Sensitivity and stability of progressive cracking in plain and reinforced cement composites

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SYNOPSIS

A general formulation of the problem of sensitivity and stability of fracture in plain and reinforced cement composites is presented. It is shown how such phenomena are subjected to very similar laws of scale. The stability of the fracturing process is due to a sufficiently high degree of redundancy in the system. The adjunctive elements, producing such redundancy, are usually fibre or bar reinforcements.

Two particular cases are eventually considered:

1. reinforced concrete beam element, subjected to bending moment;
2. masonry wall, subjected to eccentric axial force.

Their stability conditions are discussed with reference to progressive cracking.

KEYWORDS

Composite materials, fracture properties, crack growth, scale effect, fracture stability, cracking (fracturing), fibre cement composites, crack resistance, reinforced concrete, fracture sensitivity, fracture tests, failure, stresses.

INTRODUCTION

Notch sensitivity is the condition in which a notched specimen of brittle material collapses, due to the stress concentrating effect of the notch. In other words, a material is notch sensitive when the net section rupture stress is lower than the ultimate strength σ_u of an unnotched specimen. With reference to cement composites, several authors have observed that such an effect tends to disappear by decreasing the specimen size [1]. Below a certain size threshold the net section rupture stress is coincident with the ultimate strength σ_u and any stress concentration and amplification effect appears to vanish.

By now the concepts of *Fracture Mechanics* are usually applied to study the mechanical strength of notched and cracked structural elements in a more rational way. Such discipline has been developed by considering a linear elastic and homogeneous solid and defining, as generalized crack extension force, the stress-intensity factor K_I, which is a collapse parameter with rather unusual physical dimensions: (force) $\times$ (length)$^{-3/2}$ [2]. The extension of Linear Elastic Fracture Mechanics to elastic-plastic materials is a very difficult task, and many researchers have introduced new parameters which are not always clearly connected with the factor K_I (for example, J-integral, COD, etc.). Thus, more than of an extension of a theory one can speak of an inevitable renunciation of its basic concepts.

The critical value K_{IC} of the stress-intensity factor (fracture toughness), which should be a material constant, often appears to vary greatly with the sizes of specimen and crack, especially in the case of cement composites. However, above a certain size threshold K_{IC} generally appears constant, i.e. it doesn't vary with the sizes of specimen and crack.

It therefore appears that there is a transition zone

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between fracture collapse and ultimate strength collapse, included between the two above mentioned *upper* and *lower* size thresholds. In this zone the collapse is mixed, i.e. both the collapse parameters σ and K_I are valid [3,4,5].

In fracture tests it is necessary to utilize sufficiently large specimens in order to reproduce a real fracture collapse. An analogous transition is present in Fluids Mechanics, where it is possible to reproduce the phenomenon of turbulent *separation* only if the sizes of the fluid flow are sufficiently large.

The size effects in fracture testing of cement composites are often thought to be due to non-linear phenomena in the behaviour of these materials, which generally appear when the specimen sizes are small. Such non-linear phenomena are *microcracking* and *slow crack growth*. While the former is a dissipative phenomenon, similar to the plasticity of metals and related to the ultimate strength collapse, the latter is nothing but a stable crack extension, preceding the unstable propagation [6].

The stability of fracturing process in a solid body appears to be subjected to size effects too. It has been experimentally verified how such stable processes can become unstable, or viceversa, by varying the structure size or extending the crack [7].

As far as metallic materials are concerned, the plastic flow phenomena at the crack tip seem to generate stable stages in the crack propagation. That is, a certain control may be performed by the plastic zone at the crack tip, in terms of the crack depth, geometry and the size of the cracked body. The Tearing Modulus Theory [7], for instance, intends to explain the fracture stability in ductile materials, without however utilizing the concepts of Linear Elastic Fracture Mechanics at all.

A very similar function of control over the fracturing process is carried out by fibres and bars in cement composites and reinforced concrete. It is also possible to observe some large cracks in concrete dams which have been stabilized, once a compressive zone of the

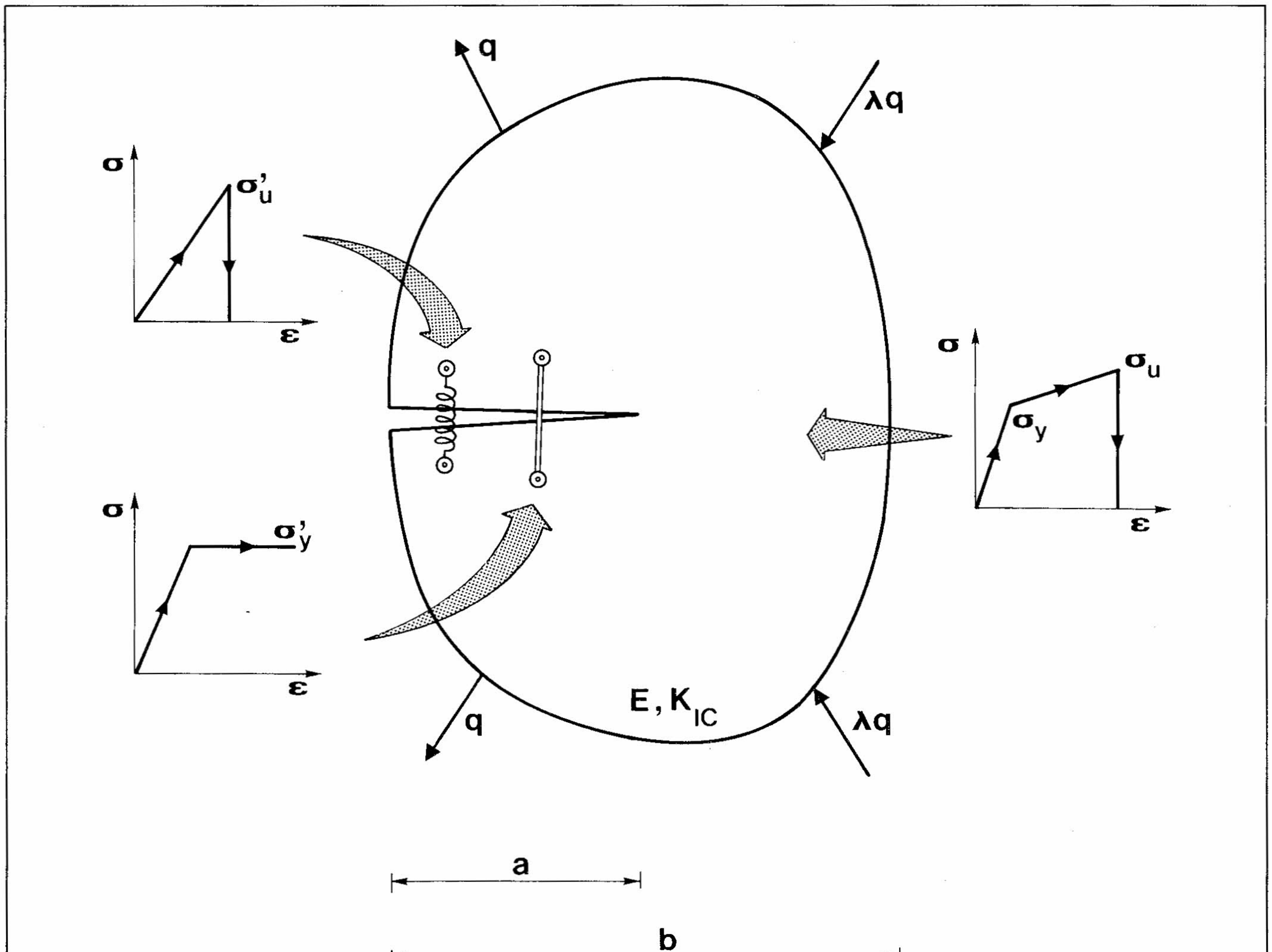

Figure 1 Mechanical model of a cracked reinforced body

structure had been reached [8]. Another stable fracturing process is fatigue fracture. This can be considered to be a combined effect of fracture and shake-down, which follows laws of scale which are not yet clarified.

However, the stability of fracturing process is frequently due to a sufficiently high degree of redundancy in the system [9]. The adjunctive elements, producing redundancy and stability, develop plastic zone or collapse (fibres, bars) as a function of a stress parameter $\sigma = FL^{-2}$, while the brittle crack propagation is governed by the factor $K_I = FL^{-3/2}$. It is precisely from this co-existence of collapses controlled by factors with different physical dimensions, that the scale effects rise, both in *sensitivity* and in *stability* of fracturing processes.

FRACTURE SENSITIVITY

Consider the general case of a symmetrical body of elastic-linear hardening material, symmetrically cracked and loaded (Figure 1). Then let elastic and elastic-plastic tie-rods perpendicular to the crack line be present. Such elements make the system redundant and stabilize the fracturing process. They can represent the plastic zone at the crack tip (Dugdale's model [10]) or external reinforcing elements, as fibres or bars. Other stabilizing factors can be compressive loadings, which tend to close the crack's sides ($K_I \leq 0$).

The collapse load q_{CR} will generally be a function of all the physical quantities involved in the problem:

$$q_{CR} = f(\sigma_Y, \sigma_u, E, K_{IC}; \sigma'_Y, \sigma'_u, E'; b, a, r_i), \qquad (1)$$

where σ_Y, σ_u, E, K_{IC} are the mechanical properties of the basic material (Figure 1), σ'_Y, σ'_u, E' those of the stabilizing elements, b is a characteristic size of the body, a is the crack length, r_i are the ratios between other characteristic sizes of the system and the reference size b. If Buckingham's Theorem for physical similitude and scale modelling is applied and σ_u and b are considered as fundamental quantities, then [3,4]:

$$\frac{q_{CR}}{\sigma_u^\alpha b^\beta} = \varphi\left(\frac{K_{IC}}{\sigma_u b^{1/2}}, \frac{a}{b}; \frac{\sigma_Y}{\sigma_u}, \frac{E}{\sigma_u}, \frac{\sigma'_Y}{\sigma_u}, \frac{\sigma'_u}{\sigma_u}, \frac{E'}{\sigma_u}; r_i\right). \qquad (2)$$

The expression (2) underlines the dependence of the non-dimensional collapse load on the non-dimensional number $s = K_{IC}/\sigma_u b^{1/2}$ (Brittleness Number), where both the mechanical properties of the basic material and the size of the body appear. Thus it is easy to understand how the transition, mentioned earlier, from a collapse due to the overcoming of K_{IC} to a collapse due to the overcoming of σ_u, is controlled by such a number, as well as by the other non-dimensional ratios appearing in equation (2).

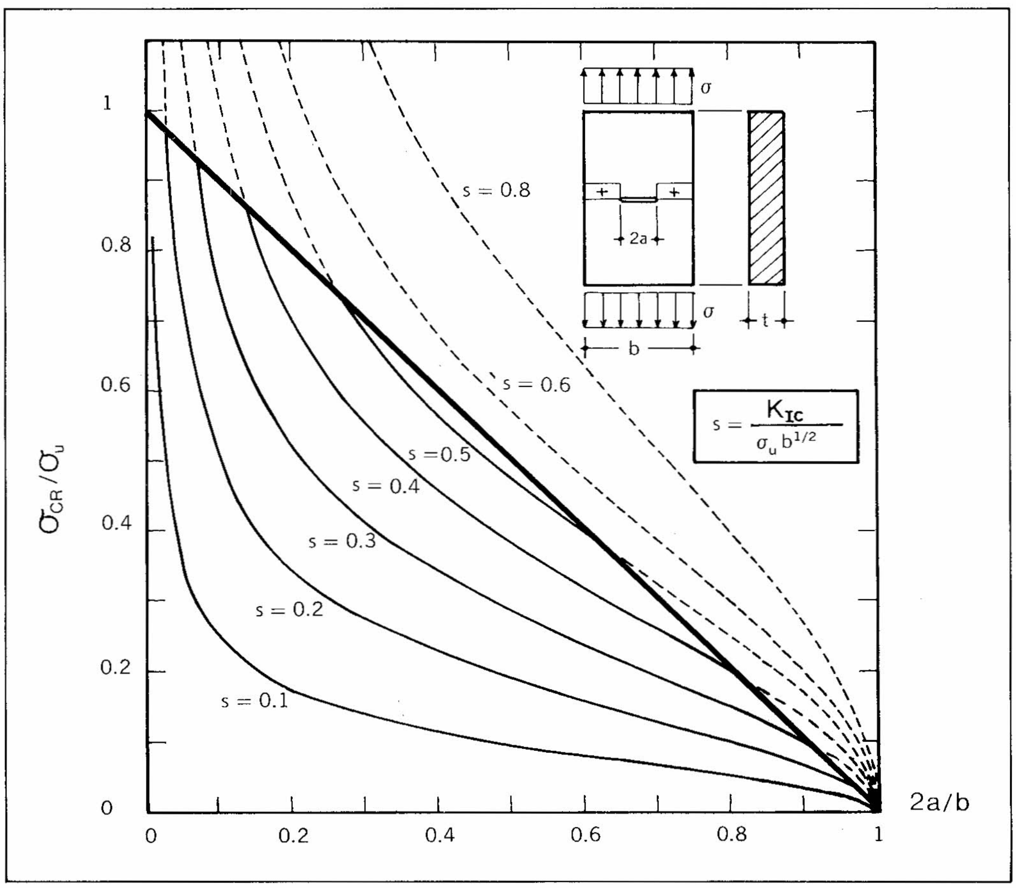

Figure 2
Non-dimensional loads of fracture collapse and ultimate strength collapse (tension test) [4]

The true function φ is very difficult to be obtained in the general case. However, if a series of structural geometries, simple and without reinforcing elements, is considered, it is possible to theoretically justify the scale effects, both for the notch sensitivity and for the K_{IC} variation. In all the elementary cases considered by the author [3,4], namely, the priority of fracture collapse with large structural sizes and the priority of ultimate strength collapse (or plastic flow collapse) with small structural sizes has been verified. Therefore it transpires, as a general rule extensible to all the geometries, that the fracture phenomenon — or rather the 'separation' of a solid — is reproducible only above certain sizes, and, in any way, for cracks which are neither too small nor too large (Figures 2, 3, 4).

Hillerborg [11] has defined a length characteristic for the material:

$$\ell_{CH} = \frac{E\mathscr{G}_{IC}}{\sigma^2_u} \approx \frac{K^2_{IC}}{\sigma^2_u} \approx s^2\, b, \tag{3}$$

which is connected with the size of the microcracked zone at the crack tip when the crack extends. He asserts that, 'as the crack advances, the zone with plane stress (i.e. the microcracked zone) may increase in width, which may lead to an increase in resistance against crack growth', i.e. to an increase of K_{IC} (R-curve) [12]. However, it is important to observe that it is unnecessary to apply non-linear constitutive laws, in order to explain the scale effects in fracture testing of cement composites; it is sufficient to consider both the collapses related to σ_u and K_{IC} [4].

In Figure 5 the values of fracture toughness K_{IC} are reported, which have been obtained by the author [13,14] for four different aggregate materials, varying the initial crack depth. It is possible to observe that, for marble and concretes, K_{IC} increases for $a/b \lesssim 0.3$ and decreases for higher values of such ratio, after achieving a maximum. This is a recurring fact in fracture testing, even for metallic materials and when the different crack depths are always related to the same specimen and to the same crack, which is made progressively to advance in a stable manner. The curves $K_{IC}(a)$ or $K_{IC}(\Delta a)$ are usually called R-curves. In the author's opinion, the K_{IC} increase by increasing the crack length is not due to the expansion of the plastic or microcracked zone at the crack tip, i.e. to the development of an increasing energy-absorbing fracture process, but, as has been already said, to more fundamental reasons of transition from a collapse to another mode. K_{IC} decreasing with increasing relative crack depth has also been observed by other investigators [15,16].

In Figure 6 the values of the ratio σ_N/σ_u, between the net section rupture stress and the ultimate strength,

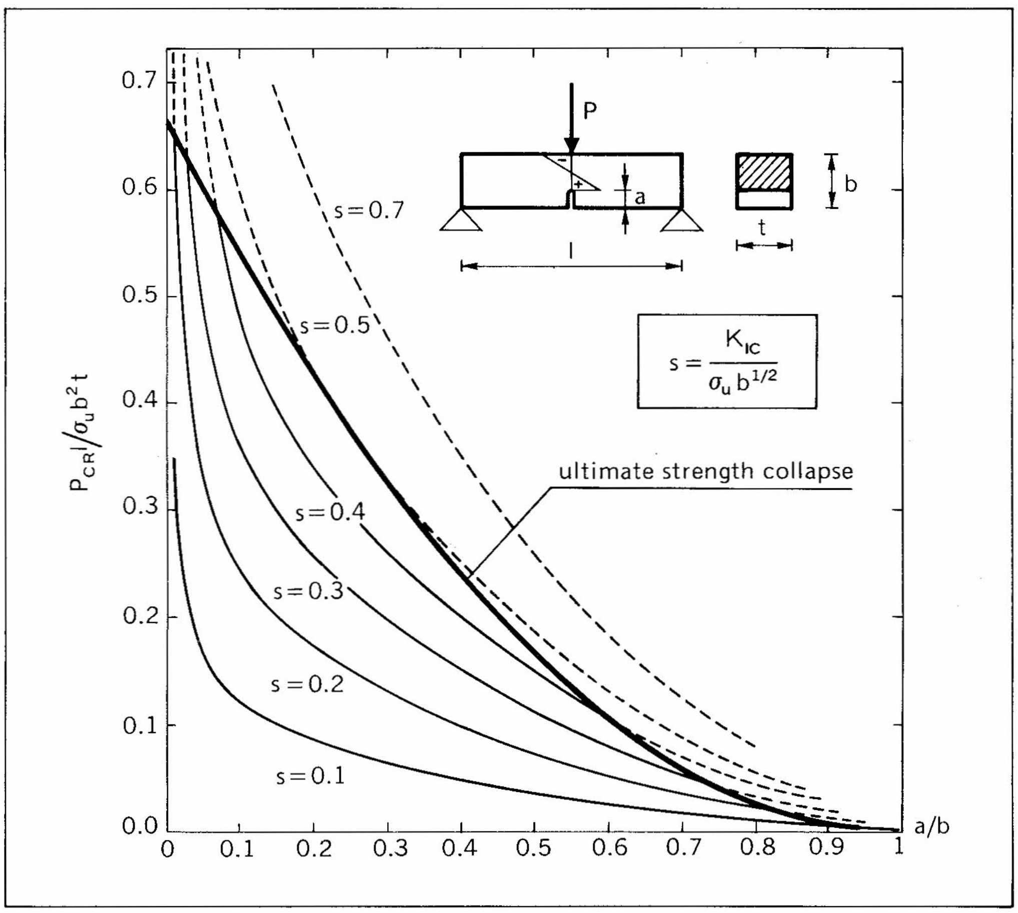

Figure 3
Non-dimensional loads of fracture collapse and ultimate strength collapse (three point bending test) [4]

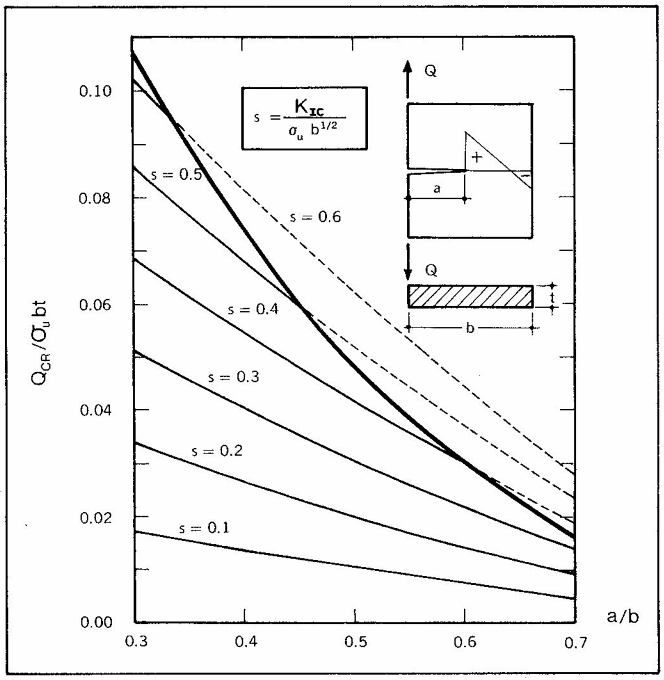

Figure 4 Non-dimensional loads of fracture collapse and ultimate strength collapse (compact test) [4]

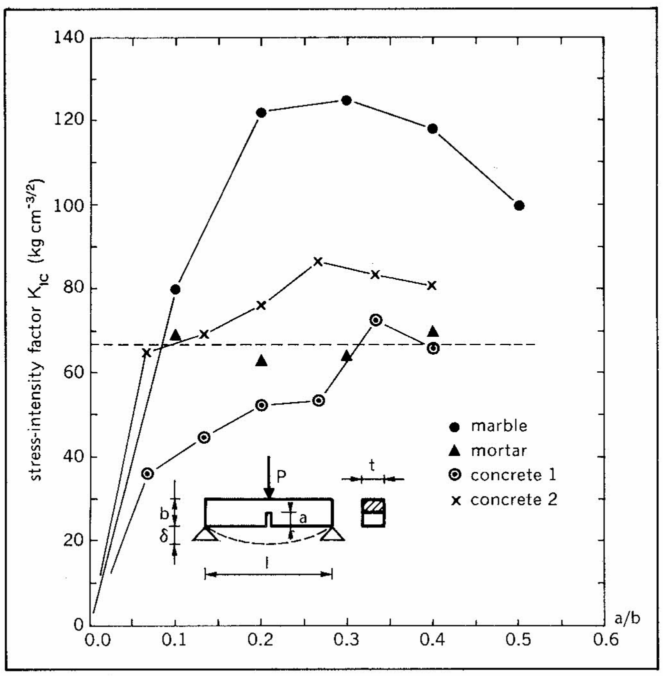

Figure 5 Experimental values of fracture toughness K_{IC} against relative crack depth [14]

related to the previously mentioned tests, are reported. Observe how the sizes of the specimens have been sufficient so that stress concentration effects are evident ($\sigma_N/\sigma_u < 1$). On the other hand they have not been sufficient for a real 'separation' collapse.

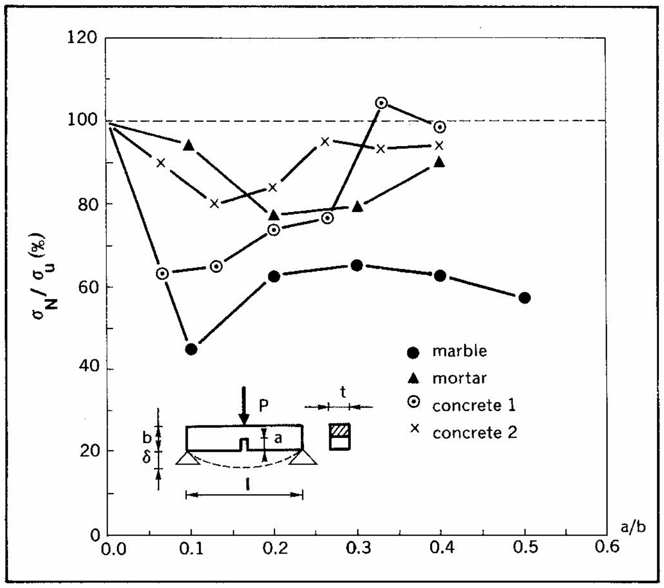

Figure 6 Experimental values of ratio σ_N/σ_u against relative crack depth [14]

Thus it is possible to conclude that notch sensitivity ($\sigma_N/\sigma_u < 1$) is a necessary condition, but not a sufficient one to have a pure fracture collapse ($K_I = K_{IC}$) [1]; similarly the increasing-decreasing variation of K_{IC} (a/b) is a necessary but not a sufficient condition to have a pure ultimate strength collapse ($\sigma_N = \sigma_u$). When both these two conditions are satisfied, as happens in most cases, then a *mixed collapse* occurs.

Such deductions have been confirmed by an analogous numerical and experimental analysis carried out at the Hydraulic and Structural Research Centre of the Italian National Power Agency (ENEL-CRIS) [17]. In that case the structural geometry of three point bending test was discretized by a Finite Element mesh (Figure 7). It was unnecessary to apply a particular mesh grading, since singular elements have been utilized around the crack tip, for which even the stress-intensity factors appear among the nodal parameters. The differences between K_I values computed in this way, and K_I values coming from the well-known ASTM-formula [18], were shown to be not significant (1 to 4%).

FRACTURE STABILITY

Consider the cracked body of Figure 1 again. If function (2) is partially derived with respect to crack length a, we have:

$$\frac{\dfrac{\partial q_{CR}}{\partial a}}{\sigma_u^{\alpha}\, b^{\beta-1}} = \varphi'\left(\frac{K_{IC}}{\sigma_u b^{1/2}}, \frac{a}{b}; \frac{\sigma_y}{\sigma_u}, \frac{E}{\sigma_u}, \frac{\sigma'_y}{\sigma_u}, \frac{\sigma'_c}{\sigma_u}, \frac{E'}{\sigma_u}; r_i\right), \qquad (4)$$

where φ' denotes the partial derivative of function φ with respect to the ratio a/b. The fracturing process of the considered system will be *stable* for positive values of

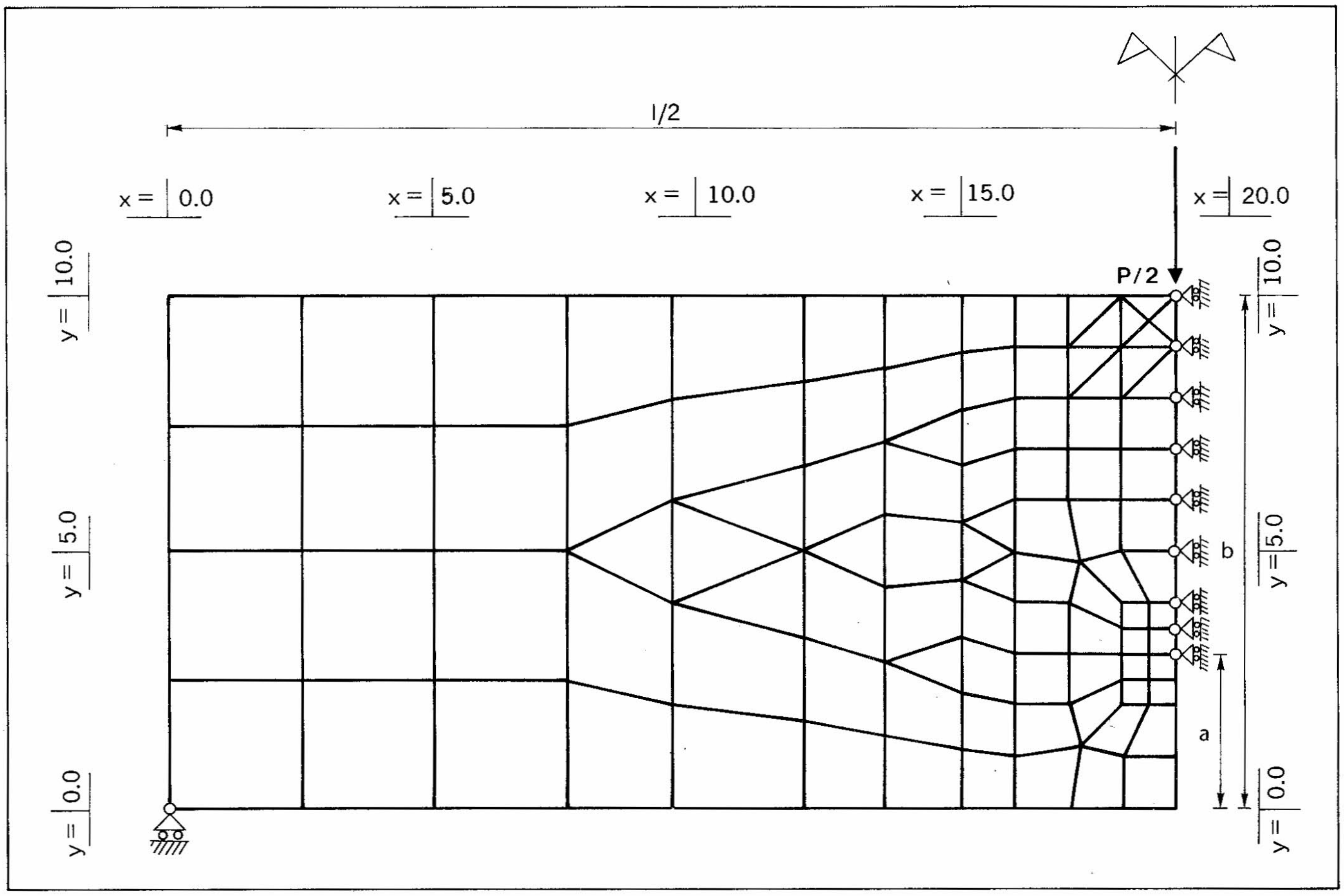

Figure 7 **Finite element discretization for three point bending test [17]**

expression (4), i.e. when the collapse load increases while the crack advances; it will be *unstable* in the opposite case.

Thus it is easy to realize that the *stability* of fracturing process follows laws of scale analogous to those followed by *sensitivity* and examined earlier.

Consider, as a particular case, the reinforced concrete beam element of Figure 8, subjected to the bending moment M. The redundant force transmitted by the reinforcement to the beam element can be estimated by means of a rotation congruence condition [19]. In the field of Linear Elastic Fracture Mechanics, such a force increases linearly by increasing the external bending moment, until the limit force $\sigma_y A_s$ (A_s = steel area) is achieved. From this point onwards a perfectly plastic behaviour of the reinforcement can be considered. Once the bending moment of steel plastic flow has been exceeded, the cracked beam element presents a linear-hardening behaviour, until the crack advances. It is possible to show how the stability of the process of concrete fracture and concurrent steel plastic flow depends on the mechanical and geometrical (scale included) properties of the beam.

In this relatively simple case equation (2) particularizes as follows [19]:

$$\frac{M_{CR}}{\sigma_u b^3} = \varphi\left(s, \frac{a}{b}; \frac{\sigma_y}{\sigma_u}; \frac{t}{b}, \frac{h}{b}, \frac{A_s}{A}\right)$$

$$= \psi\left(\frac{1}{s}\frac{\sigma_y}{\sigma_u}\frac{A_s}{A}, \frac{a}{b}, \frac{h}{b}\right)\frac{t}{b}\, s\,, \qquad (5)$$

$$= \psi\left(N_P, \frac{a}{b}, \frac{h}{b}\right)\frac{t}{b}\, s.$$

Function ψ is represented in Figure 8, against the relative crack depth a/b for varying non-dimensional number $N_P = \frac{1}{s}\frac{\sigma_y}{\sigma_u}\frac{A_s}{A}$ and for h/b = 1/20.

For N_P values close to zero, that is for low reinforced beams or for very small cross-sections, the fracture moment decreases while the crack extends, and then a typical phenomenon of unstable fracture occurs. For higher N_P values, a stable branch follows the unstable one of the curve, which describes the crack extension against the applied load. Already for $N_P = 1$ the minimum of the curve is evident and takes place for $\frac{a}{b} \simeq 0.35$.

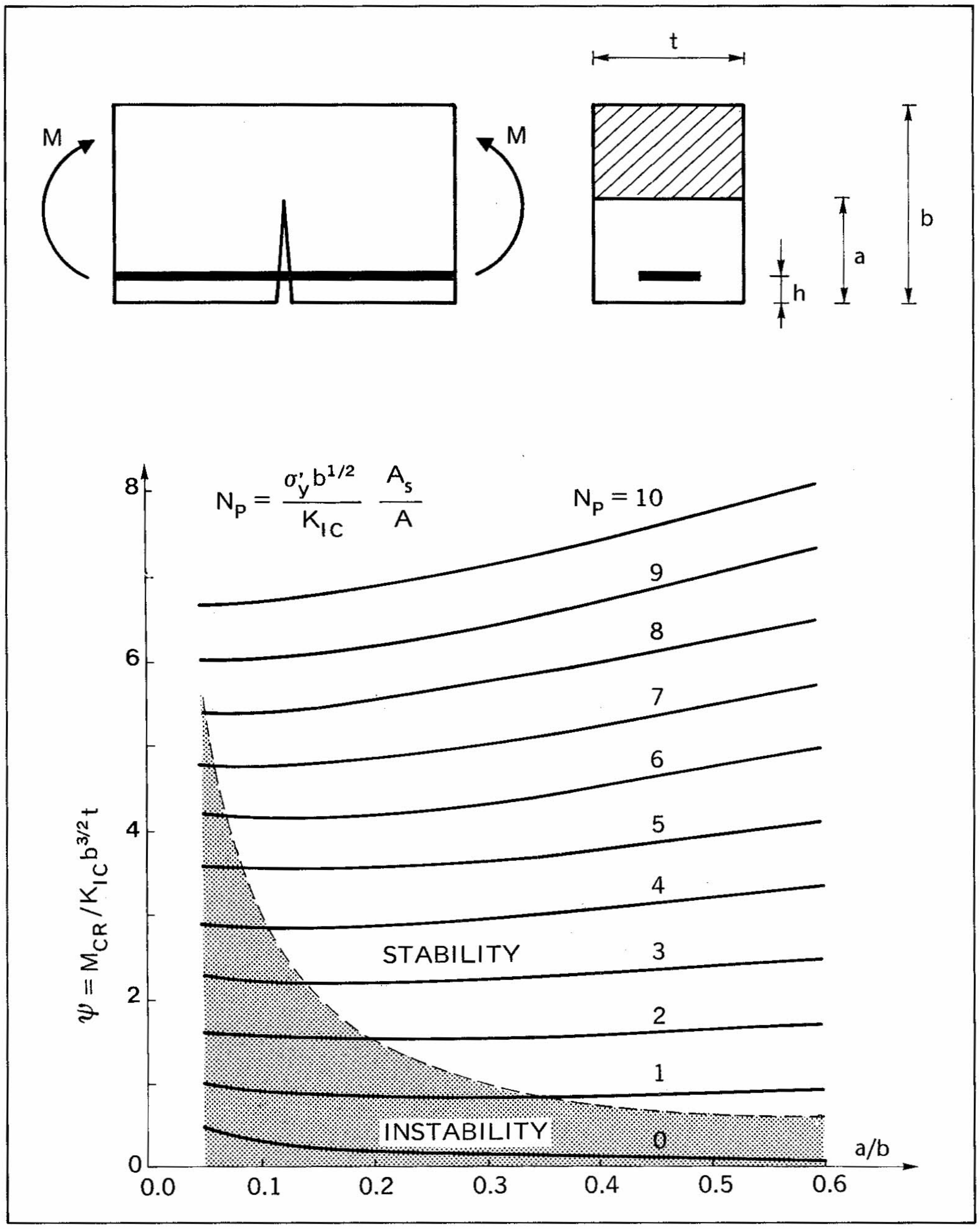

Figure 8
Stability conditions of progressive fracture in a reinforced concrete beam element [19]

For still higher N_P values, the value a/b, for which the minimum occurs, is lower, while the stable branch becomes steeper and steeper. For $N_P \geqslant 8.5$ the unstable branch completely disappears and only the stable branch remains.

The locus minimorum is represented by a broken line in Figure 8. This line divides the diagram into two zones: the upper zone is where the fracturing process is stable, while the lower one is where the process is unstable. Therefore, it is possible to assert that the fracturing process becomes stable only when either the beam is sufficiently reinforced or, the steel percentage being constant, when the cross-section is sufficiently large, and when the crack is sufficiently deep.

Increasing the ratio h/b, curves very similar to those of Figure 8 can be obtained. However they go down, i.e. fracture occurs for lower moments, since the reinforcement, being internal, resists to a lesser extent. On the other hand the broken line goes up, i.e. the stable zone of the diagram shrinks.

In the previously proposed analysis three potential collapses, co-existent with those of concrete fracture and steel plastic flow, have not been considered. They are tensile and crushing collapses of concrete and slip between reinforcement and concrete. The last one can be simulated considering a steel yield strength σ_y lower than the real one. Therefore from the curves of Figure 8 it transpires that pulling-out of reinforcement is an

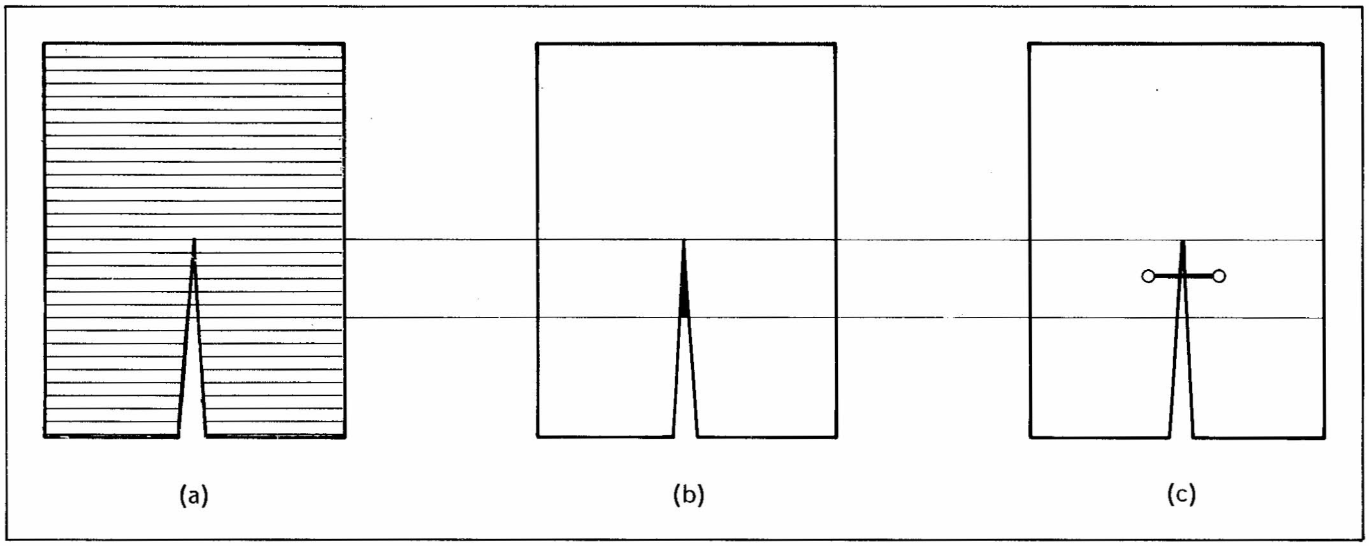

Figure 9 Reinforcing elements in a cracked body:
a) not pulled-out fibres;
b) Dugdale's plastic zone at the crack tip;
c) fictitious tie-rod

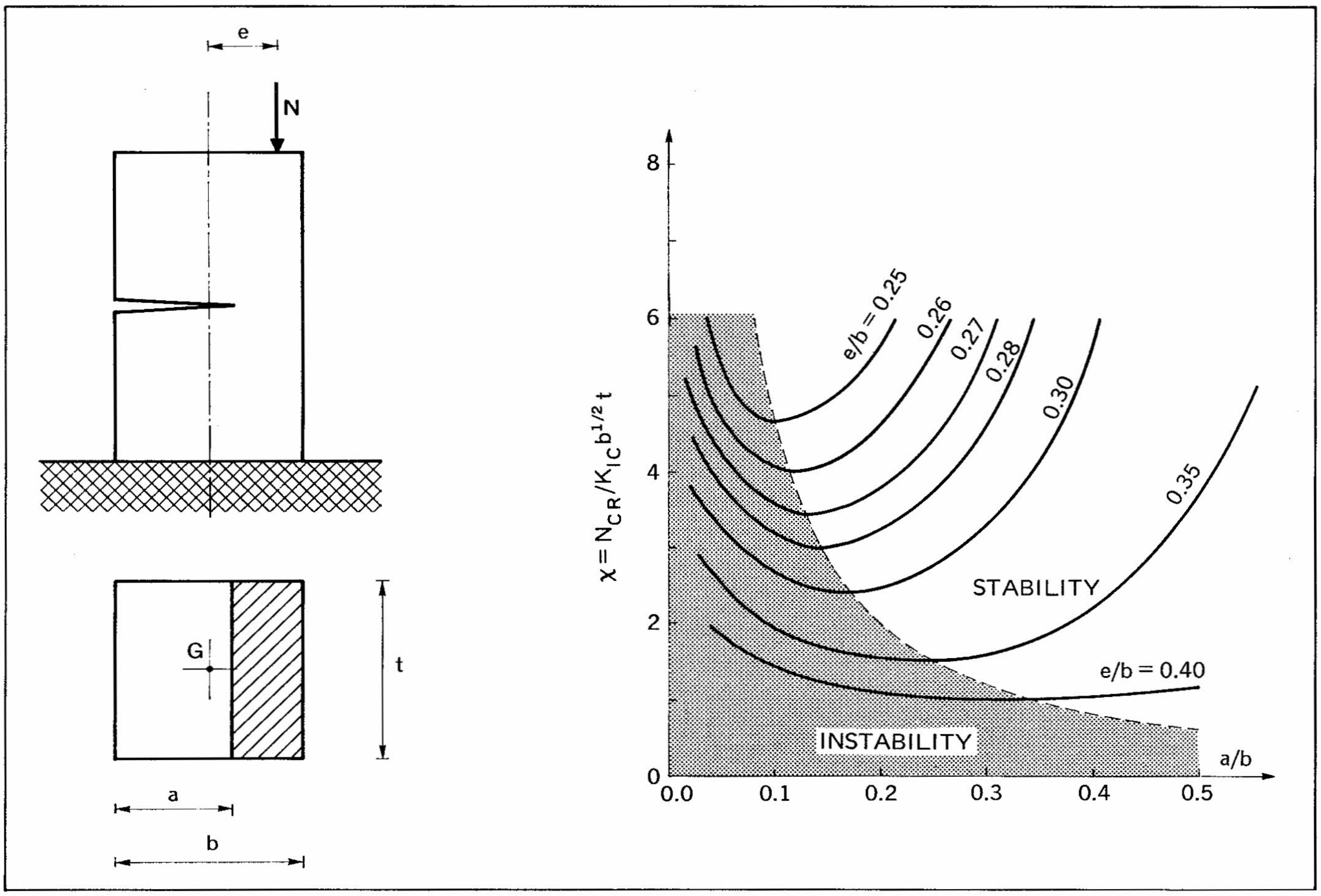

Figure 10 Stability conditions of progressive fracture in a masonry wall [20]

instabilizing phenomenon for fracturing processes of reinforced concrete beams.

In fibre reinforced concretes the reinforcing elements are numerous and then an analytical computation of stability is practically impossible. However, it is necessary to observe that, unlike in conventional reinforced concrete, fibre pulling-out is complete and involves almost all the fibres crossed by the crack (Figure 9-a). During the fracturing process the effectively reinforcing elements will only be the fibres sufficiently close to the crack tip. The stability of such a process can then be described by means of a mechanical model with a single reinforcing element running together with the crack tip (Figure 9-c). This element will represent the integrated effect due to all the active fibres. For ductile materials completely similar remarks are valid, if Dugdale's plastic zone [10] is considered as a reinforcing element carried by the crack tip (Figure 9-b).

Consider now, as a second particular case, the wall of Figure 10, subjected to eccentric axial force. In this case equation (2) particularizes as follows [20]:

$$\frac{N_{CR}}{\sigma_u b^2} = \varphi\left(s, \frac{a}{b}; \frac{t}{b}, \frac{e}{b}\right) = \chi\left(\frac{a}{b}, \frac{e}{b}\right)\frac{t}{b}\, s. \qquad (6)$$

Function χ is represented in Figure 10, against the relative crack depth a/b and for varying eccentricity e/b. Observe how in this case the stability of progressive cracking depends only on geometrical factors and then scale effects are lacking, i.e. the Brittleness Number s is not included among the actual variables of function χ, since the cause of instability is not the redundancy of the system but an external compression.

For small eccentricities e/b the progressive cracking becomes stable with small crack depths. Increasing e/b the stable branch is reached for larger and larger crack depths a/b. This is consistent, since the crack has to run a longer and longer way to achieve the compressive zone of the wall. For $e/b \rightarrow \infty$ the stable branch tends to disappear, since the instabilizing bending moment becomes more and more relevant in comparison with the stabilizing compression.

CONCLUSIONS

1. Below a certain size threshold any stress concentration and amplification effect vanishes (σ_N = constant = σ_u).

2. Above a certain size threshold fracture toughness appears as a material constant (K_{IC} = constant).

3. Between the above mentioned thresholds there is a transition zone, from ultimate strength collapse to fracture collapse. In such a zone the structure is notch sensitive ($\sigma_N < \sigma_u$), however fracture toughness varies with the sizes of structure and crack ($K_{IC} \neq$ constant).

4. Even the *stability* of progressive cracking is subjected to size effects, as well as the crack *sensitivity* of structures.

5. The stability of progressive cracking is generally due to compressive loadings or to a sufficiently high degree of redundancy in the system. The adjunctive elements, producing redundancy and stability, can be external, as in the case of fibre and bar reinforcements, or internal, as in the case of Dugdale's plastic zone at the crack tip.

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