕ10	0.775	456	0.898	25.74 27.62	64.01 65.89	77.31 75.81
			(*c*) Beam size C					
0	150 × 400	0	0.000	0	0	44.12 36.28	0.00 0.00	0.00 0.00
1	150 × 400	2ϕ4	0.043	637	0.098	36.67 —	15.69 —	8.40 —
2	150 × 400	4ϕ5	0.128	569	0.261	39.97 37.49	33.08 31.64	25.50 23.29
3	150 × 400	4ϕ8	0.327	441	0.521	45.50 40.79	55.69 52.16	66.54 63.46
4	150 × 400	4ϕ10	0.517	456	0.847	49.03 48.83	87.66 81.18	99.18 96.12

h and, therefore, measure 600, 1,200, 2,400 mm for the specimens A, B, and C, respectively (see Fig. 2).

The specimens were marked in the following way:

- By varying the beam size:
  - (A) Beam depth h = 100 mm (b = 150 mm; L = 600 mm).
  - (B) Beam depth h = 200 mm (b = 150 mm; L = 1,200 mm).
  - (C) Beam depth h = 400 mm (b = 150 mm; L = 2,400 mm).
- By varying the brittleness class:
  - (0) Brittleness number N_P = 0.00 (no reinforcement).
  - (1) Brittleness number $N_P \approx 0.10$ (on the average).
  - (2) Brittleness number $N_P \approx 0.26$ (on the average).
  - (3) Brittleness number $N_P \approx 0.53$ (on the average).
  - (4) Brittleness number $N_P \approx 0.87$ (on the average).

The content of steel depends on the beam size and on the brittleness number (see Eq. 1). It is reported for each beam in Table 1. No shear reinforcement was present.

The distance of the bars from the lower beam edge is, in each case, equal to one-tenth of the total beam depth. For each beam size (A,B,C) and for each brittleness class (0,1,2,3,4), two reinforced concrete beams were realized, with a total of 30 specimens. All the beams were initially unnotched and uncracked. The following results, when not otherwise specified, are related to the average value of each case contemplated by the experimental investigation.

Testing Procedure

The experiments were carried out at the Department of Structural Engineering of the Politecnico di Torino. The three-point-bending tests were realized by a servocontrolled machine. The beams were supported by a cylindrical roller and a spherical connection at the two extremities. The load was applied through a hydraulic actuator, and the loading process was controlled by a strain gage (type DD1), placed on the lower beam edge parallel to the beam axis and symmetrical with respect to the force. Its length was equal to the beam depth, i.e., 100, 200, or 400 mm, for beam sizes A, B and C, respectively (see Fig. 2). The sensitivity of the strain gage utilized is 1 mV/1 V/1 mm, for a feed-voltage of 5 V. The strain rate was imposed at a constant and very low value. On the average, the crack formation in the middle of the beam was achieved after about 7 min and the steel yielding after about 45 min.

Transducers with a sensitivity of 1 mV/0.01 mm were used to measure the central deflection. The latter was referred to a bar, connected with the concrete beam at the middle of the depth, corresponding to the two supports. Such a device is shown in Fig. 3, for a reinforced concrete beam of depth

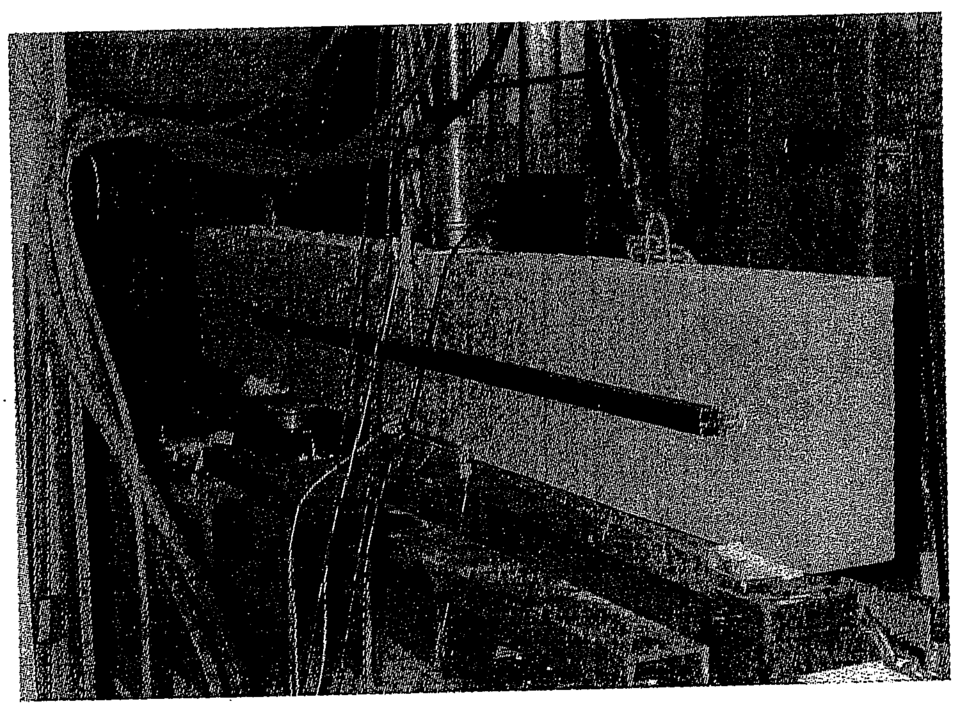

FIG. 3. Testing Apparatus and Deflection Reference Bar Connected with Concrete Beam

h = 400 mm (size C). Deflection and strain-gage deformation were plotted automatically as functions of the applied load.

The load of first cracking was detected by means of a brittle enamel, applied in the zone where the first crack formation is expected. The loads of first-crack formation, of steel yielding and of final collapse, are summarized in Table 1.

INTERPRETATION

The load-deflection diagrams are plotted in Figs. 4(*a*)–(*c*) for each beam size and brittleness class (each curve is related to a single specimen of the two considered). As is possible to verify in Table 1, the peak or first-cracking load is decidedly lower than the steel-yielding load only in the cases 3 and 4, i.e., for high brittleness numbers N_P. In cases 0 and 1, the opposite result is clearly obtained. On the other hand, case 2 demonstrates a transition condition between hyperstrength and plastic collapse, the two critical loads being very close.

Specimen C0 (h = 400 mm, no reinforcement) presents a clear snap-back behavior, the softening branch assuming even a positive slope [see Fig. 4(*c*)]. It was possible to follow such a branch, since the loading process was con-

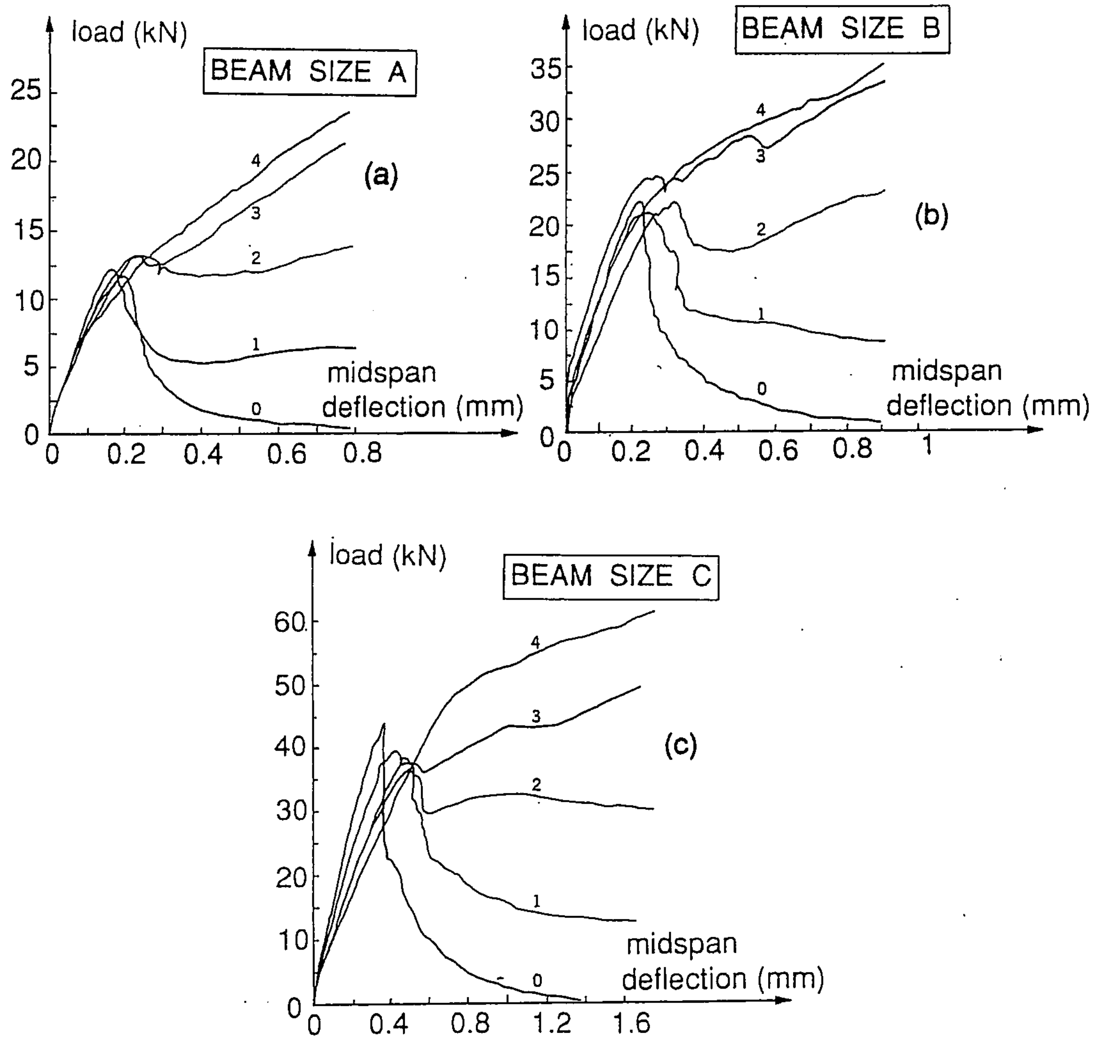

FIG. 4. Load versus Deflection Diagrams for Reinforced Concrete Beams: (*a*) Beam Depth h = 100 mm; (*b*) Beam Depth h = 200 mm; (*c*) Beam Depth h = 400 mm

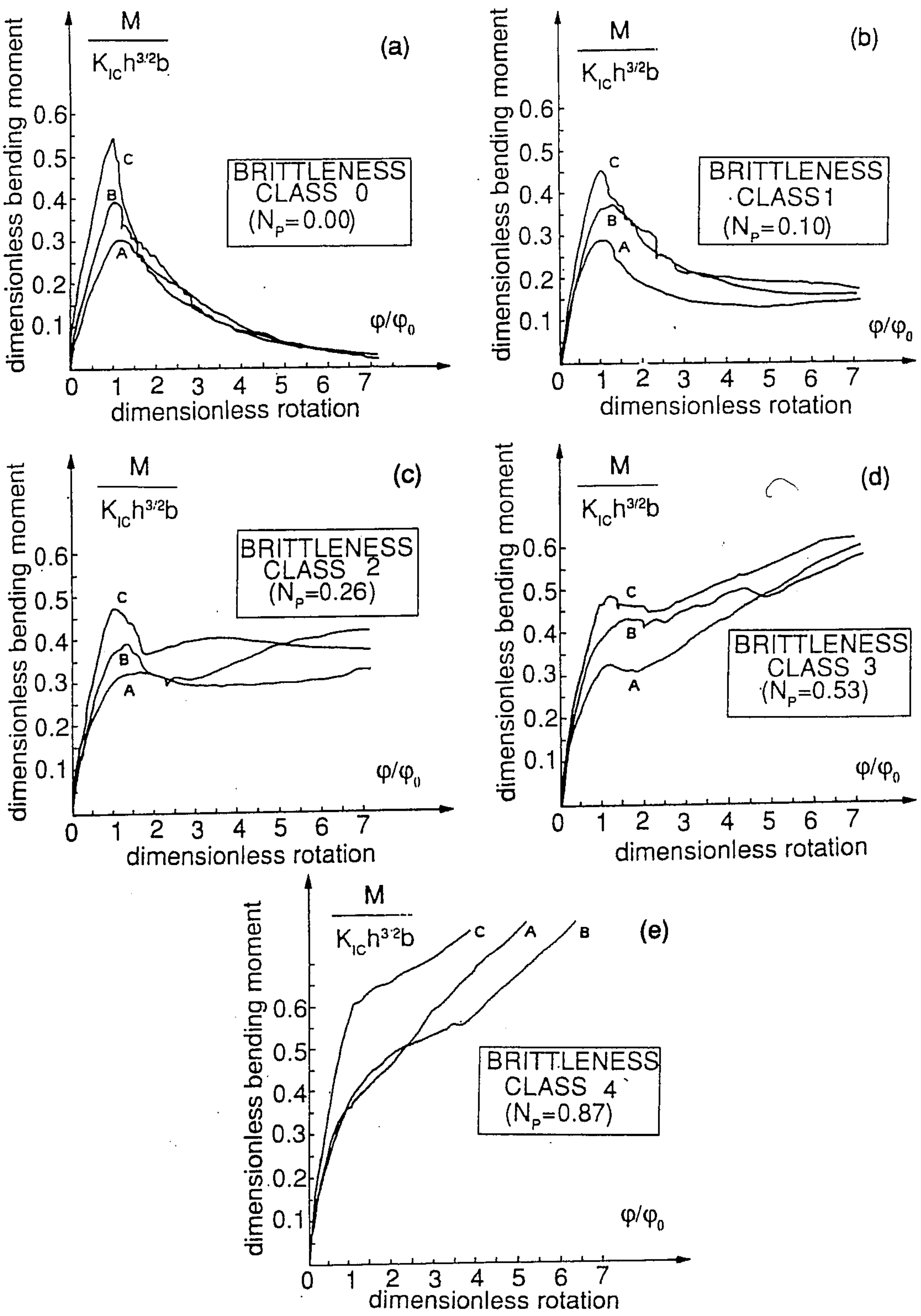

FIG. 5. Dimensionless Bending Moment versus Rotation Diagrams: Beam Depth (A) h = 100 mm; Beam Depth (B) h = 200 mm; Beam Depth (C) h = 400 mm

trolled by a monotonically increasing function of time, i.e., the crack-mouth-opening displacement. If the controlling parameter had been the central deflection, a sudden drop in the loading capacity and an unstable, fast crack propagation would have occurred (Carpinteri 1985; Levi et al. 1987).

The dimensionless bending moment versus rotation diagrams are plotted in Figs. 5(*a*)–(*e*), for each brittleness class and beam size. The local rotation is nondimensionalized with respect to the value ϕ_0 recorded at the first cracking, and is related to the central beam element of length equal to the beam depth h. The bending moment, on the other hand, is nondimensionalized

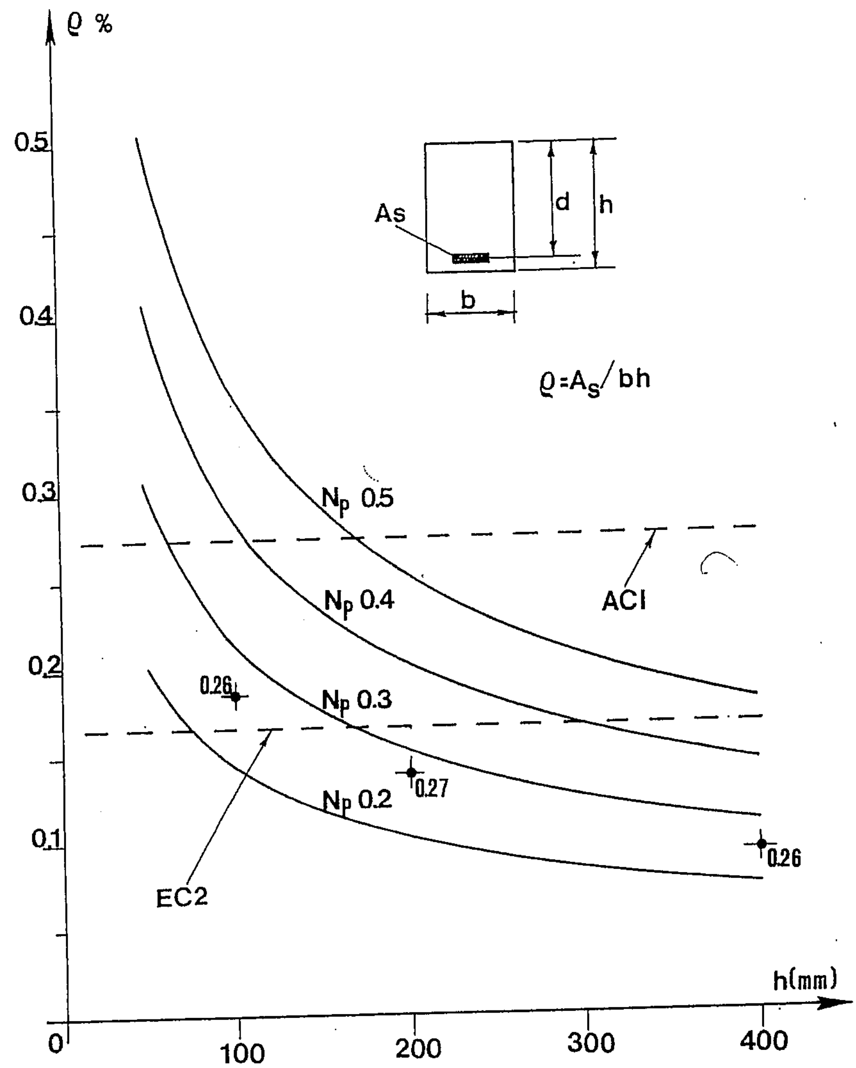

FIG. 6. Steel Percentage against Beam Depth, Brittleness Number N_P Being Constant (f_y = 569 N/mm^2)

with respect to concrete fracture toughness K_{IC} and beam depth h: $M/K_{IC}h^{3/2}b$.

The diagrams in Fig. 5 are significant only for $\phi/\phi_0 > 1$, the strain softening and curvature localization occurring only after the first cracking. The dimensionless peak moment does not appear to be the same, when the brittleness class is the same and the beam depth is varied. This is due to the absence of an initial crack or notch. On the other hand, the postpeak branches are very close to each other and present the same shape for each selected brittleness class. The size-scale similarity seems to govern the postpeak behavior, especially for low brittleness numbers N_P (class 0, 1, 2, 3), and for large beam depths h (sizes B, C).

Therefore, the demand transpires of analyzing the postpeak and ductile behavior of low reinforced high-strength concrete beams, through the concepts of fracture mechanics. The possibility of extrapolating predictions from small to large scales is entrusted to the nondimensional (brittleness) number N_P—see Eq. 1—where, in addition to the traditional geometrical and mechanical parameters, even the concrete fracture toughness K_{IC}, or the concrete fracture energy G_F, appears.

Fig. 5 reveals that a particular value of number N_P does exist, for which the steel-yielding moment is approximately equal to the first cracking mo-

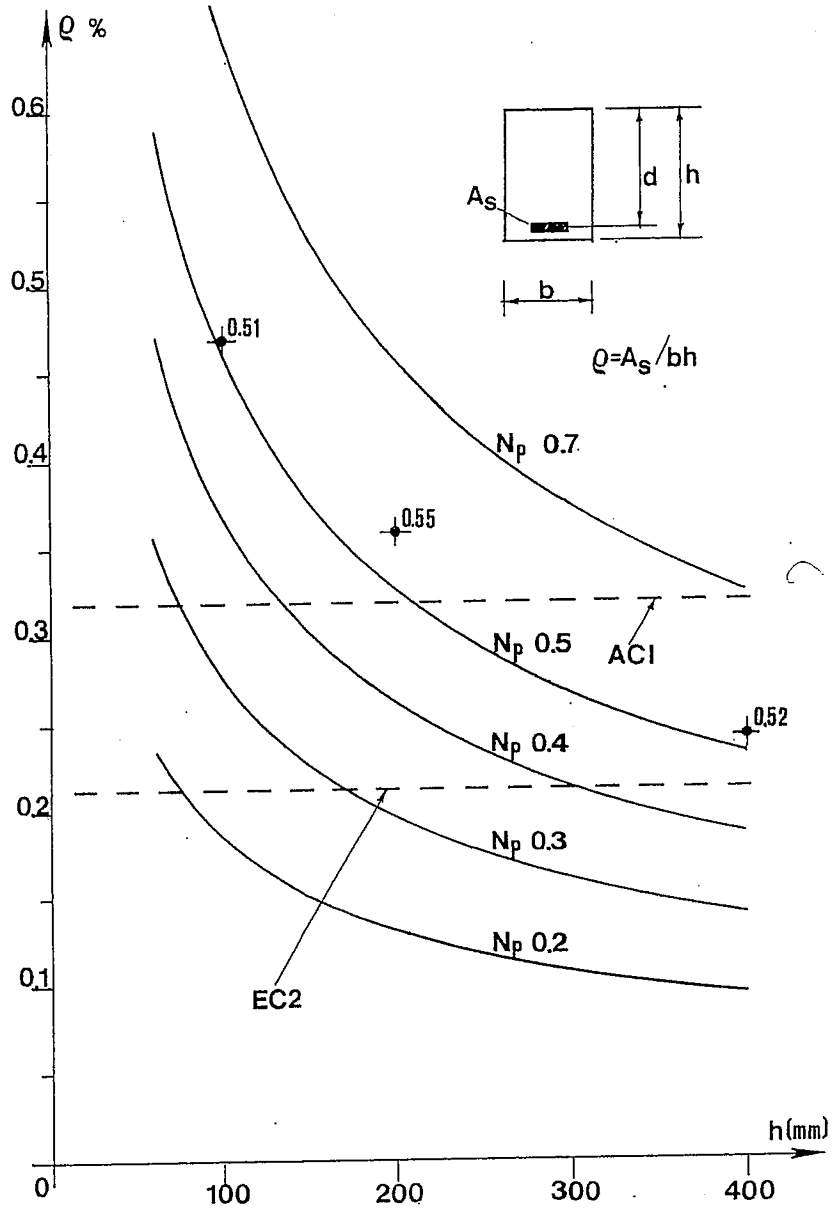

FIG. 7. Steel Percentage against Beam Depth, Brittleness Number N_P Being Constant (f_y = 441 N/mm^2)

ment: $N_P \approx 0.26$. Such a condition can be assumed as that defining the minimum amount of reinforcement (Jaccoud and Charif 1986; Levi 1985; Fehling and König 1986; König 1986). It is evident that, if number N_P is fixed, the steel percentage must be inversely proportional to the beam depth. The brittleness curves N_P = constant are plotted in Figs. 6 and 7, i.e., the steel percentage versus the beam depth. The diagrams are related to the different yield strengths of the utilized bars. In Figs. 6 and 7, the limit values of the American Concrete Institute ("Building Code" 1983) and of the Eurocode EC2 ("Design" 1988) are reported. They are constant and then represented by horizontal lines. It is therefore evident that the formulas suggested by the standard codes are inadequate, at least for high-strength concrete. The steel percentages provided by the codes are conservative for large beam depths, whereas they tend to be insufficient for small beam depths. Also, the experimental results obtained for $N_P \approx 0.26$ (transition value between brittleness and ductility) and $N_P \approx 0.53$, are reported in Figs. 6 and 7.

Currently, the writers are testing concretes with normal strength, R_{ck} =

25 and 40 N/mm², to verify whether the brittleness number N_P can be utilized even for normal concrete in the determination of the minimum reinforcement for structures in flexure. As a matter of fact, in normal concrete the crushing collapse of the compression zone often prevails over the tensile collapse. On the other hand, similar size effects also have been observed recently for normal concrete (Hilleborg 1988).

CONCLUSIONS

1. A dimensional analysis criterion to evaluate the minimum amount of reinforcement in high-strength concrete is proposed. Due to the different physical dimensions of steel-yield strength and concrete fracture toughness, a dimensionless brittleness number N_P is defined in Eq. 1, and is able to describe the size effects on the failure process.
2. The brittleness of reinforced concrete beams increases by increasing the size or decreasing the steel content. On the other hand, a physically similar behavior is revealed in cases where the brittleness number N_P is the same. Only when the steel percentage is inversely proportional to the square root of the beam depth is the mechanical behavior reproduced.
3. A particular value of number N_P does exist, for which the steel-yielding moment is equal to the first cracking moment. Such a condition is assumed as that defining the minimum amount of reinforcement.
4. The minimum steel percentage tends to be inversely proportional to the beam depth, whereas the current standard codes suggest, for direct loading, values independent of the beam depth. It follows that the formulas provided by the codes are inadequate, at least for high-strength concrete.

ACKNOWLEDGMENTS

The financial support of the Department of Public Education and the National Research Council is gratefully acknowledged by the writers.

APPENDIX I. REFERENCES

Bosco, C., Carpinteri, A., and Debernardi, P. G. (1988). "Fracture of reinforced concrete: Scale effect and snap-back instability." Int. Conf. on Fracture and Damage of Concrete and Rock, Inst. of Mechanics, Technical University of Vienna, Vienna, Austria.

"Building code—Requirements for reinforced concrete." (1983). *ACI-318-83*, American Concrete Institute, Detroit, Mich.

Carpinteri, A. (1981). "A fracture mechanics model for reinforced concrete collapse." Colloquium on Advanced Mechanics of Reinforced Concrete, International Association of Bridge and Structural Engineers.

Carpinteri, A. (1984). "Stability of Fracturing Process in RC Beams." *J. Struct. Engrg.*, ASCE, 110(3), 544–558.

Carpinteri, A., and Carpinteri, A. (1984). "Hysteretic behavior of RC beams." *J. Struct. Engrg.*, ASCE, 110(9), 2073–2084.

Carpinteri, A. (1985). "Interpretation on the Griffith instability as a bifurcation of the global equilibrium." Advanced Research Workshop on Application of Fracture Mechanics to Cementitious Composites, North Atlantic Treaty Organization, Evanston (Illinois), S. P. Shah, ed., 287–316.

"Design of concrete structures, part 1." (1988). *Eurocode No. 2*, Commission of European Communities.

Fehling, E., König, G., and Scheidler, D. (1986). "Crack width control and tension stiffening." *DARMSTADT CONCRETE—Ann. J. Concr. and Concr. Struct.*, Institut fur Massivbau, Technische Hochschule Darmstadt, Darmstadt, West Germany, 1, 65–86.
Hilleborg, A. (1988). "Fracture mechanics concepts applied to moment capacity and rotational capacity of reinforced concrete beams." Conf. on Fracture and Damage of Concrete and Rock, Inst. of Mechanics, Technical University of Vienna, Vienna, Austria.
Jaccoud, J. P., and Charif, H. (1986). "Armature minimal pour le contrôle de la fissuration." Rapport final des essais série ⟨⟨C⟩⟩, *Ecôle Polytéchnique Fédérale de Lausanne,* Lausanne, Suisse, 114 (in French).
König, G. (1986). "Restraint crack-width control and minimum reinforcement in thick concrete members." *DARMSTADT CONCRETE—Ann. J. Concr. and Concr. Struct.*, Institut fur Massivbau, Technische Hochschule Darmstadt, Darmstadt, West Germany, 1, 9–22.
Levi, F. (1985). "On minimum reinforcement in concrete structures." *J. Struct. Engrg.*, ASCE, 111(12), 2791–2796.
Levi, F., Bosco, C., and Debernardi, P. G. (1987). "Two aspects of the behavior of slightly reinforced structures." *CEB Bulletin d'Information n° 185,* Comité Euro-International du Beton, 37–50.
Szalai, K. (1988). "Principle of dimensioning of slightly reinforced concrete structures." 26th Plenary Session, Comité Euro-International du Beton, Dubrovnik, Yugoslavia.

APPENDIX II. NOTATION

The following symbols are used in this paper:

A = cross-sectional area of the beams;
A_S = total cross-sectional area of reinforcement;
b = total width of cross section;
d = vertical distance between reinforcement and upper edge of cross section;
E = Young's modulus of concrete;
f_y = yield strength of reinforcement bars;
G_F = concrete fracture energy;
h = total depth of cross section;
L = distance between supports (span);
l = total length of the beam;
K_{IC} = critical value of the stress-intensity factor;
N_P = brittleness number;
P = applied load (at midspan);
R_{cm} = average (cubic) strength of concrete;
R_{ck} = characteristic (cubic) strength of concrete;
δ = deflection at midspan;
φ = local rotation (at midspan);
φ_0 = local rotation at first cracking;
ρ = percentage of steel referred to the total cross section (A_S/A); and
ϕ = bar diameter.