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Procedia Structural Integrity 42 (2022) 321–327

Structural Integrity Procedia

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23 European Conference on Fracture – ECF23

Multi-Fractal Scaling Law applied to VHCF with an emphasis on statistical fluctuations

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Abstract

In recent decades, ultrasonic testing machines have allowed to investigate the very-high cycle fatigue (VHCF) range, featuring feasible gigacycle fatigue testing in a very short time, although the influence of frequency is still partly controversial. The fatigue behaviour depends on the random distribution of initial defects, whatever the stress range amplitude is. As a consequence, the structural size comes into play and the so called "size-effects" are usually observed. On the other hand, the scaling is not uniform when sufficiently large dimensional ranges are analysed. To this aim, the multi-fractal formalism was adopted to explain the observed specimen-size effect in the VHCF regime when a wide dimensional range is investigated. In addition, the statistical dispersion of experimental results was accounted by adopting different probabilistic functions. Among them, the Generalized Extreme Value Distribution Type-I was selected. In this way, it was possible to derive the analytical relationship for probabilistic-scale dependent P-S-N-b curves.

Subsequently, the proposed model was used to analyse the experimental data obtained in an experimental campaign carried out at Politecnico di Torino by the present authors. More in detail, ultrasonic fully reversed tension–compression fatigue tests in the very-high cycle fatigue (VHCF) range were conducted on a set of hourglass and dog-bone specimens made of EN AW-6082 aluminium alloy, with a diameter in the middle cross-section ranging from 3 mm up to 30 mm.The comparison between the theoretichal model and the experimental data allowed to demonstrate the ability of Multi-Fractal Scaling Law (MFSL) to provide objective values of the fatigue strength of full-size components subjected to VHCF and to guarantee safe structural design.

Keywords: Ultrasonic fatigue tests; Size effects; Statistical analysis; P-S-N-*b* curves

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10.1016/j.prostr.2022.12.039

1. Introduction

Nowadays, ultrasonic fatigue testing machines are the principal tool to explore the fatigue behaviour of metallic materials in the very-high cycle fatigue (VHCF) domain Fitzka et al. (2021), since it is possible to carry out tests on specimens beyond one billion cycles in a reasonable testing time.

In fact, machinery components may experience Very-High-Cycle Fatigue (VHCF) during their service life in many industrial applications. Therefore, it is of paramount importance to guarantee a reliable design against VHCF failure. More recently, the concept of very-high-cycle low-amplitude fatigue has been applied to explain the collapse of the Morandi bridge (Italy) Invernizzi et al. (2019, 2020a,b). In fact, according to Invernizzi et al. Invernizzi et al. (2022) the failure of the stay cables of Polcevera viaduct may have been triggered from the combined effect of gigacycle fatigue and corrosion.

An open issue in the gigacycle fatigue field concerns the specimen size effect on the very-high cycle fatigue response. In fact, a reliable prediction of the VHCF lifetime of metallic components has to consider the detrimental effect of the structural size Wen et al. (2021). On the other hand, due to the need to guarantee a resonance frequency of the specimens very close to the working frequency of the UFTMs, it is more complex to test specimens with very different risk-volumes. Thus, due to these technological limitations, an appropriate theoretical model for the prediction of the specimen-size effect on the VHCF strength is needed to achieve the required structural integrity and reliability.

The first attempt to investigate the statistical size effect on the VHCF response was done by Furuya Furuya (2008, 2010, 2011), who performed VHCF tests on high-strength steel samples with different risk-volumes. From this experimental campaign, it was found that, the larger are the specimens, the lower the fatigue resistance results to be due to the appearance of larger inclusions in the fish-eye fracture origin. Few years later, the influence of size-effects was also investigated by Tridello et al. (2020) on Gaussian and hourglass specimens made of H13 ESR steel, from which emerged a trend similar to that observed by Furuya, i.e. a decrement in the VHCF resistance by increasing the specimen size Tridello (2019); Tridello et al. (2020).

More recently, the authors of the present contribution carried out a new and very peculiar experimental campaign on aluminium alloy samples. For the first time, VHCF ultrasonic tests were performed on hourglass and dog-bone samples spanning over one full order of magnitude in the diameter of the middle cross-section Montagnoli (2021); Invernizzi et al. (2022).

In the following, the multi-fractal model is adopted to interpret the observed specimen-size effect in the VHCF regime when a wide dimensional range is investigated. Furthermore, the theoretical model was equipped with a probabilistic treatment, so that the statistical dispersion of fatigue experimental results was considered. Eventually, a comparison between the proposed theoretical model and the experimental data obtained by the authors of the present contribution was performed, which allowed to demonstrate the ability of the MFSL to predict the specimen size effect on the VHCF resistance.

Nomenclature

$\Delta\sigma_{0;\,50\%}$	intercept of the median S-N curve
$N_{50\%}$	median fatigue life
b	characteristic specimen size
l_{ch}	material characteristic length
$\Delta\sigma$	stress range
n	exponent of the Basquin's law
$\Delta\sigma^{\infty}_{0;\,50\%}$	intercept of the median S-N curve for very large specimen sizes
P	probability of failure
$\bar{N}$	random variable
$\bar{\mu}$	location parameter
$\bar{\beta}$	scale parameter
N	number of cycles to failure
F	empirical cumulative distribution function

m	sample size

2. MFSL for the assessment of P-S-*N*-*b*

In this section, the influence of structural size on the fatigue resistance in the VHCF range is investigated by adopting a theoretical model based on the geometrical multi-fractality, which can be considered as the obvious extension of the self-similar fractal concept Carpinteri (1994).

A typical feature of a self-similar fractal is its scale-invariance, in the sense that similar morphologies appear in a wide range of scales of observation due to the absence of a characteristic length of the fractal domain Carpinteri et al. (1995). In other words, a self-similar fractal exhibits a uniform scaling at all the scales. In this context, the damaged specimen ligament can be described as a lacunar mono-fractal set with a non-integer Hausdorff dimension lower than 2, providing negative scaling of the mechanical parameter defined over it.

On the other hand, the existence of an internal characteristic length is a obstacle for the development of a perfect self-similar scaling on the whole scale range Carpinteri et al. (1996). The internal characteristic length turns out to be dependent on the material microstructure and the minimum size of the inherent defects embedded in the specimens. This non-uniform scaling of fractals is defined in the Literature as "self-affinity", which implies a continuous transition from an extreme disordered (fractal) regime for smaller scales to an ordered (Euclidean) regime for larger scales Carpinteri and Chiaia (1997). From a practical point of view, it can be stated that if the negative uniform scaling law on the mechanical quantity is extrapolated to very large scales, an absurd null value will be predicted for this parameter. Therefore, the existence of this internal characteristic length implies that for very large specimen sizes the mechanical parameter will assume a non-zero positive value. Therefore, according to the geometrical multi-fractal concepts, the following expression for the intercept of the median S-N curve in the bi-logarithmic plane, $\Delta\sigma_{0;\,50\%}$, can be put forward:

$$\Delta\sigma_{0;\,50\%} = \Delta\sigma^{\infty}_{0;\,50\%}\left(1+\frac{l_{ch}}{b}\right)^{1/2}. \tag{1}$$

$\Delta\sigma^{\infty}_{0;\,50\%}$ is the intercept of the median S-N curve obtained for very large sizes, b is the characteristic specimen size, and l_{ch} is the material characteristic length, which provides information about the microstructural disorder Carpinteri et al. (2020); Carpinteri and Montagnoli (2019). Furthermore, for the materials that do not show an evident fatigue limit, a power-law type equation can be used to describe the median Wöhler's curve:

$$N_{50\%} = \left(\frac{\Delta\sigma_{0;\,50\%}}{\Delta\sigma}\right)^{n}, \tag{2}$$

where n is the exponent of the stress-life power-law equation and $N_{50\%}$ is the median fatigue life for the applied stress range $\Delta\sigma$. Therefore, by substituting Eq. 1 in Eq. 2, it is possible to derive the following analytical relationship:

$$N_{50\%} = \left(\frac{\Delta\sigma^{\infty}_{0;\,50\%}}{\Delta\sigma}\right)^{n}\left(1+\frac{l_{ch}}{b}\right)^{n/2}, \tag{3}$$

which predicts a decrement in the very-high cycle fatigue life by increasing the specimen size, being the stress range the same, although such decrement is not constant with the scale of observation. In fact, this relationship connects the two asymptotic behaviours for smaller and larger specimens Carpinteri and Montagnoli (2020). For very small specimens, the maximum possible disorder is reached and an oblique asymptote, with a slope equal to $-1/2$, is obtained Montagnoli et al. (2020). On the contrary, for very large specimens, the dependence on the specimen size disappears and the lowest (asymptotic) value of the nominal VHCF resistance is reached. With easy steps, Eq. 3 can be also rewritten in the following way:

$$\Delta\sigma = \frac{\Delta\sigma^{\infty}_{0;\,50\%}}{{N_{50\%}}^{1/n}}\left(1+\frac{l_{ch}}{b}\right)^{1/2}, \tag{4}$$

where the very-high cycle fatigue strength is expressed as a function both of the median number of cycles to failure and the specimen size. This equation implies a non-uniform decrement in the VHCF resistance by increasing the specimen size, being N the same. In other words, a continuous variation of the fractal dimension with the scale of observation is predicted by Eq. 4, which implies the disappearance of the scale effect for very large specimen sizes.

On the other hand, the design of structural components against the fatigue failures should deal with the large dispersion on the fatigue experimental data of the material investigated in the very-high cycle fatigue regime Freire Júnior et al. (2014); Invernizzi et al. (2021). In other words, it is needed to consider the fatigue life as a stochastic variable, making it possible the construction of the probabilistic stress-life curves. In the following, the statistical dispersion on the experimental dataset will be taken into account by modelling the normalised fatigue life, $\bar{N}$, with Generalized Extreme Value (GEV) distribution Type-I:

$$P(\bar{N}) = \exp\left[-\exp\left(-\frac{\bar{N}-\bar{\mu}}{\bar{\beta}}\right)\right], \tag{5}$$

where $\bar{\mu}$ and $\bar{\beta}$ are the location parameter and the scale parameter, respectively. Furthermore, it is worth to emphasize that in Eq. 3 it has been implicitly assumed that the parameters of the GEV distribution type-I are considered constant for each stress level. This new random variable is derived by the ratio between the number of cycles to failure N, experimentally obtained for a certain stress range $\Delta\bar{\sigma}$, and the corresponding theoretical median fatigue life, which is assessed according to Eq 2 by imposing $\Delta\sigma = \Delta\bar{\sigma}$. In this way, it is possible to compute the analytical expression of the probabilistic stress-life curves:

$$N = \{\bar{\mu} + \bar{\beta}\,[-\ln(-\ln(P))]\}\left(\frac{\Delta\sigma_{0;\,50\%}}{\Delta\sigma}\right)^{n}, \tag{6}$$

which, by recalling Eq. 2, can be also expressed as follows:

$$N = \{\bar{\mu} + \bar{\beta}\,[-\ln(-\ln(P))]\}\,N_{50\%}. \tag{7}$$

Finally, the relationship for the specimen-size dependent probabilistic stress-life curves can be obtained by substituting Eq. 3 in Eq. 7:

$$N = \{\bar{\mu} + \bar{\beta}\,[-\ln(-\ln(P))]\}\left(\frac{\Delta\sigma^{\infty}_{0;\,50\%}}{\Delta\sigma}\right)^{n}\left(1+\frac{l_{ch}}{b}\right)^{n/2}, \tag{8}$$

which yields to a non-uniform vertical downward translation of P-S-N curves by increasing the specimen size. In other words, Eq. 6 predicts a decrement in the VHCF life with the specimen size, for a fixed stress range and probability of survival.

3. Comparison of the model to an experimental dataset

In this section, fatigue data of an ultrasonic fatigue campaign investigating the size effect on aluminium alloy samples spanning over a wide dimensional range are analysed according to the model proposed in the previous section.

In this experimental campaign, hourglass and dog-bone specimens made of aluminium alloy EN-AW6082 T6 were tested with an ultrasonic fatigue testing machine under fully-reversed constant amplitude conditions. More in detail, the ultrasonic fatigue tests were conducted on specimens with diameters in the middle cross-section ranging between 3 mm and 30 mm, which were tested up to failure or up to 10^{10} cycles. The experimental results showed an evident influence of structural size on the very-high cycle fatigue resistance, although such effect was not constant with the specimen size. In fact, a transition in terms of decrement in the VHCF strength was observed between small scales, where the size effect was more pronounced, and larger scales, where the size effect was vanishing (see Montagnoli (2021) for more details). Therefore, a non-linear regression of the experimental data provided the values of the three out of five free-parameters, i.e. $\Delta\sigma^{\infty}_{0;\,50\%}$, n, and l_{ch}. More in detail, the best-fitting of the experimental data yielded to a value of 759 MPa for the coefficient of power-law of the median S-N curve, n was found equal to 19.7, whereas 1.9

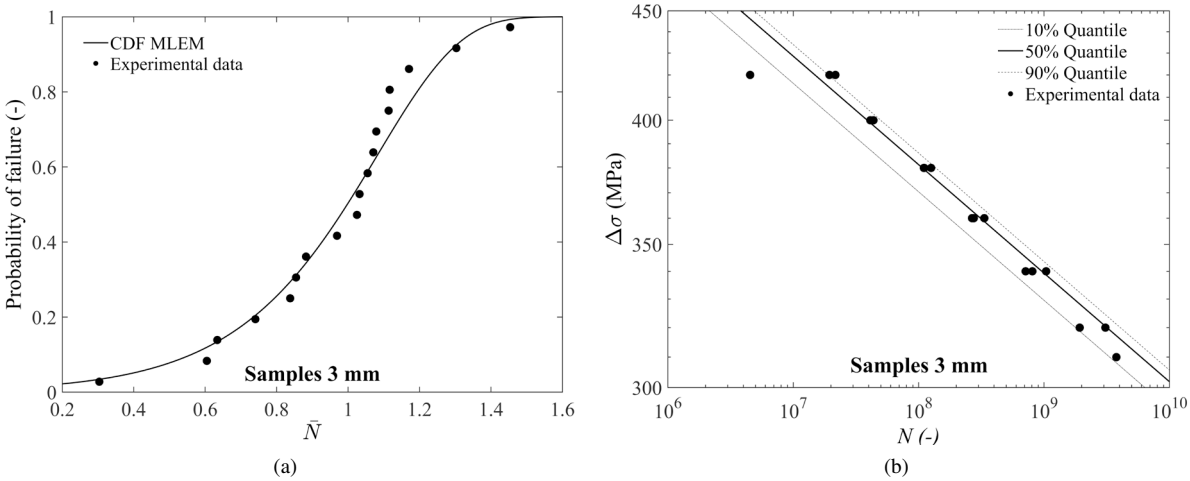

Fig. 1: Specimen of 3 mm in diameter: (a) CDF; (b) P-S-N curves

mm was the material characteristic length. It is worth to emphasize that this value is in perfect accordance with the experimental evidence that for specimen sizes larger than 12 mm the decrement in the VHCF resistance disappears.

Subsequently, the location and the scale coefficients of Generalized Extreme Value (GEV) distribution Type-I were obtained by adopting the Maximum Likelihood Method. After that, the evaluation of GEV Type-I CDF adherence to the experimental data was carried out by means of the application of four different goodness-of-fit (GoF) statistics tests. In all cases, the GoF statistics tests did not reject the null hypothesis that the experimental data came from Generalized Extreme Value (GEV) distribution Type-I at the 5% significance level.

In Figs. 1(a) and 2(a) the estimated CDF for the specimens of 3 and 30 mm in diameter are plotted against the experimental data. The empirical cumulative probability of failure was evaluated according to Bernard's median rank for the ith sorted element:

$$F = \frac{(i - 0.3)}{(m + 0.4)}, \tag{9}$$

where m is the sample size. Eventually, in Figs. 1(b) and 2(b) the estimated probabilistic stress-life curves corresponding to the αth quantiles, 10%, 50%, and 90% for the specimens of 3 and 30 mm in diameter are shown. According to Figs. 1(b) and 2(b), the estimated curves are in good agreement with the experimental data, from which it emerges that the proposed approach is able to correctly predict the specimen size effect on the VHCF resistance and to provide a reliable estimation of the probabilistic stress-life curves.

4. Conclusions

In the present contribution the multi-fractal model, equipped with probabilistic treatment of the statistical dispersion of fatigue experimental results, was adopted to interpret the observed specimen-size effect in the VHCF regime when a wide dimensional range is investigated. The proposed model was formulated upon the hypothesis that the material ligament is represented by a lacunar multi-fractal set. In this way, the non-uniform negative scaling on the VHCF resistance by increasing the specimen size can be captured. In addition, the model is able to predict the VHCF life under various probabilities of survival of structural components with different dimensions. Eventually, the proposed model was compared to experimental data obtained in an experimental campaign carried out at Politecnico di Torino by the present authors. The comparison between the theoretical model and the experimental data allowed to demonstrate the ability of Multi-Fractal Scaling Law (MFSL) to provide reliable values of the very-high cycle fatigue strength of full-size components subjected to VHCF.

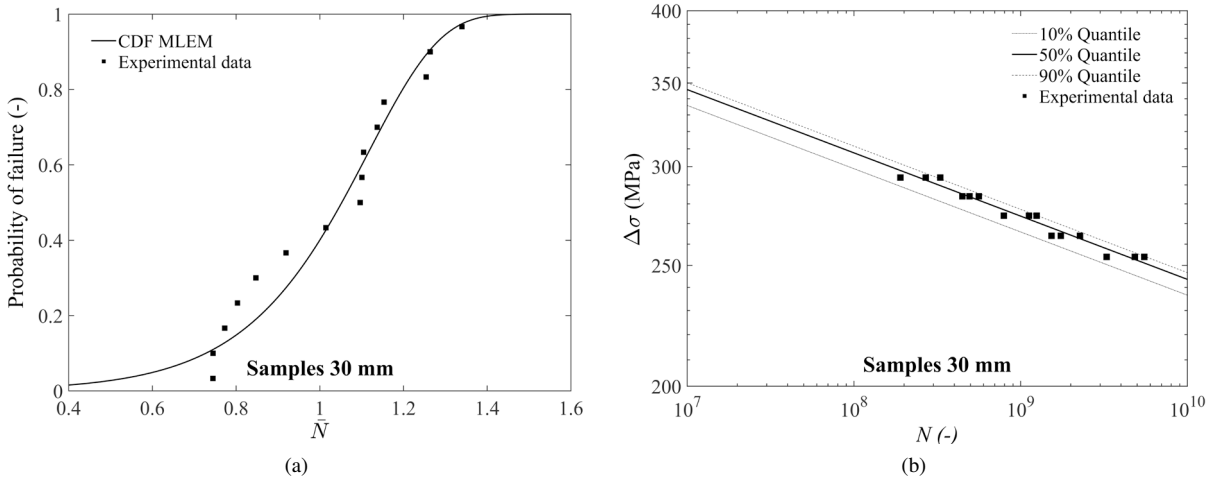

Fig. 2: Specimen of 30 mm in diameter: (a) CDF; (b) P-S-N curves

Acknowledgements

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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