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NOTCH SENSITIVITY IN FRACTURE TESTING OF AGGREGATIVE MATERIALS

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Abstract—The notch sensitivity effects on fracture testing of aggregative materials, and more generally of brittle materials, the specimen and crack sizes varying, have been studied based on the Dimensional Analysis.

Such effects are due to the co-existence of two different structural crises, induced by generalized forces with different physical dimensions, and to the finiteness of specimen sizes.

The application of Buckingham's Theorem allows the definition of a non-dimensional parameter s (the test brittleness number), which governs the notch sensitivity phenomenon.

Some recurring experimental inconsistencies are thus explained, such as:

(1) The increase or decrease of fracture toughness K_{Ic} by increasing the crack length;
(2) The increase of K_{Ic} by increasing the specimen sizes;
(3) The variability of K_{Ic} by varying the test typology.

1. INTRODUCTION

THE DEFINITION of suitable fracture parameters, which measure the crack extension capability, together with adjustments in the experimental techniques, meet two requirements, which arise at different moments. The first requirement is qualitative and arises during the project, when the fracture toughness of the building material is to be characterized. The second is quantitative and arises during the quality control, when the risk of a structural crack is to be verified.

For concretes, and more generally for heterogeneous materials, the fracture toughness definition is problematic, because of the presence of several materials with different physical properties and the complexity of the crack extension phenomenon. In any case the global fracture toughness of a composite material is a weighted mean of the partial fracture toughnesses relating respectively to matrix, aggregate and bond between matrix and aggregate[1].

The toughness weighting criterion will depend on the concrete structure and on the fundamental fracture mechanisms (Fig. 1):

(1) Debonding between aggregate and matrix;
(2) Branching into the matrix;
(3) Microcracking in the matrix;
(4) Bridging of microcracks;
(5) Crack arrest at the interface.

The first four elementary events are negative, while the last is positive with regard to the concrete

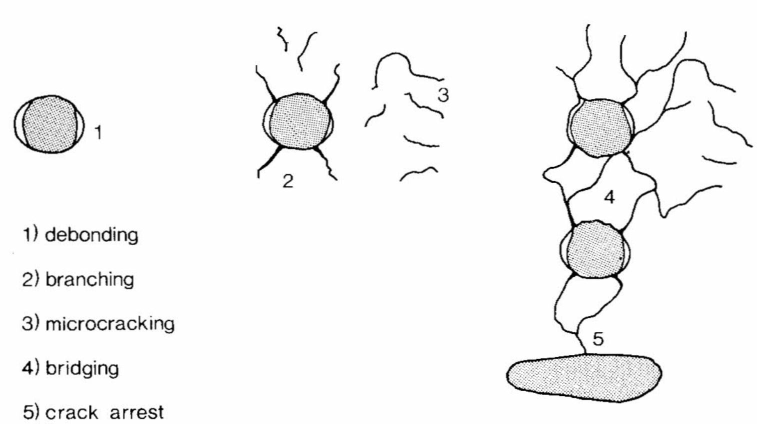

Fig. 1. Fundamental fracture mechanisms in concrete.

strength. Debonding is regulated by the fracture toughness at the interface[2, 3]; branching, microcracking and bridging are regulated by the matrix fracture toughness; finally crack arrest by the aggregate fracture toughness.

As is well-known, the stress-intensity factor K_I is the amplification factor of the stress field which develops at the crack tip and is due to the symmetrical loads:

$$\{\sigma\} = \frac{K_I}{\sqrt{r}}\{F(\vartheta)\}. \tag{1}$$

While the vector $\{F(\vartheta)\}$, which is the stress field profile, and the singularity $r^{-1/2}$, concern the mathematical Irwin's model[4], the stress-intensity factor K_I depends on the geometry of structure, crack and external loadings. The factor K_I derives from a linear elastic analysis; therefore the experimental determination of its critical value K_{Ic} is valid only when the crack tip plastic zone is small, in relation to the specimen and crack sizes (*condition of small scale yielding*). Thus such limitation allows K_{Ic} tests only for materials with a linear behaviour up to the fracture conditions (high strength steels, rocks, mortars).

The experimental determination of fracture toughness K_{Ic} for metallic materials is regulated by the "*Standard Method of Test for Plane Strain Fracture Toughness of Metallic Materials*" ASTM E 399-74[5]. This is the result of extensive experimental research particularly regarding high strength alloys. Such standard method sets a lower limit to the specimen and crack sizes, in order to respect the small scale yielding condition[6].

Meanwhile several authors have taken an interest in fracture testing of *aggregative materials*[8–31], since this interest was stimulated by Kaplan in 1961[7]. They have tried to standardize experimental techniques, but the results have often shown disagreements and inconsistences. The critical factor K_{Ic} often appears as a function of specimen size and crack length, instead of as a material constant, as also happens for aluminum[32–34] and titanium alloys[35].

The variability of K_{Ic} for aggregative materials has been charged to non-linear effects, as microcracking at the crack tip and slow crack growth, which reveal themselves prior to the unstable crack propagation. The non-linear effects are evident for ductile metallic materials, but they have not always been found in the load-displacement diagrams of aggregative materials.

As the factor K_I is valid only in the linear elastic field and the fracture crisis, for certain metallic alloys and certain specimen sizes, occurring in the non-linear field, the experimental determination of another fracture parameter, holding also in the non-linear field, was suggested[36–38]. Such new parameter is the *J-integral*, which is equal to the total potential energy-release rate of the system associated with a unit virtual crack extension[39–41]. The experimental determination of the energetic parameter J_{Ic} for aggregative materials, in accordance with the method suggested by Begley and Landes[36], has not been carried out yet, probably because of the almost complete lack of softening shown by the experimental curves. One of the purposes of the research, which the author is conducting, is to verify the advantages and the limits of the J-integral application to the aggregative materials.

As has been said, many authors have pointed out the K_{Ic} dependence on the specimen sizes, as well as on its thickness[42], and on the crack length. They have then noticed how, for sufficiently large specimen sizes and for neither too small nor too large crack lengths[32], the fracture toughness K_{Ic} appears constant. The justification that is often given for these phenomena is anything but rigid: the variability of K_{Ic} could be caused by *non-linear phenomena*, as plasticity in metallic materials and microcracking in aggregative materials.

In the present paper the *Dimensional Analysis*, and more precisely Buckingham's Theorem for physical similitude and scale modeling, is applied to Linear Elastic Fracture Mechanics; so a non-dimensional parameter is defined, which governs the notch sensitivity phenomenon and later on will be called *brittleness number*. Such parameter depends on the material fracture toughness K_{Ic}, on its ultimate strength σ_u and on a linear size b, characteristic for the geometry of the considered specimen or structure.

The Dimensional Analysis application is justified by the co-existence of two different structural crises, induced by generalized forces with different physical dimensions: collapse at ultimate stress ($\sigma = FL^{-2}$) and crack extension crisis ($K_I = FL^{-3/2}$). The experimental inconsistences are so explained, considering the simple ultimate stress collapse as a crisis independent of the crack tip stress concentration.

2. DIMENSIONAL ANALYSIS APPLICATION TO LINEAR ELASTIC FRACTURE MECHANICS

As in Hydraulics a fluid flow and its characteristic dynamic phenomena can be reproduced by a scale model, only if the model sizes are not lower than certain critical values, so in Solids Mechanics the mechanical behaviour of a structure and its collapse mechanisms can be reproduced by a scale model (specimen), only if the model sizes are not too small. This is what happens for a cracked body with two different potential crises:

(1) The *collapse at ultimate stress*, induced by the highest normal stress σ in the body, considering the crack only as a weakening for the cross section and not as a stress concentrator;

(2) The *crack propagation collapse*, induced by the stress intensity factor K_I, supposing structure and loadings as symmetrical with respect to crack line.

The presence of two generalized forces with different physical dimensions, $\sigma = [FL^{-2}]$ and $K_I = [FL^{-3/2}]$, imposes the application of Dimensional Analysis, in order to define non-dimensional parameters, synthetically governing the structural collapse and showing which crisis mechanism is coming before the other. Afterwards *Buckingham's Theorem* for similitude and modeling will be applied to the simplest case: a symmetrical body, symmetrically loaded with respect to crack line, of homogeneous, isotropic and linear elastic material, with ultimate strength σ_u and fracture toughness K_{Ic} (Fig. 2). However the fundamental Buckingham's Theorem could be clearly applied also to the more general case of a body of heterogeneous and elastic-plastic material. But in this case the results would be less significant even though more general.

Let q_0 be the measure of the crisis load, for collapse at ultimate stress or for crack propagation, in the case of the cracked structure of Fig. 2. Such load is a function of several variables:

$$q_0 = f(q_1, q_2, \ldots, q_n; r_1, r_2, \ldots, r_m), \tag{2}$$

where q_i are physical quantities with different dimensions and r_i are non-dimensional numbers. Each quantity with certain dimensions appears just once in function f. For example, q_1 could be the ultimate strength σ_u and r_1 the ratio between the yield strength σ_y and the ultimate strength; q_2 could be a linear size b of the structure (Fig. 2), $r_2, r_3, \ldots$ the ratios between the other sizes, characteristic for the considered structural geometry, and the size b; then q_3 could be the fracture toughness K_{Ic} and so on.

In the case of *composite material*, like concrete, among the non-dimensional numbers would also appear the ratios between aggregate and matrix ultimate strength, between aggregate and matrix fracture toughness, between bond and matrix fracture toughness, the volumetric ratio (aggregate/matrix) and the ratio between the mean size of the aggregate and the characteristic size b. Generally the function f is not analytically obtainable, as in the simple case which will be considered. An empirical relationship f would however be obtainable, if a lot of results were available, varying the parameters of the problem.

Consider now two dimensionally independent quantities q_1 and q_2 (statical case). As clarified in the Appendix, they can be considered as fundamental, i.e. it is possible that the product $q_1^{\alpha_{10}} q_2^{\alpha_{20}}$, for suitable values of α_{10} and α_{20}, has the same dimensions as q_0. Then the product $q_1^{\alpha_{13}} q_2^{\alpha_{23}}$ can have the same dimensions as q_3, for suitable values of α_{13} and α_{23}, and so on.... Therefore function (2) can be transformed as follows:

$$\frac{q_0}{q_1^{\alpha_{10}} q_2^{\alpha_{20}}} = \varphi\left(q_1, q_2, \frac{q_3}{q_1^{\alpha_{13}} q_2^{\alpha_{23}}}, \ldots, \frac{q_n}{q_1^{\alpha_{1n}} q_2^{\alpha_{2n}}}; r_1, r_2, \ldots, r_m\right). \tag{3}$$

Function f becomes φ because of the performed non-dimensionalizations.

If the unit of measure of q_1 changes, φ doesn't vary, being a non-dimensional number. Thus φ is not really a function of q_1; analogously it is not a function of q_2. It is only a function of $(n-2+m)$ non-dimensional numbers:

$$\frac{q_0}{q_1^{\alpha_{10}} q_2^{\alpha_{20}}} = \varphi(N_3, N_4, \ldots, N_n; r_1, r_2, \ldots, r_m). \tag{4}$$

In the simplest case of homogeneous and linear elastic material (Fig. 2):

$$q_0 = f\left(\sigma_u, b, K_{Ic}; \frac{a}{b}, \frac{t}{b}, \frac{l}{b}\right), \tag{5}$$

where also the crack length a appears among the geometric parameters. Considering σ_u and b as fundamental quantities, (5) becomes:

$$\frac{q_0}{\sigma_u^\alpha b^\beta} = \varphi\left(\frac{K_{Ic}}{\sigma_u b^{1/2}}; \frac{a}{b}, \frac{t}{b}, \frac{1}{b}\right), \tag{6}$$

where the function φ depends on the geometry of structure and external loads.

As far as the most usual fracture test geometries are concerned, length 1 is not always really present as argument in function φ. When it is, it is possible however to separate function φ into two functions φ_1 and φ_2, such as:

$$\frac{q_0}{\sigma_u^\alpha b^\beta} = \varphi_1\left(s; \frac{a}{b}\right)\varphi_2\left(\frac{t}{b}, \frac{1}{b}\right). \tag{7}$$

Function φ_1, which is the more interesting and governs the notch sensitivity phenomenon, is a function of the non-dimensional number:

$$s = \frac{K_{Ic}}{\sigma_u b^{1/2}}, \tag{8}$$

and of the relative crack length a/b. Afterwards upper bounds of function φ_1, concerning the most usual fracture tests, will be obtained and diagrammatized as function of a/b, varying the brittleness number s.

For the *tension test* (TT, Fig. 3) the stress-intensity factor is [45]:

$$K_I = \sigma\sqrt{(\pi a)}\left(\sec\frac{\pi a}{b}\right)^{1/2}. \tag{9}$$

From (9) it is possible to obtain the crack extension load:

$$\frac{\sigma_{CR}}{\sigma_u} = s\left(\frac{\cos\dfrac{\pi}{2}\dfrac{2a}{b}}{\dfrac{\pi}{2}\dfrac{2a}{b}}\right)^{1/2} \tag{10}$$

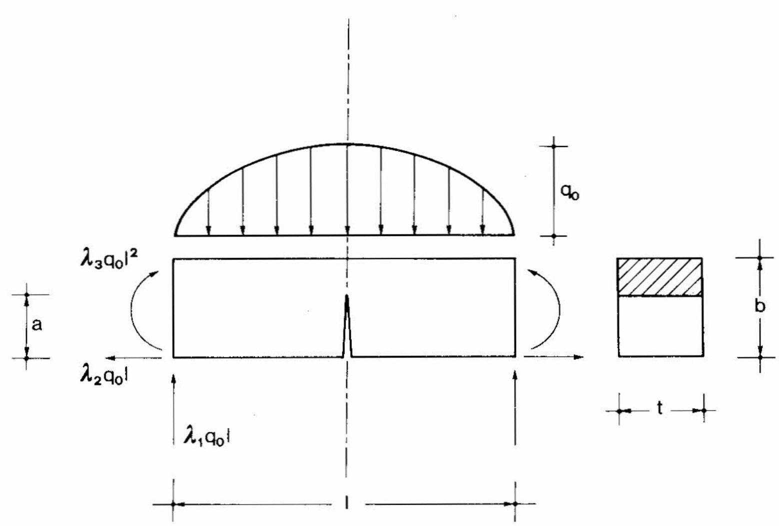

Fig. 2. Symmetrical body, symmetrically loaded with respect to crack line.

On the other hand the load of collapse at ultimate strength, relating to the crack section, is:

$$\frac{\sigma_{CR}}{\sigma_u} = 1 - \frac{2a}{b}. \tag{11}$$

Such load is really only an upper bound, because of the co-existing fracture collapse. In Fig. 3 the non-dimensional load of fracture collapse is reported as a function of the relative crack length $2a/b$, varying the brittleness number s. The load of collapse at ultimate strength is also reported, and it is evident how the fracture tests completely lose their meaning for s higher than the critical value $s_0 \simeq 0.54$, ultimate strength collapse occurring in this case. For values $s \leq 0.54$ the fracture tests are significant only with cracks of intermediate length.

For the *four points bending test* (FPB, Fig. 4) the stress-intensity factor K_I is [46, 47]:

$$K_I = \frac{6M}{tb^{3/2}} g\left(\frac{a}{b}\right),$$

where:

$$g\left(\frac{a}{b}\right) = \left(\frac{a}{b}\right)^{1/2} \left[2 - 2.435\left(\frac{a}{b}\right) + 10.19\left(\frac{a}{b}\right)^2 - 7.912\left(\frac{a}{b}\right)^3 + 27.03\left(\frac{a}{b}\right)^6 - 16.51\left(\frac{a}{b}\right)^7\right]$$

for $0 \leq a/b \leq 0.5$;

$$g\left(\frac{a}{b}\right) = 0.663 \Big/ \left(1 - \frac{a}{b}\right)^{3/2} \tag{12}$$

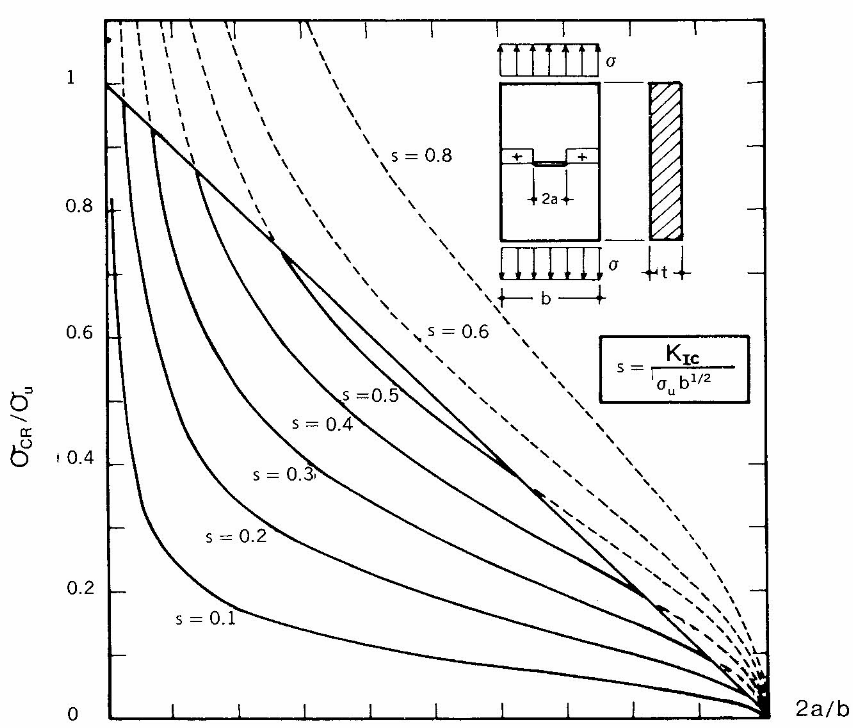

Fig. 3. Crack extension load (varying the brittleness number s) and load of collapse at ultimate strength, against relative crack length (TT).

for $0.5 < a/b < 1$. From (12) the crack extension moment can be obtained:

$$\frac{M_{CR}}{\sigma_u b^2 t} = \frac{s}{6g\left(\frac{a}{b}\right)}. \tag{13}$$

The moment of ultimate strength collapse is:

$$\frac{M_{CR}}{\sigma_u b^2 t} = \frac{1}{6}\left(1 - \frac{a}{b}\right)^2. \tag{14}$$

In Fig. 4 diagrams are represented, which are analogous to those of Fig. 3. Also in this case for $s \geqslant 0.54$, fracture tests are not significant, since the fracture collapse doesn't come before the ultimate strength collapse for any relative crack length.

For the *three points bending test* (TPB, Fig. 5) the stress-intensity factor K_I is[5]:

$$K_I = \frac{P1}{tb^{3/2}} f\left(\frac{a}{b}\right),$$

where:

$$f\left(\frac{a}{b}\right) = 2.9\left(\frac{a}{b}\right)^{1/2} - 4.6\left(\frac{a}{b}\right)^{3/2} + 21.8\left(\frac{a}{b}\right)^{5/2} - 37.6\left(\frac{a}{b}\right)^{7/2} + 38.7\left(\frac{a}{b}\right)^{9/2}. \tag{15}$$

From (15) the crack extension force can be obtained:

$$\frac{P_{CR}1}{\sigma_u b^2 t} = \frac{s}{f\left(\frac{a}{b}\right)}. \tag{16}$$

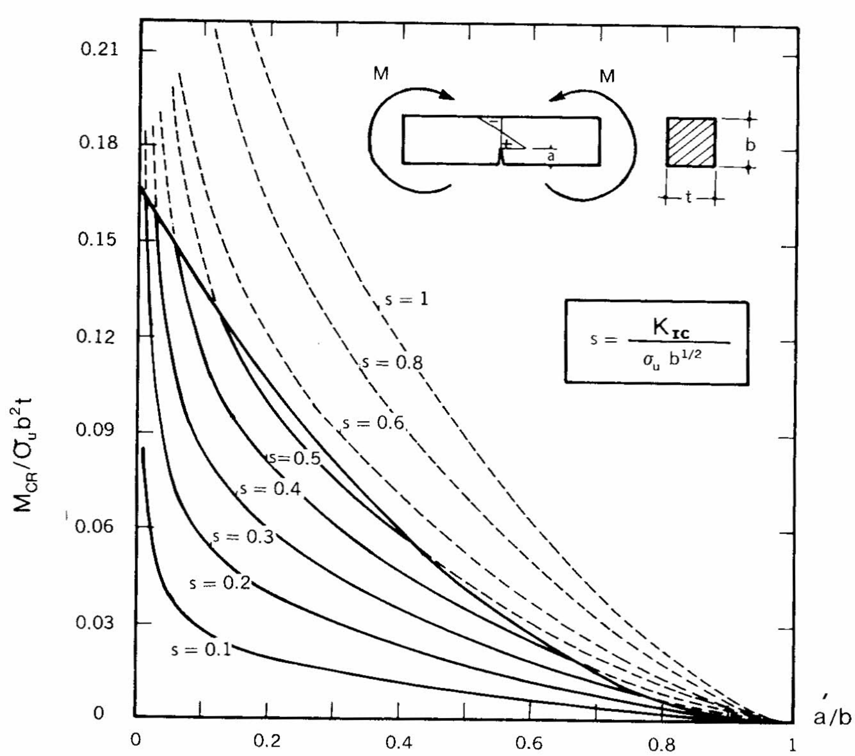

Fig. 4. Crack extension moment (varying the brittleness number s) and moment of collapse at ultimate strength, against relative crack depth (FPB).

The force of ultimate strength collapse is:

$$\frac{P_{CR}l}{\sigma_u b^2 t} = \frac{2}{3}\left(1 - \frac{a}{b}\right)^2. \tag{17}$$

In Fig. 5 diagrams are reported which are analogous to the previous ones. In this case for $s \geqslant 0.50$, the fracture tests are not significant; in fact the fracture curve $s = 0.50$ is tangent to the ultimate strength curve. Being $0.50 < 0.54$, it is possible to assert that, for simple geometric reasons, the two previously considered tests (TT and FPB) are more favourable than TPB as far as the notch sensitivity is concerned.

For the *compact test* (CT, Fig. 6) the stress-intensity factor K_I is [5]:

$$K_I = \frac{Q}{tb^{1/2}} h\left(\frac{a}{b}\right),$$

where:

$$h\left(\frac{a}{b}\right) = 29.6\left(\frac{a}{b}\right)^{1/2} - 185.5\left(\frac{a}{b}\right)^{3/2} + 655.7\left(\frac{a}{b}\right)^{5/2} - 1017\left(\frac{a}{b}\right)^{7/2} + 638.9\left(\frac{a}{b}\right)^{9/2} \tag{18}$$

for $0.3 \leqslant a/b \leqslant 0.7$. From (18) the crack extension force is:

$$\frac{Q_{CR}}{\sigma_u bt} = \frac{s}{h\left(\frac{a}{b}\right)}, \tag{19}$$

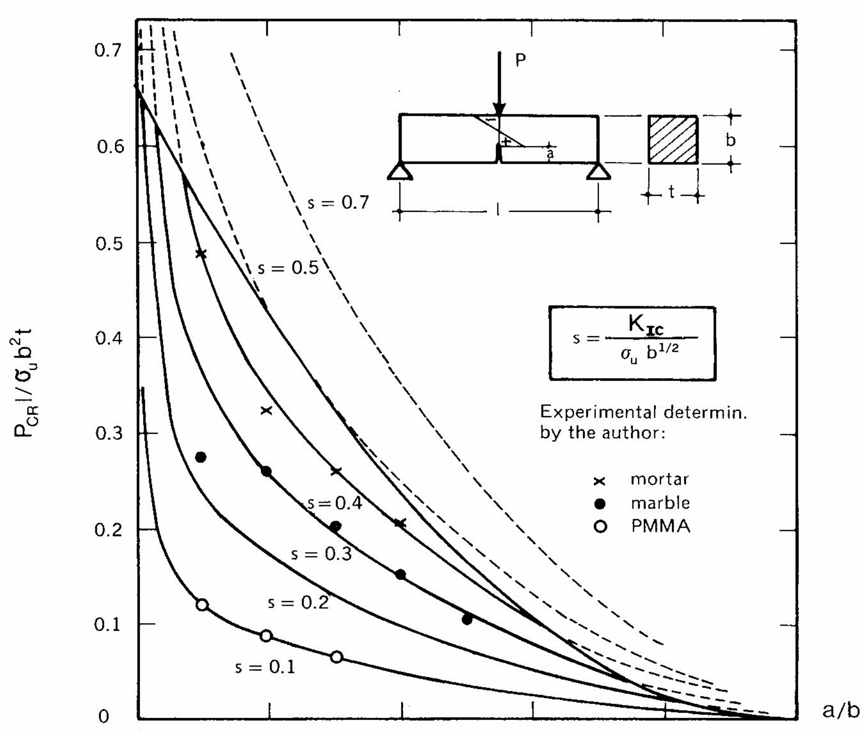

Fig. 5. Crack extension force (varying the brittleness number s) and force of collapse at ultimate strength, against relative crack depth (TPB).

while the ultimate strength collapse in the weakened cross section due to the eccentric load occurs for:

$$\frac{Q_{CR}}{\sigma_u bt} = \frac{\left(1-\frac{a}{b}\right)^2}{2\frac{a}{b}+4}. \tag{20}$$

For $s \geq 0.63$ the fracture tests are not significant (Fig. 6). Therefore the compact test is reasonably favourable with respect to notch sensitivity. That is, also for high brittleness numbers s, i.e. materials being equal, for small specimens, the fracture collapse can occur, on condition that the cracks are not too long. For example for $s = 0.55$ the relative crack length must be $a/b \leq 0.4$. It is curious that the condition of small scale yielding, according to ASTM E 399, requires exactly the contrary, i.e. sufficiently long cracks: $a/b > 0.45$ [5].

The diagrams, which have been shown, can be useful especially during the test project. Having chosen the fracture test geometry, knowing the ultimate strength value σ_u and foreseeing the toughness magnitude, it is possible to choose a specimen width b so that the tests are significant, i.e. so that $s \leq s_0$, where s_0 is the limit value depending on the test typology. When the size b is decided, i.e. the test brittleness number s, it is necessary to pay attention to fix the cracks' lengths inside a certain interval.

This interval is of increasing amplitude with the decreasing of the brittleness number s, and is individualized by the intersections of the fracture curves s = constant with the ultimate strength curve. When, for $s = s_0$, the fracture curve intersects the ultimate strength curve in two coinciding points, i.e. they are tangent, then the interval becomes a point.

In Fig. 7 the loci on the plane (s, a/b) are represented, where the fracture tests are significant. There are always two opposite requirements: the first one is economical and aims at setting an upper bound to the specimen sizes; the second is technical and aims at setting a lower bound to the specimen sizes. An optimum solution could theoretically be achieved, choosing the highest brittleness number s_0 (i.e. the lowest specimen size) consistent with significant results. But in this case to the economical advantage of a smaller specimen corresponds the disadvantage of a unique crack length allowing significant results.

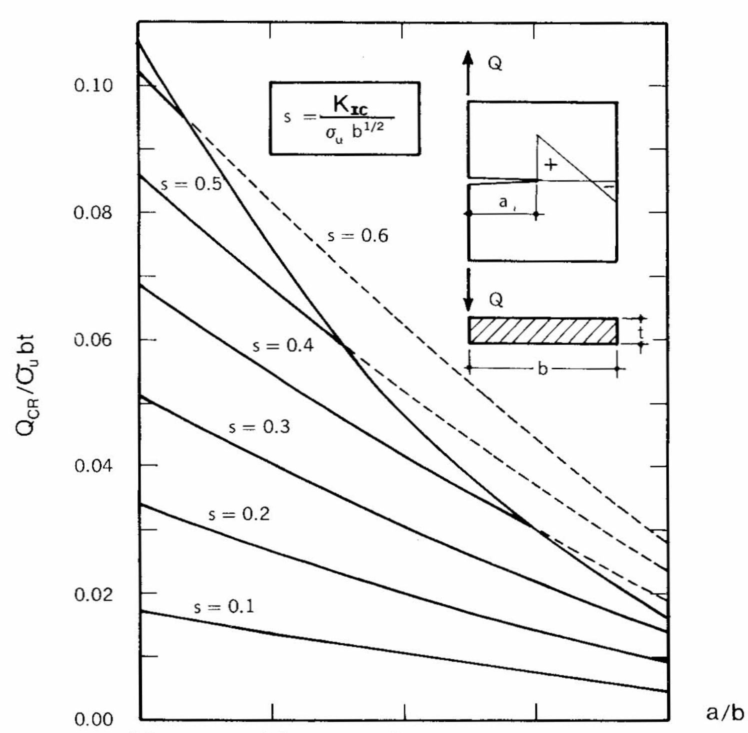

Fig. 6. Crack extension force (varying the brittleness number s) and force of collapse at ultimate strength, against relative crack depth (CT).

3. DISCUSSION

Attention must always be paid to specimen size effects on the fracture toughness determination for aggregative materials, and to the results extrapolation to larger structures[48–50]. Some interesting laws of similitude have been expressed by Atkins *et al.*[51, 52], but without considering a second collapse, independent of the fracture collapse. In the case of plastic materials Goldstein *et al.*[53] have defined the "material scale length" $L = (K_{Ic}/\sigma_y)^2$, characteristic for a body with infinite sizes, without considering the notch sensitivity effects due to the finite structural geometry.

A first idea of the material notch sensitivity can be provided by the ratio between the experimental load of failure and the calculated load of ultimate strength (or yield strength). Such ratio is called "*specimen strength ratio*"[54, 55, 21]; when it is lower than unit it means that there is a stress concentration effect in addition to the section weakening effect. The specimen strength ratio however depends on the specimen sizes, therefore it is not an absolute feature which also includes the size effects, as the brittleness number does.

Brown[15] has performed fracture tests to measure the toughness of cement pastes and mortars, varying the crack length. He notices how the paste fracture toughness appears independent of the crack length, while the mortar fracture toughness appears to increase with the crack length. Since plastic effects are missing, Brown, like Kaplan[7], justifies the K_{Ic} variability with the slow crack growth (stable propagation) occurring before the unstable propagation. In Fig. 8 the K_{Ic} values, obtained by Brown for mortar, are reported in function of the relative crack length. The two sets of results relate to FPB tests with specimens 38 × 38 × 250 mm, and to double cantilever beam tests (DCB) with specimens 50 × 100 × 350 mm. Supposing a fracture toughness $K_{Ic} = 75$ kp cm$^{-3/2}$, equal to the DCB maximum, and an ultimate strength $\sigma_u = 57$ kp cm^{-2}, it is possible to obtain the brittleness numbers of the two testing sets:

$$s\,(\mathrm{FPB}) = \frac{75}{57 \times (3.8)^{1/2}} = 0.67; \qquad s\,(\mathrm{DCB}) = \frac{75}{57 \times (35)^{1/2}} = 0.22.$$

It is evident that the brittleness number relating to the FPB tests is too high (Fig. 4); in fact: $s\,(\mathrm{FPB}) = 0.67 > s_0\,(\mathrm{FPB}) = 0.54$. Thus the curve $K_{Ic}(a/b)$ appears bell-shaped, and the reported K_{Ic} values relate to an ultimate strength collapse rather than a fracture collapse. The fracture curve $s = 0.67$ being upper and not intersecting the ultimate strength curve (Fig. 4), it is possible to assert that the established K_{Ic} values are lower than the real one. If higher specimen sizes had been chosen, such as

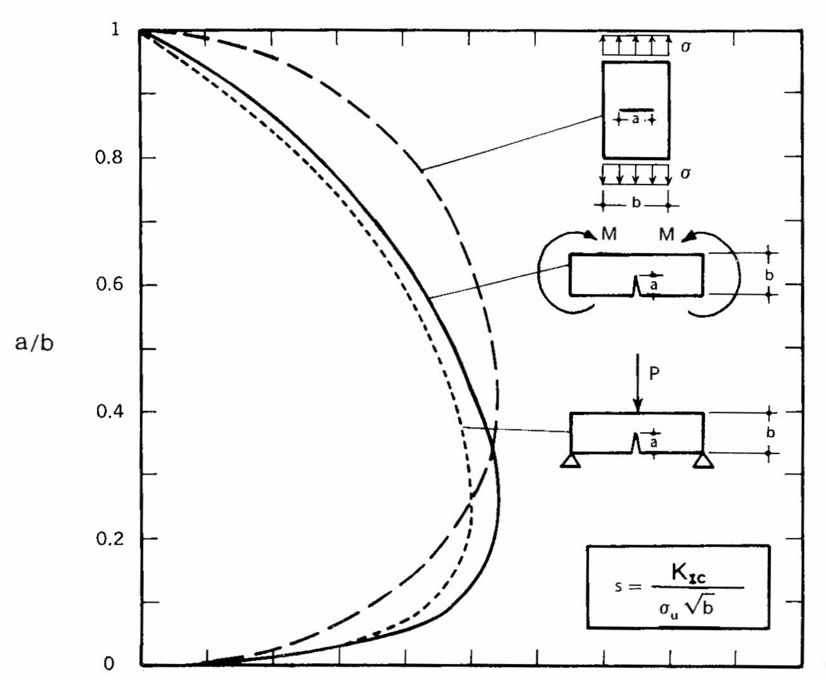

Fig. 7. Loci on the plane (s, a/b), where the fracture tests are significant.

$s < 0.54$, the experimental diagram would probably have shown a middle flat zone, representing the fracture collapse and thus significant K_{Ic} values.

For the DCB tests the specimen sizes seem to be higher than those relating to $s = s_0$. The calculations for the DCB test have not been performed but the very low standard deviation of the tests with the two longest cracks seems to be proof in favour of the preceding statement. In Fig. 8 the "fictitious K_{Ic}" curve, obtained by the brittleness number theory, is also reported (only for the FPB tests). The good results obtained by Brown for the cement paste, with specimens of the same size, are due to the lower value of the ratio (K_{Ic}/σ_u) of such material, which is reflected in the number s.

Schmidt[18] has also pointed out size effects of toughness increasing with specimen and crack sizes, in TPB tests on Indiana limestone. He justifies the K_{Ic} variability with the microcracking at the crack tip occurring before the unstable propagation. In Fig. 9 the K_{Ic} values by Schmidt are reported in function of the relative crack depth. Sizes of the largest specimens were $102 \times 102 \times 406$ mm, while the ultimate strength was 781 psi. As before, assuming K_{Ic} as the value 900 psi $\sqrt{(\text{in})}$ (Fig. 9), which Schmidt calls limit value, it is possible to obtain the test brittleness number:

$$s = \frac{900}{781 \times (4)^{1/2}} = 0.58.$$

In this case again s is too high (Fig. 5):

$$s\,(\text{TPB}) = 0.58 > s_0\,(\text{TPB}) = 0.50.$$

Thus the curve $K_{Ic}(a/b)$ is bell-shaped again, without a flat part, since the ultimate strength collapse always comes before the crack extension collapse. The value $K_{Ic} = 900$ psi $\sqrt{(\text{in})}$ is thus not exact, and the test number s is really higher than 0.58.

Barr and Bear[16, 22] have determined the fracture toughness of a concrete using CNRBB specimens (circumferentially notched round bars under bending). The results indicate that K_{Ic} is increasing with the notch depth; thus only the increasing branch of the bell-shaped curve has been found. Then Barr and Bear[56] have performed split-cube tests, i.e. eccentrically compressed specimen tests, similar to CT tests (eccentric tension). In this case the results indicate that K_{Ic} is decreasing with the notch depth; thus only the decreasing branch of the bell-shaped curve has been found. The same decreasing branch is found by Hilsdorf[21] performing CT tests on cement paste. In fact the curves of Figs. 3 and 6, emphasize as, for merely geometric reasons, CT tests can also provide the decreasing branch with not

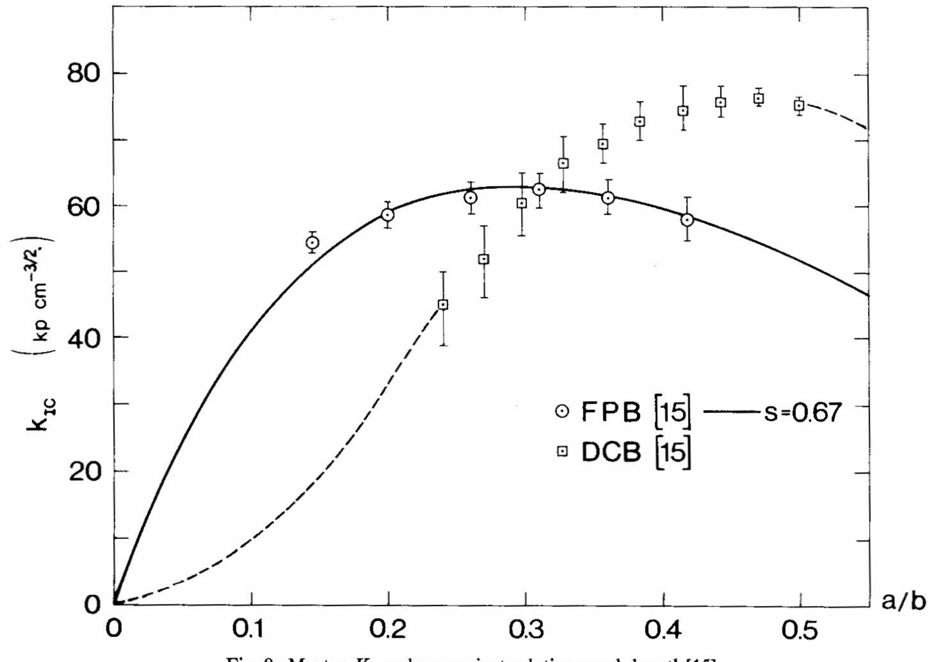

Fig. 8. Mortar K_{Ic} values against relative crack length[15].

very large relative crack depths, while the tension tests can provide the decreasing branch only with very long cracks.

Hardrath *et al.*[57] have reported K_{Ic} values against the relative crack length, for TT tests on hiduminium alloy H-48. In Fig. 10 the experimental results are reported for three different specimen widths: $b = 50$, 100, 200 mm, with the corresponding bell-shaped curves. Assuming $K_{Ic} = 95\ \mathrm{MN\ m^{-3/2}}$, and it being known that $\sigma_y = 445\ \mathrm{MN\ m^{-2}}$ and $\sigma_u = 500\ \mathrm{MN\ m^{-2}}$, it is possible to have the following brittleness numbers.

	σ_y	σ_u
$b = 50$ mm	$s = 0.95$	$s = 0.85$
$b = 100$ mm	$s = 0.67$	$s = 0.60$
$b = 200$ mm	$s = 0.48$	$s = 0.42$

Only the tests with $b = 200$ mm can be significant; in fact s_0 (TT) = 0.54 (Fig. 3). But in the case of the two shortest cracks ($b = 200$ mm) the tests do not appear significant (Fig. 10). In fact a plastic flow collapse really occurs in the cracked section as in the remaining tests.

In Fig. 5 some experimental results by the author are finally reported. They have been obtained by TPB tests on Carrara marble, mortar and PMMA (three specimens for each crack depth). The PMMA results (specimen sizes: $50 \times 50 \times 400$ mm) fall exactly on the fracture curve $s = 0.1$, showing the high notch sensitivity of PMMA, in spite of the relatively small specimen cross section. The mortar results (specimen sizes: $150 \times 150 \times 600$ mm) approach the curve $s = 0.4$, providing the mean value $K_{Ic} = 66.23\ \mathrm{kp\ cm^{-3/2}}$ and the standard deviation $\Delta K_{Ic}/K_{Ic} = 5.30\%$. The considerable cross section allows significative results, i.e. the collapse mechanism is the crack propagation and not the ultimate strength overcoming. The marble results (specimen sizes: $100 \times 100 \times 300$ mm) approach the curve $s = 0.3$, providing the mean value $K_{Ic} = 109.16\ \mathrm{kp\ cm^{-3/2}}$ and the standard deviation $\Delta K_{Ic}/K_{Ic} = 16.96\%$. It is interesting to observe how, increasing the material ratio (K_{Ic}/σ_u), it is necessary to use larger and larger specimens, to obtain significant results (i.e. $s < s_0$). The mortar tests ($s = 0.4$) would have been significant up to the crack depth $a/b \simeq 0.6$, and not above it.

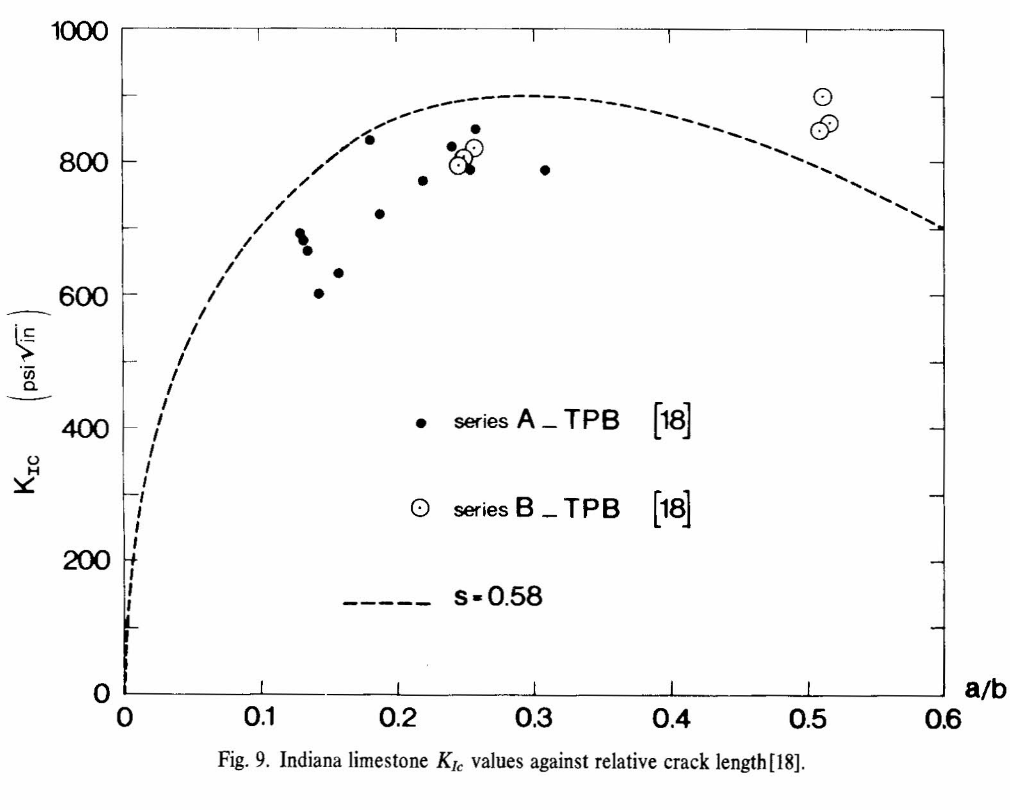

Fig. 9. Indiana limestone K_{Ic} values against relative crack length[18].

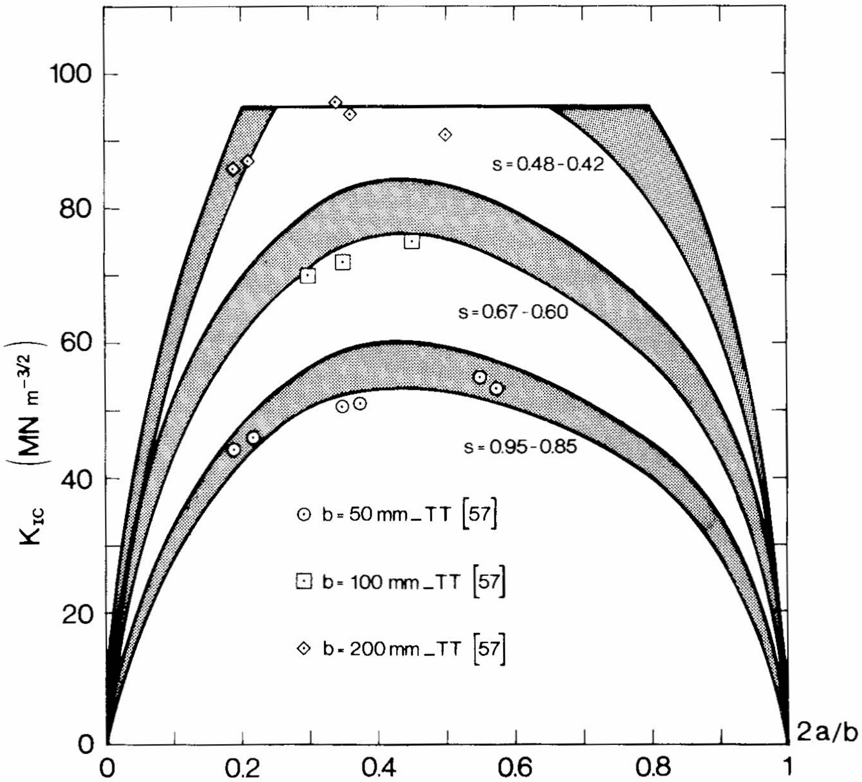

Fig. 10. Hiduminium H-48 alloy K_{Ic} values against relative crack length[57].

4. CONCLUDING REMARKS

For the most usual fracture tests the non-dimensional load of fracture collapse has been reported as a function of the ralative crack length, varying the brittleness number s. Such load has been compared with the load of ultimate strength collapse and it has been possible to observe how, for too high brittleness numbers s (i.e. too small specimens), the fracture tests are completely meaningless, the ultimate strength collapse occurring first. For lower s numbers the fracture tests are significant, only if they are performed with intermediate length cracks. Ultimate strength collapse coming before fracture collapse means that the energy release for crack extension is not sufficient to provide the surface energy of the new free surface growing at failure.

For the elastic–plastic materials, like ductile metallic alloys, the small scale yielding condition implies the notch sensitivity condition. Since the size of the crack tip plastic zone, at the fracture conditions, is approximately independent of the specimen and crack geometry[6, 58], it is possible to assert that the small scale yielding condition is the same as setting a lower bound to the specimen and crack sizes[5]. For the linear elastic materials, the small scale yielding condition, which is always satisfied because of tending to zero of the plastic zone size, would in any case imply the notch sensitivity condition. It would be true if the ultimate strength σ_u were infinite, as often is implicitly assumed in Fracture Mechanics[44], i.e. if the unique potential collapse were that of fracture.

Assuming the co-existence of two potential collapses, the fracture collapse and the ultimate collapse, it is possible to show that, for brittle materials such as rocks, mortars and concretes also, the notch sensitivity is not an intrinsic material property, but it depends on the sizes of the structure where the crack is.

Naturally the present research, performed with the help of Dimensional Analysis, and intending to clarify the notch sensitivity phenomenon in fracture testing of aggregative materials, can be extended to the metallic materials, showing a sufficiently brittle behaviour, i.e. linear up to failure. At the moment the author is working to generalize the brittleness number theory to the following cases:

(1) Elastic–perfectly plastic material;

(2) Elastic-work-hardening material.

Effects totally similar to those examined in the present paper are found in fact in a series of recent experimental works contained in ASTM-STP 668 (1979), where the critical values of the J-integral are determined for several metallic alloys [59–62]. In particular bell-shaped curves have been found like those previously discussed. The J_{Ic} parameter, holding in elastic–plastic field (large scale yielding), is variable with the specimen and crack sizes too. Therefore it is evident that, in addition to the non-linear effects, there are more fundamental reasons originating the notch sensitivity phenomenon.

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APPENDIX

Let the fundamental mechanical quantities be: length $[L]$, force $[F]$, time $[T]$. Consider a certain quantity $Q = [L^\alpha F^\beta T^\gamma]$. It is known that, if the length unit of measure is multiplied by λ, the force unit of measure by φ, the time unit of measure by τ, the unit measuring the quantity Q results multiplied by $\lambda^\alpha \varphi^\beta \tau^\gamma$. Vice versa, if q is the measure of Q in the units (L, F, T), in the new units $(\lambda L, \varphi F, \tau T)$ the measure results divided by $\lambda^\alpha \varphi^\beta \tau^\gamma$. Consider now three mechanical quantities Q_1, Q_2, Q_3:

$$
\begin{aligned}
Q_1 &= [L^{\alpha_1} F^{\beta_1} T^{\gamma_1}];\\
Q_2 &= [L^{\alpha_2} F^{\beta_2} T^{\gamma_2}];\\
Q_3 &= [L^{\alpha_3} F^{\beta_3} T^{\gamma_3}].
\end{aligned}
\tag{A1}
$$

If the units of measure (L, F, T) are multiplied by λ, φ, τ, the units measuring Q_1, Q_2, Q_3 are multiplied by χ_1, χ_2, χ_3, so that:

$$
\begin{aligned}
\chi_1 &= \lambda^{\alpha_1} \varphi^{\beta_1} \tau^{\gamma_1};\\
\chi_2 &= \lambda^{\alpha_2} \varphi^{\beta_2} \tau^{\gamma_2};\\
\chi_3 &= \lambda^{\alpha_3} \varphi^{\beta_3} \tau^{\gamma_3}.
\end{aligned}
\tag{A2}
$$

The three quantities Q_1, Q_2, Q_3 are said to be *fundamental* if, given χ_1, χ_2, χ_3, it is possible to univocally determine λ, φ, τ. From (A2) it follows:

$$
\begin{aligned}
\lg \chi_1 &= \alpha_1 \lg \lambda + \beta_1 \lg \varphi + \gamma_1 \lg \tau;\\
\lg \chi_2 &= \alpha_2 \lg \lambda + \beta_2 \lg \varphi + \gamma_2 \lg \tau;\\
\lg \chi_3 &= \alpha_3 \lg \lambda + \beta_3 \lg \varphi + \gamma_3 \lg \tau.
\end{aligned}
\tag{A3}
$$

This linear system has the unknowns $\lg \lambda, \lg \varphi, \lg \tau$, i.e. λ, φ, τ, the coefficients $\alpha_i, \beta_i, \gamma_i$ and the known terms $\lg \chi_i$, for $i = 1, 2, 3$. Thus it admits one and only one solution, if and only if the coefficients' determinant is different from zero:

$$
D = \begin{vmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{vmatrix} \neq 0.
\tag{A4}
$$

The previous condition being satisfied, the three quantities Q_1, Q_2, Q_3 are said to be *dimensionally independent* and therefore they can be taken as fundamental quantities.

Another completely equivalent definition of dimensionally independent quantities is the following one: three quantities Q_1, Q_2, Q_3 are dimensionally independent, when any quantity $Q_0 = [L^{\alpha_0} F^{\beta_0} T^{\gamma_0}]$ can have the same physical dimensions as the product

$Q_1^{\alpha_{10}} Q_2^{\alpha_{20}} Q_3^{\alpha_{30}}$, for suitable and univocally determinable values of α_{10}, α_{20}, α_{30}. In fact, recalling (A1), follows:

$$\begin{aligned} \alpha_0 &= \alpha_1\alpha_{10} + \alpha_2\alpha_{20} + \alpha_3\alpha_{30}; \\ \beta_0 &= \beta_1\alpha_{10} + \beta_2\alpha_{20} + \beta_3\alpha_{30}; \\ \gamma_0 &= \gamma_1\alpha_{10} + \gamma_2\alpha_{20} + \gamma_3\alpha_{30}, \end{aligned} \tag{A5}$$

which is a linear system with unknowns α_{10}, α_{20}, α_{30}, and has as coefficient matrix the transposed matrix D^T of the system (A3). As for system (A3), so system (A5) admits one and only one solution, if and only if the condition (A4) holds.

There are only two fundamental quantities in the statical field: length $[L]$ and force $[F]$, since time is missing. Based on the above reported considerations, it is possible for example to assert that stress $\sigma = [L^{-2}F]$, length $[L]$ and time $[T]$, are dimensionally independent:

$$D = \begin{vmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = -1 \neq 0.$$