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Multiple snap-back instabilities in progressive microcracking coalescence

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ABSTRACT

This work describes an application of the Crack Length Control Scheme to analyze the damage evolution in solids containing a distribution of collinear microcracks. By means of a Dual Boundary Element Method numerical procedure, a model of crack growth is developed according to Linear Elastic Fracture Mechanics. A controlled crack propagation is obtained increasing the macrocrack length in the loading process. Some illustrative problems are shown, related to finite plates in plane strain conditions with one row of evenly spaced collinear cracks. The snap-back branches of the load-displacement curve are numerically plotted, and the interaction effects of the microcracks on damage evolution are analyzed.

1. Introduction

Cracked solids often show an unstable structural behavior which is represented by a negative slope of the load-displacement softening branch [14–16,18,15–18]. This means that the load must decrease to obtain a stable crack propagation. Moreover, the global behavior can range from ductile to brittle (Fig. 1), depending on material properties, structure geometry, and loading conditions [1,9,10,17,21–23,26,33,34]. In extremely brittle cases, crack propagation occurs suddenly with a catastrophic drop in the load carrying capacity, and the load-displacement softening branch assumes a positive slope. If the loading process is controlled by the displacement, the curve presents a discontinuity, and the representative point drops on the lower branch with negative slope. This means that both load and displacement must decrease to obtain a controlled crack propagation [11,19].

Such a phenomenon, the so-called snap-back instability, was deeply investigated with reference to the crack growth analysis in elastic-softening materials [15,16,18,20,21,24,31]. In the framework of Linear Elastic Fracture Mechanics (LEFM) it reproduces the classical Griffith instability for very brittle systems, depending on structural geometry and material properties [11,13,16,35].

From the point of view of post-critical equilibrium states, snap-back phenomena are some of the most crucial problems in nonlinear structural analysis. Using Finite Element Method (FEM), the nonlinear solution is based on an iterative procedure that can abort according to the adopted control parameter. If the system is controlled by the load, the snapping phenomena can arise and the possible descending post-peak response cannot be plotted. On the other hand, if an arbitrary displacement component is selected as the controlling parameter, whereas the corresponding load value is considered as unknown, the virtual branch response related to the snap-back instabilities will not be manifest.

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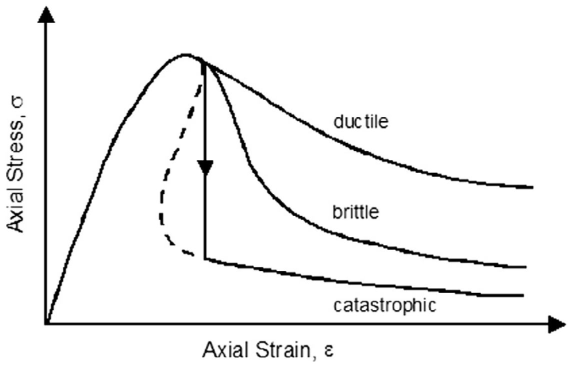

Fig. 1. Ductile, brittle, and catastrophic (snap-back) response.

In order to overcome these problems, with reference to a cracked homogeneous solid, the snap-back branch can be followed and controlled only if both load and displacement decrease. The same branch may be captured only if the loading process is controlled by a monotonically increasing function after the maximum load is reached. In the case of damage evolution analysis, such a monotonically increasing parameter can be represented by the crack mouth opening or sliding displacement, as well as the crack length itself.

Based on such considerations, the Crack Length Control Scheme (CLCS) has been proposed and proved to be appropriate in Fracture Mechanics problems [11,24]. Moreover, the indirect displacement control scheme, developed by Rots and de Borst [41], refers to a driving parameter that coincide with the Crack Mouth Sliding Displacement (CMSD). On the other hand, CLCS uses the crack length itself in Mode I [15,16,18,24,31,42] as well as in Mixed Mode loading conditions [29,2,42].

By means of FEM, various simulations of brittle behavior in homogeneous finite plates have been performed under CLCS [22–24]. If multi-cracked plates are considered, where a crack distribution is assumed, the length of the forming macrocrack is a strictly monotonic increasing function [8,31].

In this work, an application of CLCS is described, to analyze the damage evolution of solids containing a distribution of collinear interacting cracks. By means of a procedure based on the Dual Boundary Element Method (DBEM) [7,8,26,29,30–32], a numerical algorithm for cracking coalescence is developed according to the LEFM criteria [40,43]. A slow and controlled damage propagation is obtained increasing the crack length in the incremental loading process. With reference to finite plates in plane strain conditions with one row of evenly spaced collinear cracks, some illustrative problems are shown. The multiple snap-back branches of the load-displacement curve are numerically plotted, and the effects of the crack interaction on damage evolution are analyzed.

2. Local and global snap-back instabilities

Let us consider the case of a finite plate of side 2 h and width 2b with a central crack of length 2a, and subjected to an uniaxial traction, σ, as shown in Fig. 2a. The crack is perpendicular to the direction of traction. The loading process is controlled by the crack length, which is a quantity growing by definition (CLCS).

Fig. 2b shows the trend of the global stress-strain response curve. Note that the control parameter is expressed as a nondimensional ratio between the crack half-length, a, and the half-width of the plate, b.

The stress-strain diagram shows a linear elastic behavior up to point A, at which the maximum load is reached. After point A, the stress-strain curve could be represented only controlling the test by the crack length growth. It is interesting to remark that, starting the test with the ratio $a/b = 0.1$, an unstable global snap-back behavior emerges (branch AB presents a positive softening slope).

Setting b as constant, and varying the crack length a, there is a change in the global behavior of the plate: by increasing the ratio a/b, the snap-back instability becomes less and less accentuated. For values $a/b > 0.3$, the snap-back phenomenon disappears, and the post-peak curve shows only negative slopes (normal softening).

In addition, let consider the case of a specimen in which reinforced fibers are embedded in the matrix (i.e., the case of a fiber reinforced concrete beam), and with an edge crack, as illustrated in Fig. 3a. A load P is applied, opening the two faces of the crack that propagates inside the bulk of the specimen. Propagation will occur alternately within the matrix and through the fibers [3–6,12,25,27,28]. The loading process is controlled by the crack length growth.

In Fig. 3c, the relative load-displacement diagram is reported. An immediate recognition of the previously treated global instability phenomena, here in terms of local behavior, is then possible. Eventually, if n fibers are present in the specimen, there will be n local snap-back phenomena in the structural response.

In the post-peak portion of the curve, the peaks represent the state of incipient propagation of the crack, when each individual fiber crossed by the crack tip has been deformed plastically, while the positive slopes in the descending branches represent the unstable crack propagation in the brittle matrix and between two consecutive fibers. In other words, when the

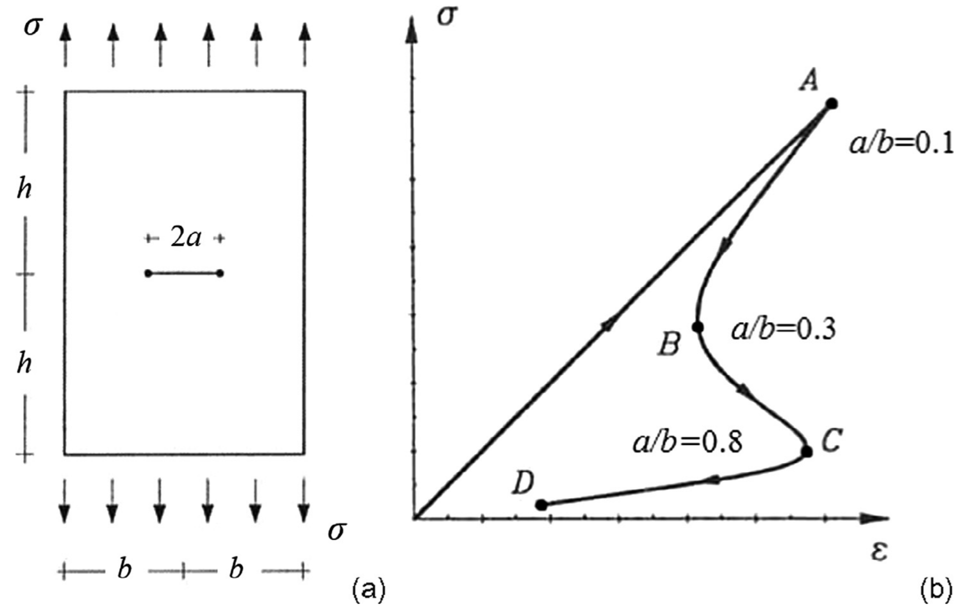

Fig. 2. Finite plate of length 2 h and width 2b with a central crack of length 2a, subjected to uniaxial traction (a); global stress-strain response diagram (b).

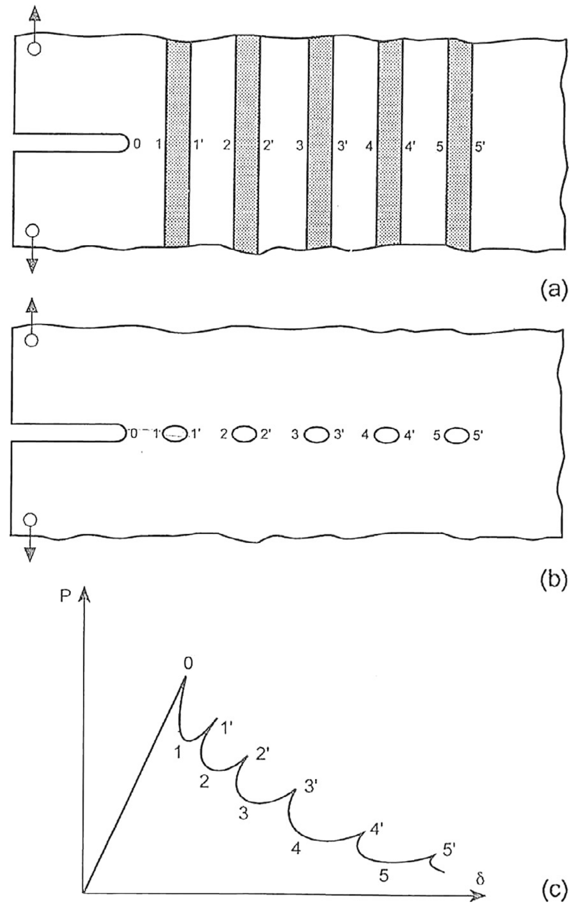

Fig. 3. Regular distribution of reinforcing fibers in a brittle matrix specimen with an edge crack (a); Brittle matrix specimen with an edge crack and collinear microcracks (b); Load-displacement diagram (c).

crack propagates within the matrix, the local behavior is unstable and the load-displacement curve is softening with a positive slope (e.g., 0–1 in Fig. 3a,c). Then, when the crack meets a fiber, its growth is stopped until a new critical condition is reached in the matrix.

This will result in an increase in both load, P, and displacement, δ (e.g., 1–1′ in Fig. 3a,c).

As a last example, let us consider a specimen with an edge crack, containing a distribution of collinear microcracks as illustrated in Fig. 3b.

As previously stated, when the crack propagates within the matrix, the behavior is usually unstable, and the load-displacement curve is softening with a positive slope (e.g., 0–1 in Fig. 3b,c).

Finally, when the coalescence between the dominant crack and a collinear microcrack occurs, the load increases until the material fracture toughness is reached again (e.g., 1–1′ in Fig. 3b,c).

3. Effects of pre-existing damage: Ductile-to-brittle transition

In this section, several case studies are presented, concerning a finite plate of length $2b$ and width $2b$ with a central dominant crack of length $2a_0$, subjected to an uniaxial traction, σ, as shown in Fig. 4 [44]. In the ligament we consider a different number n of initial collinear microcracks, of length $2a$ each, and spaced with the same distance s between the corresponding tips. The ratio of the sum of the lengths of the microcracks to the ligament is defined as a measure of the pre-existing damage D (D = 0, means undamaged ligament, i.e., absence of microcracks, whereas D = 1 means total separation). The problem is treated as symmetrical also in its evolution.

By means of a model based on DBEM, a numerical algorithm for cracking coalescence is developed according to the LEFM criteria. A slow and controlled crack propagation is obtained increasing the macrocrack length in the incremental loading process. For each increment of the crack extension, the Dual Boundary Element Method is applied to perform a stress analysis, and the stress intensity factors are computed through the J-integral [39]. The crack extension is automatically discretized with new boundary elements, and so remeshing is not required by virtue of the single-region representation of cracked structures, an intrinsic feature of the DBEM [39,40]. At the beginning of each numerical simulation, the crack is represented by two coincident boundary lines. Since the problem appears to be symmetric, the maximum number of boundary elements considered in the following simulations is 52, while the minimum increment value of the crack extension is assumed as $0.002b$.

In Figs. 5–14, the stress-strain global response diagrams of different numerical tests are reported, in which the geometry of Fig. 4 is considered by varying the relative length of the dominant crack (a_0/b), the damage ratio (D), and the relative length of the collinear microcracks (a/b). In this way, the snap-back branches of the load vs. displacement curve are numerically captured, and the effects of the crack interaction on fracture evolution are analyzed.

In Table 1, the values of the parameters a_0/b, D, a/b contemplated in the case studies are summarized.

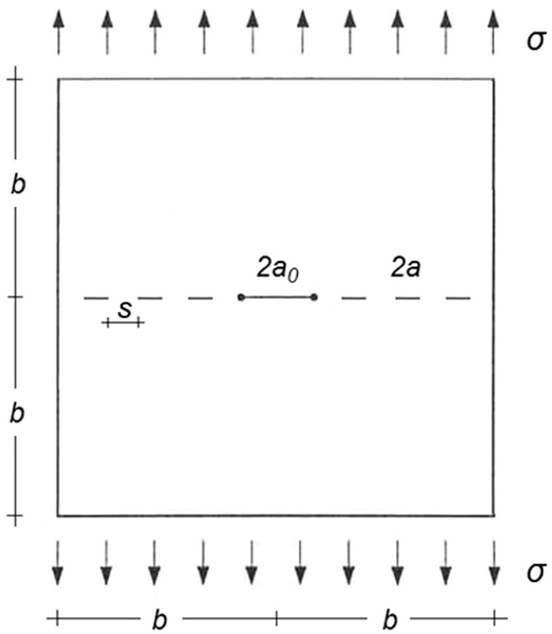

Fig. 4. Finite plate subjected to uniaxial traction, with a central dominant crack and a distribution of collinear microcracks.

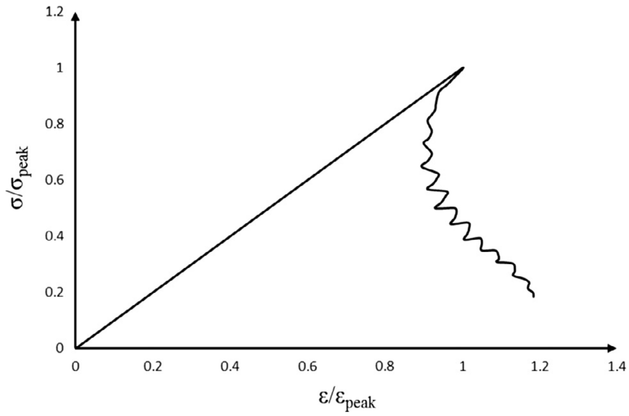

Fig. 5. Case study A: a_0/b = 0.1; D = 30%; a/b = 0.01.

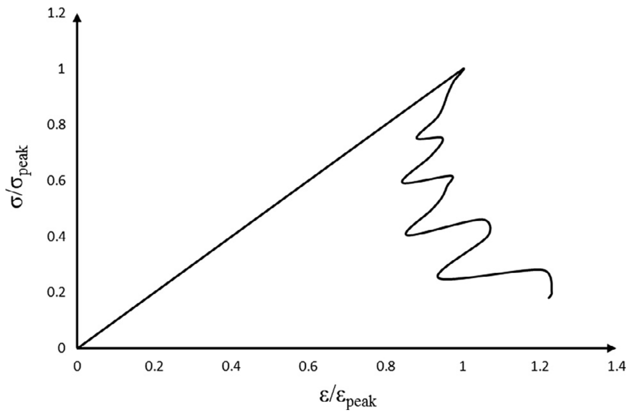

Fig. 6. Case study B: a_0/b = 0.1; D = 30%; a/b = 0.03.

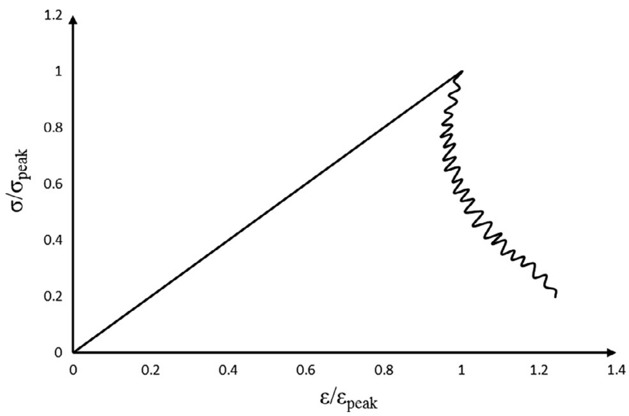

Fig. 7. Case study C: a_0/b = 0.1; D = 50%; a/b = 0.01.

It is worth noting that the parameter a_0/b is particularly relevant in the considered analyses. By comparing the five cases related to a_0/b = 0.1 (Cases A-E), with those related to a_0/b = 0.3 (Cases F-L), a remarkable transition from a global snap-back instability to a global softening behavior is evidenced. By decreasing the length of the dominant crack, a_0, and leaving unchanged the damage ratio, D, an increase in the global brittleness of the specimen is observed.

Moreover, by increasing the damage ratio, D, we can observe that:

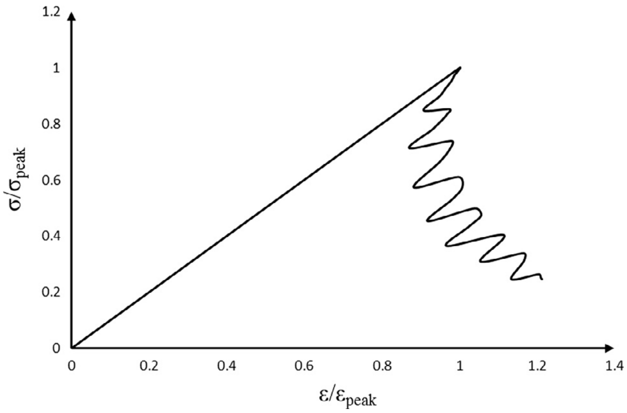

Fig. 8. Case study D: $a_0/b = 0.1$; D = 50%; $a/b = 0.03$.

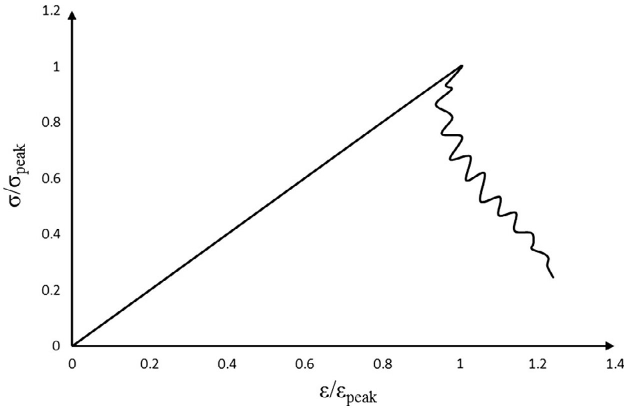

Fig. 9. Case study E: $a_0/b = 0.1$; D = 70%; $a/b = 0.03$.

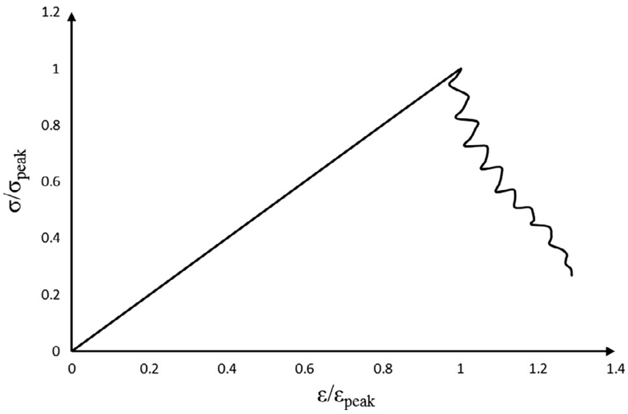

Fig. 10. Case study F: $a_0/b = 0.3$; D = 30%; $a/b = 0.01$.

(i). When the dominant crack is small ($a_0/b = 0.1$), the severity of the global snap-back phenomenon decreases. For D = 0.3 (Figs. 5,6), the loading drop is about 60%, whereas for D = 0.5 (Figs. 7,8), the loading drop is about 40%, and for D = 0.7 (Figs. 9), the loading drop is about 30% of the peak load.

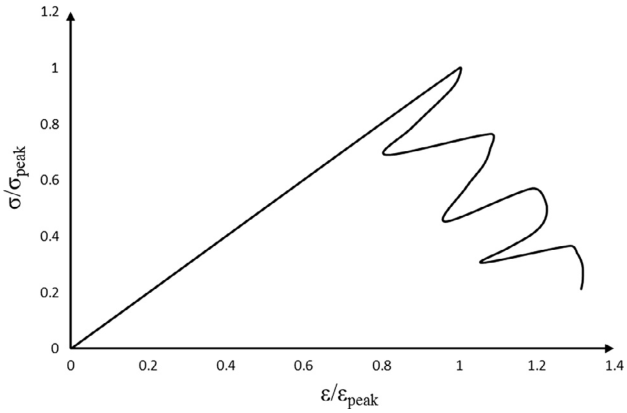

Fig. 11. Case study G: $a_0/b = 0.3$; D = 30%; $a/b = 0.03$.

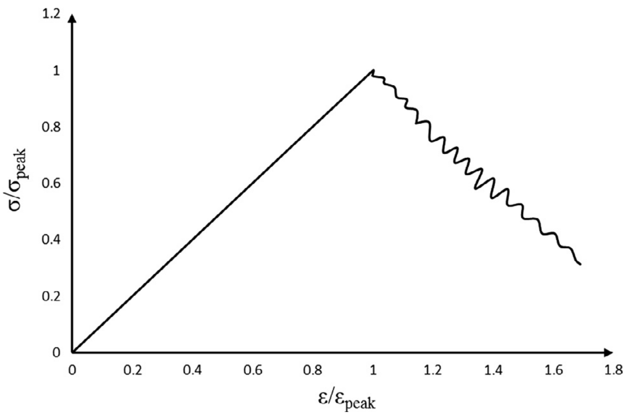

Fig. 12. Case study H: $a_0/b = 0.3$; D = 50%; $a/b = 0.01$.

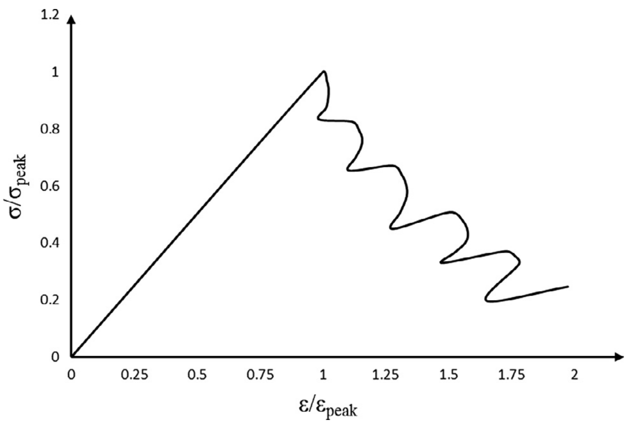

Fig. 13. Case study I: $a_0/b = 0.3$; D = 50%; $a/b = 0.03$.

(ii). When $a_0/b = 0.3$, the average slope of the softening branch in the stress-strain diagrams decreases. For D = 0.3 (Figs. 10,11), the average slope in the dimensionless digram is about 3. For D = 0.5 (Figs. 12,13), the average slope is about 1, whereas for D = 0.7 (Figs. 14), the average slope in the dimensionless digram is about 0.8.

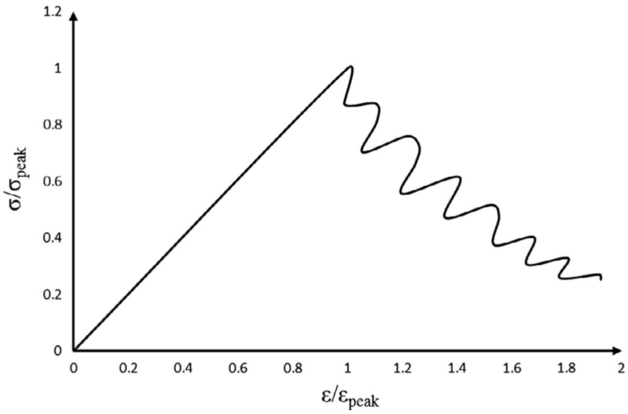

Fig. 14. Case study L: $a_0/b = 0.3$; D = 70%; $a/b = 0.03$.

Table 1
Case studies.

Case Study	a_0/b	D	a/b
A	0.1	0.3	0.01
B	0.1	0.3	0.03
C	0.1	0.5	0.01
D	0.1	0.5	0.03
E	0.1	0.7	0.03
F	0.3	0.3	0.01
G	0.3	0.3	0.03
H	0.3	0.5	0.01
I	0.3	0.5	0.03
L	0.3	0.7	0.03

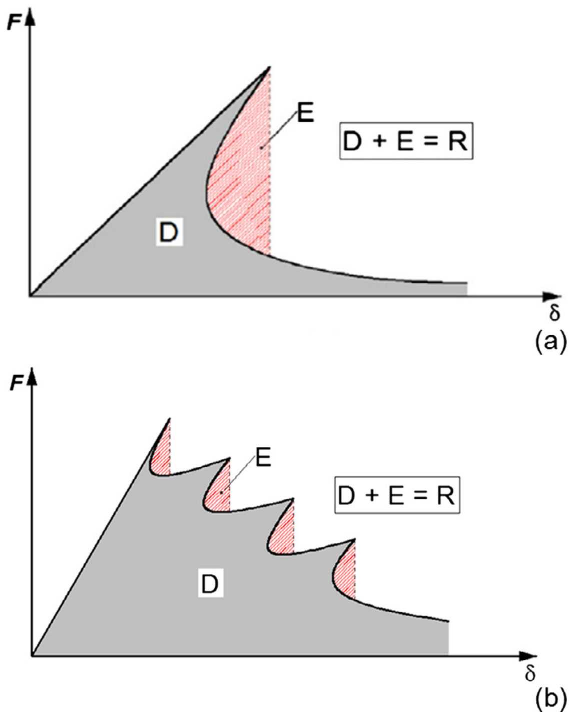

Fig. 15. Load-displacement curves representing: (a) a catastrophic behavior (single snap-back); (b) a global softening behavior perturbed by multiple local instabilities (snap-back). The grey areas identify the dissipated energy, D, whereas the dashed ones represent the emitted energy, E. The total released energy, R, is the summation of the two previous ones: D + E = R.

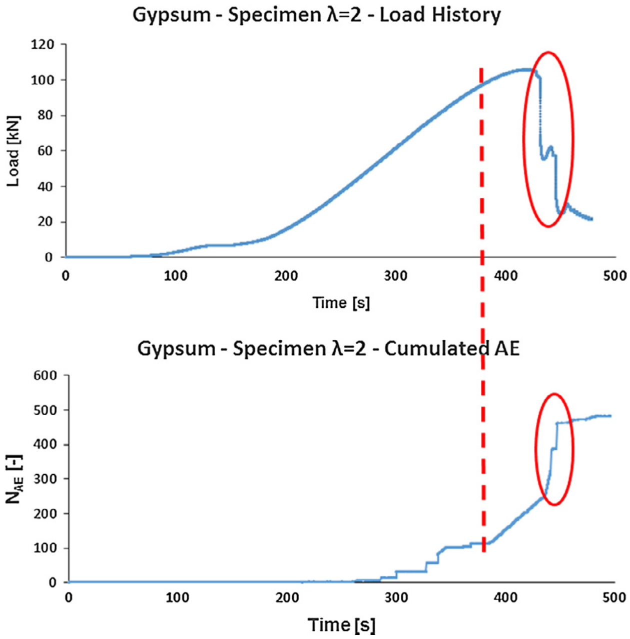

Fig. 16. Stress-strain diagram and AE cumulated curve for a gypsum specimen of slenderness 2 subjected to a compression test. An evident correlation emerges between the loading drops (snap-back instabilities) in the stress-strain diagram, and the AE bursts (emitted energy, E) in the cumulated curve.

Therefore, the post-critical softening branch tends to decrease its slope by increasing the initial damage ratio, D.

A sharp global snap-back instability is identified in the case studies with small damage ratios.

This is to say that a structural element almost undamaged may present a brittle behavior, whereas a comparatively more damaged one appears to be ductile. In other words, a damaged ligament can avoid snap-back instabilities presenting a post-peak softening response.

As regards the influence of the length of the microcracks, it can be observed that, varying the ratio a/b and leaving unchanged a_0/b and D, it means to modify only the microscopic aspects of the damage distribution in the ligament, but not its severity. In each diagram obtained above, local snap-back instabilities are present. In general, n local instabilities are consequent to n microcracks in the ligament. As the ratio a/b changes, there is no change in the global response, although it modifies the number and relevance of the local snap-back phenomena.

4. Conclusions

The fracture evolution in solids containing a distribution of collinear interacting cracks is analyzed. By means of a model based on the Dual Boundary Element Method, a numerical algorithm for cracking growth and coalescence is developed according to Linear Elastic Fracture Mechanics criteria. A slow and controlled crack propagation is obtained increasing the macrocrack length in the incremental loading process (Crack Length Control Scheme). With reference to finite plates with one row of evenly spaced collinear cracks in plane strain conditions, some illustrative problems are shown. The snap-back branches of the load vs. displacement curve are numerically captured, and the effects of the crack interaction on fracture evolution are analyzed. The length of the dominant crack deeply affects the global response of the structural element, mainly driving the ductile-to-brittle transition. The initial damage ratio of the ligament also influences the post-peak regime in terms of ductility, while the length of the collinear microcracks affects only locally the curve, modifying the number and relevance of local snap-back phenomena.

Moreover, from an energetic point of view, according to very recent interpretations [37], the relationship between the microcracks coalescence and the emitted energy analyzed by the Acoustic Emissions (AE), is represented by the area subtended by each snap-back branch (Fig. 15). In fact, in a loading process, the total energy released during the test, R, is equal to the sum of the dissipated energy, D (grey area in Fig. 15), plus the emitted energy, E, during the snap-back phenomena (pink[1] area in Fig. 15).

As shown by a series of compression tests on different natural materials, a correlation exists between AE signal energy [36,38] and the energy emitted during snap-back instabilities in the structural response. As can be seen in Fig. 16, for a gypsum specimen of slenderness 2, subjected to a compression test, the correlation between the loading drops (snap-back instabilities) in the stress-strain diagram and the AE bursts (emitted energy, E) in the cumulated curve is evident.

[1] For interpretation of color in Fig. 15, the reader is referred to the web version of this article.

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