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# Brittle materials and stress concentrations: are they able to withstand?

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**Abstract**

The combination of brittleness in materials and stress concentrations may lead to premature fracture of structural components. To improve the modelling capability more and more sophisticated methods have to be employed. To the purpose a Finite Fracture Mechanics (FFM) criterion based on the contemporaneous fulfilment of a stress requirement and the energy balance has been proposed in the literature. This coupled approach is here refined and applied to investigate brittle fracture in rounded V-notched samples under mode I loading. The failure FFM condition is expressed by a system of two equations in two unknowns: the critical crack advance and the apparent generalized fracture toughness (i.e., the failure load). The refined criterion is validated by its implementation with various experimental data, available in the literature, related to ceramic, metallic, and plastic brittle materials.

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*Keywords:* Blunt notch, coupled criterion, brtille fracture, mode I loading

## 1. Introduction

Many approaches have been put forward to study the mode I brittle fracture of structural components containing rounded V-notches in the framework of the Theory of Critical Distances [1-7]. The main idea these criteria are based on is that failure takes place when either a stress-based quantity, or an energy-related characteristic at a finite distance from the notch tip reaches a critical value: such a distance results to be a material constant.

If the stress and energy conditions are coupled one to each other, as it happens for FFM criteria [8-12], the critical distance becomes a structural parameter. The coupled approaches were introduced to remove some

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inconsistencies related to criteria based either on stress or on energy requirements [13].

In 2002 an accurate expression for the stress field along the blunt notch bisector was developed [14]; thus, the above referred stress condition can be easily computed. What allows to develop stress-based criteria [2,3,6] in an almost-analytical way. On the contrary, as it regards the energy balance, the relationships proposed for the stress intensity factor (SIF) related to a crack stemming from a blunt V-notch tip (providing the crack driving force) are usually approximated. Although closed-form solutions can be obtained, the results are affected by some notch amplitude depending errors, the entity of which has to be discussed case-by-case [11]. An alternative way may consist in proceeding numerically, via a specific finite element analysis [1,9,10]. Of course, this implies a more laborious computation, since it is necessary to carry out a different simulation for each data set.

As it concerns approximating functions for the SIF of a crack stemming from a blunt V-notch, let us mention Lukas's formula [15], which provides an approximating function for the SIF of a crack emanating from an elliptical hole, despite its range of validity being very limited. Note that a U-notch can be considered as a limit case of an elliptical hole (see also [16]), when the minor axis to major axis length ratio tends to zero. An analytical relationship for the SIF related to a rounded tip V-notch was proposed to satisfy the asymptotic limits of very short/long cracks [11]. It was numerically verified for $\omega$ = 90°, 120°, and 150°, showing the maximum deviation (nearly 7%) at $\omega$ = 90°. Since the energy available for a crack length increment is proportional to the integral of the squared SIF, the global error is expected to increase. Even more recently, the relationship proposed in Ref. 11 was improved by adding a notch amplitude parameter $m$, the value of which was fitted by means of *ad hoc* numerical simulations [17]: in this case, the error on the SIF was estimated to be lower than 1% over the whole range $0° \leq \omega \leq 180°$, which can be considered very satisfying for practical applications.

The relationship provided in [17] will be exploited to enhance the FFM approach. The criterion consists in a system of two equations (one for the stress requirement and the other for the energy balance, respectively) in two unknowns: namely, the apparent generalized fracture toughness (i.e., the failure load) and the critical crack advance. As already hinted, the latter results to be a structural parameter, depending on the notch amplitude and on the root radius, as well as on the material properties. Dimensionless results will be presented hereby, together with the comparison with experimental data [6, 18-20].

The novelties of the present work with respect to the past ones [11-12] are substantially two: (i) a FFM implementation via the relationship proposed in [17], what allows to improve the prediction accuracy, especially on samples with small notch amplitudes, as well as to develop a unified FFM approach in the framework of blunt V-notches. Indeed, since the crack driving function cannot be integrated explicitly, numerical quadrature formulae must be employed; (ii) the comparison of FFM results with various experimental data sets, covering the whole $0° \leq \omega \leq 150°$ range.

## 2. Finite Fracture Mechanics

The FFM criterion is based on the hypothesis of a finite crack advancement $l$ and assumes the contemporaneous fulfilment of two conditions [21-22]. By referring to the coordinate system displayed in Fig. 1a, the former condition requires that the average stress $\sigma_y(x)$ upon the crack advancement $l$ is higher than the material local tensile strength, $\sigma_u$. The latter one ensures that the energy available for a crack length increment $l$ (involving the integration of the crack driving force over such a length) is higher than the energy necessary to create the new fracture surfaces. By means of Irwin's formula, these conditions, at the critical mode I point, can be put in the following form:

$$\begin{cases} \int_0^{l_c} \sigma_y(x)\,\mathrm{d}x = \sigma_u\, l_c \\ \int_0^{l_c} K_I^2(c)\,\mathrm{d}c = K_{Ic}^2\, l_c \end{cases} \tag{1}$$

$K_I(c)$ and $K_{Ic}$ being the SIFs related to a crack of length $c$ stemming from the notch root (Fig.1b) and the fracture toughness, respectively. A system of two equations in two unknowns is thus obtained, with the unknowns being the critical crack advancement and the failure load (implicitly embedded in the stress field and SIF functions). Note that the hypothesis of positive geometries (monotonic increasing $K_I(c)$ functions) has been implicitly assumed in (1).

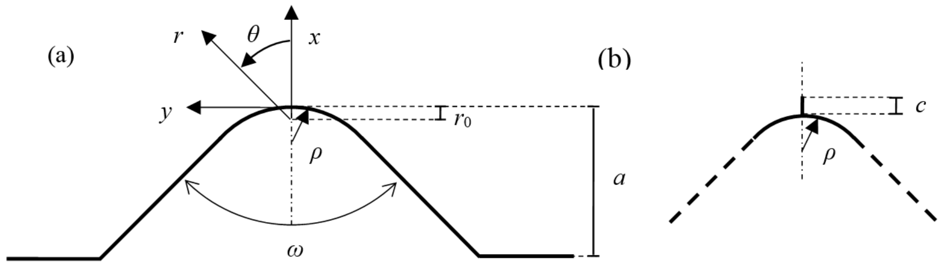


Fig. 1. (a) Rounded V-notch with Cartesian and polar coordinate systems; (b) crack of length *c* stemming from the notch tip.

*2.1 Stress field*

By assuming that the notch tip radius $\rho$ is sufficiently small in respect to the notch depth $a$, the stress field along the notch bisector could be expressed in polar coordinates as in [14]:

$$\sigma_\theta(r,0) = \frac{K_I^V}{(2\pi r)^{1-\lambda}}\left[1+\left(\frac{r_0}{r}\right)^{\lambda-\mu}\eta_\theta(0)\right] \tag{2}$$

where $K_I^V$ is the generalized stress intensity factor referring to a V-notch of depth $a$. The eigenvalues $\lambda$ and $\mu$, as well as the function $\eta_\theta(0)$, depend on the notch amplitude. The distance between the notch tip and the origin of the polar coordinates system is indicated as $r_0$ (Fig. 1):

$$r_0 = \frac{\pi-\omega}{2\pi-\omega}\rho \tag{3}$$

where $\rho$ is the notch root radius. For sharp cracks, the origin coincides with the notch root end, whatever the value of $\omega$. As the crack becomes blunter, $r_0$ increases for a given value of $\omega$.

*2.2 SIF function*

Let us consider a crack of length $c$ stemming from a blunted V-notch root (Fig. 1b). As far as the notch depth $a$ is sufficiently large with respect to $c$, the following SIF function was proposed [17],

$$K_I(c) = K_I^V \beta c^{\lambda-1/2}\left\{1+\left[\left(\frac{\beta}{\psi}\right)^{\frac{1}{1-\lambda}}\frac{r_0}{c}\right]^m\right\}^{\frac{\lambda-1}{m}} \tag{4}$$

The notch angle amplitude functions $\beta$ and $m$ are functions of $\omega$, whereas $\psi$ is equal to $1.12\sqrt{\pi}\,(1+\eta_\theta(0))\,(2\pi)^{\lambda-1}$. As said before, the parameter $m$ was fitted to improve the predictions obtained by carrying out an *ad hoc* finite element analysis [17]: Eq. (4) fulfils the asymptotic limits of very short and very long (but still small in respect to the notch depth) cracks, providing errors below 1% over the range $0° \le \omega \le 180°$.

## 3. FFM implementation

Supposing that failure takes place when the generalized SIF, $K_I^V$, reaches its critical conditions $K_{Ic}^{V,\rho}$, as expected for brittle structural behaviour, FFM can be implemented by inserting Eqs. (2) and (4) into system (1) and integrating. Note that $K_{Ic}^{V,\rho}$ represents the apparent generalized fracture toughness (i.e. the generalized fracture toughness measured as if the V-notch was sharp): it depends on the root radius, whereas the generalized SIF, $K_I^V$ does not, since it refers to a sharp V-notch. Analytical manipulations yield:

$$\begin{cases} \dfrac{K_{Ic}^{V,\rho}}{\sigma_u r_0^{1-\lambda}} = f(\overline{l}_c) \\ \dfrac{K_{Ic}^{V,\rho}}{K_{Ic} r_0^{1/2-\lambda}} = \sqrt{h(\overline{l}_c)} \end{cases} \tag{5}$$

where $\overline{l}_c = l_c / r_0$ , $\overline{c} = c / r_0$ and

$$f(\overline{l}_c) = \frac{(2\pi)^{1-\lambda}\,\overline{l}_c}{\left[\dfrac{\left(\overline{l}_c+1\right)^{\lambda}-1}{\lambda}\right] + \eta_\theta(0)\left[\dfrac{\left(\overline{l}_c+1\right)^{\mu}-1}{\mu}\right]}, \tag{6}$$

$$h(\overline{l}_c) = \frac{\overline{l}_c}{\displaystyle\int_0^{\overline{l}_c} \frac{\beta^2 \overline{c}^{2\lambda-1}}{\left\{1+\left[\left(\dfrac{\beta}{\psi}\right)^{\frac{1}{1-\lambda}} \dfrac{1}{\overline{c}}\right]^m\right\}^{\frac{2(1-\lambda)}{m}}} \mathbf{d}\overline{c}}. \tag{7}$$

Note that the presence of the parameter $m$ in Eq. (7) does not allow to get a closed form solution for the function $h$: the integral will be thus evaluated numerically according to a recursive adaptive Simpson quadrature formula. By equaling both expressions in (5) with respect to $K_{Ic}^{V,\rho}$, the system can be recast in the following form:

$$\begin{cases} \dfrac{K_{Ic}^{V,\rho}}{\sigma_u r_0^{1-\lambda}} = f(\overline{l}_c), \\ \dfrac{r_0}{l_{ch}} = \dfrac{h(\overline{l}_c)}{f^2(\overline{l}_c)}, \end{cases} \tag{8}$$

where $l_{ch} = (\mathrm{K}_{Ic} / \sigma_u)^2$ is the Irwin's length. Given the material, the notch amplitude $\omega$ and the notch root radius $\rho$, the value of the critical crack advancement $\overline{l}_c$ can be derived from the latter equation in (8). This value must then be inserted into the former equation to obtain the apparent generalized fracture toughness, $K_{Ic}^{V,\rho}$.
FFM results are presented in Fig. 2 as compared to the sharp V-notch case, according to which:

$$K_{Ic}^{V} = \lambda^{\lambda}\left[\frac{2(2\pi)^{2\lambda-1}}{\beta^2}\right]^{1-\lambda} l_{ch}^{1-\lambda}\sigma_u \tag{9}$$

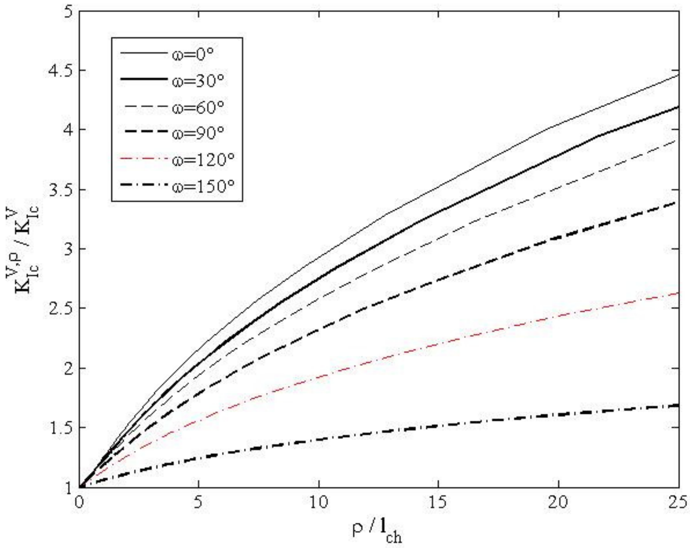


Fig. 2. FFM: dimensionless plot of the apparent generalized fracture toughness vs. notch root radius.

### 3.1. Comparison with experimental data

Let us now compare FFM predictions with experimental data available in the literature.

The firstly considered data set refers to three point bending tests on single edge U-notched specimens made of alumina [20]. The material properties are $K_{Ic} = 3.81$ MPa$\sqrt{\text{m}}$ and $\sigma_u = 290$ MPa, corresponding to $l_{ch}$~0.173 mm. FFM predictions, in terms of the apparent fracture toughness $K_{Ic}^{U} = K_{Ic}^{V=0,\rho}$ are plotted in Fig. 3, showing a very good matching. The calibration of the parameter $m$ in Eq. (5) allows to get a more flat behaviour in the neighbourhood of $\rho = 0$ with respect to the FFM approach put forward in [11].

The second data set is related to Charpy V notch specimens fabricated with as quenched AISI 4340 steel**:** they were heat treated by a 1200°C- 870°C step quenching procedure and tested under three-point bending. Although the V-notch angle related to Charpy samples is 45°, it was considered as a crack ($\omega$ = 0°) both in [18-19] and in [23]. The measured AISI 4340 steel properties were $K_{Ic} = 73.9$ MPa$\sqrt{\text{m}}$ and $\sigma_u = 1980$ MPa in [18-19] (resulting in $l_{ch}$~1.39 mm), and $K_{Ic} = 56.6$ MPa$\sqrt{\text{m}}$ and $\sigma_u = 2193$ MPa in [23] (corresponding to $l_{ch}$~0.667 mm), respectively. The material is of course less brittle than common ceramic materials, usually tested to investigate blunt V-notch brittle fracture. Indeed, it has been shown that some criteria based on a critical distance work rather well even for ductile metallic materials, when the final rupture is preceded by large-scale plastic deformations [24]. FFM theoretical predictions and experimental data are depicted in Fig. 3: predictions are reasonable except for the largest root radii ($\rho \geq 0.6$ mm), where the percent deviation grows up to nearly 27% for $\rho$ = 2.0 mm [18-19]. Being possible to exclude a change in the fracture mechanism to a more ductile one [25], it has to be concluded that in these cases the radius is not negligible with respect to the notch depth ($a$ = 2 mm): higher order terms should be taken into account in the asymptotic expansions (2) and (4).

Lastly, to face cases where $\omega > 0°$, the rounded-tip semi-circular bend specimens tested in [6] were considered. The material was PMMA, with the following measured properties: $K_{Ic} = 1.96$ MPa$\sqrt{\text{m}}, \sigma_u = 70.5$ MPa . Indeed, as observed in, [26] the value of $\sigma_u$ to be used is not the tensile strength obtained by testing plain specimens, since this last quantity can be strongly affected by the presence of micro-cracks/defects or crazing phenomena. As a matter of fact, $\sigma_u$ has to be fitted and may be equal to even twice the one related to plain specimens. In the present work, we assume $\sigma_u = 90$ MPa , this value being fitted on FFM predictions. As a result, we have $l_{ch}$~0.474 mm. Results are presented in Figs. 4 and 5 for what concerns $\omega = 30°$ and 150°. The agreement is satisfactory.

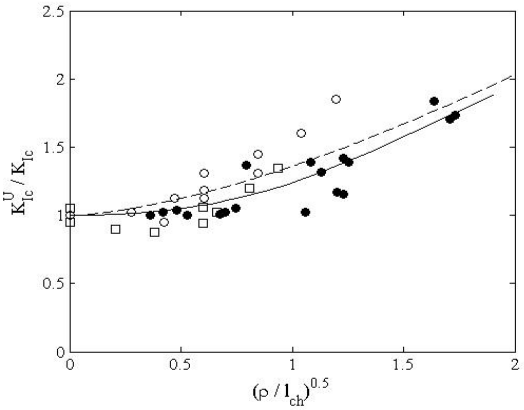


Fig. 3. Dimensionless apparent fracture toughness versus dimensionless square root of the notch root radius according to FFM (continuous line): experimental data refer to alumina [20] (filled circles), to as- quenched AISI 4340 steel tested in [18] (circles) and in [23] (squares). The dotted line refers to the model presented in [11].

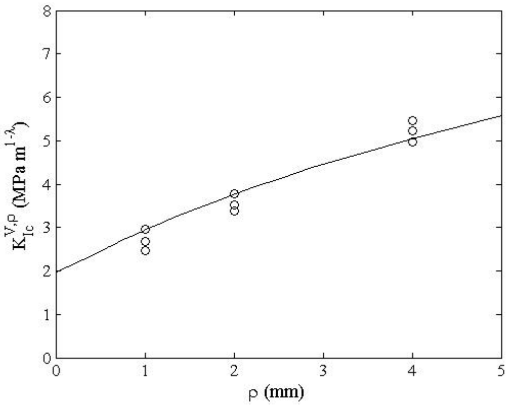


Fig. 4. FFM failure predictions on PMMA blunted V-notched samples [6] referring to $\omega = 30°$: theoretical results and experimental data

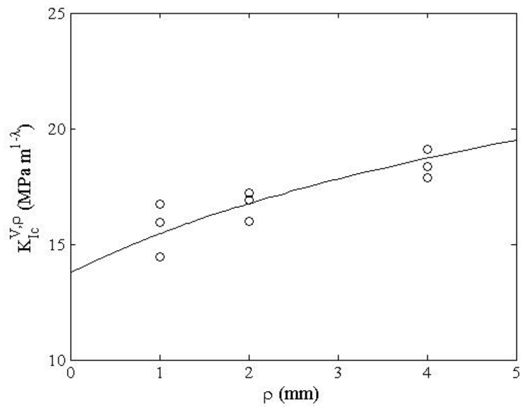


Fig. 5. FFM failure predictions on PMMA blunted V-notched samples [6] referring to $\omega = 150°$: theoretical results and experimental data.

## 4. Conclusions

The coupled stress and energy criterion of FFM was applied to investigate mode I brittle fracture on blunt V-notched elements, involving different notch radii and different materials. The implementation of the expression for the SIF stemming from a round V-notch proposed in [17] allowed to improve estimate accuracy. Indeed, theoretical predictions vary slightly for opening angles $0° \leq \omega \leq 90°$, due to the relative slow variation of the parameters governing the stress field and the SIF functions over this range.

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