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Numerical simulation of the fracturing processes in masonry arches

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Masonry arch structures, and, more generally, vaulted structures, are traditionally assessed using a well- established approach, such as linear elasticity or limit analysis, whereby system behaviour at the intermediate stage – which occurs when the material's tensile strength has been exceeded but the collapse mechanism has not yet formed – is disregarded. A more accurate interpretation requires a thorough analysis that can take into account the intermediate cracking stage and uses a constitutive law providing a closer approximation to the actual behaviour of the material. In this paper, an evolutionary fracturing process analysis for the stability assessment of masonry arches is presented. This method makes it possible to capture the damaging process that takes place when the conditions evaluated by means of linear elastic analysis no longer apply and before the conditions assessed through limit analysis set in. Furthermore, the way the thrust line is affected by the opening of cracks and the redistribution of internal stresses can be checked numerically. The results obtained with the described approach are compared with a numerical simulation performed with the finite element code Diana (TNO, The Netherlands) adopting discrete cracking with cohesive laws. Finally, the case study of the arch of the Mosca Bridge over the Dora River in Turin, Italy, is described.

1 Introduction

Albeit well established, the linear elastic and/or limit analysis approaches traditionally used to assess the behaviour of arch structures, and, more generally, vaulted structures, often leave the structural engineer at a loss [1]. While linear elastic analysis is applicable when the thrust line stays within the central kern to prevent tension arising and not to exceed the limits of elastic theory, limit analysis provides a thrust line between hinges, which lies everywhere within the masonry of the arch ring. Therefore, neither analysis can capture the intermediate damaging stage, which occurs during the loading process immediately before and after the conditions addressed by the previous schemes. There is a need for a more sophisticated computational method that uses a real constitutive law of the material and enables the cracking stages between elastic behaviour and final collapse to be taken into account [2]. Such a method is introduced in this work by adopting an elastic-softening schematisation for masonry and, more generally, for quasibrittle materials. As Hillerborg et al. [3] have shown, considering an elastic-softening constitutive law for the material is equivalent to using an elastic constitutive law coupled with a fracture process according to linear elastic fracture mechanics (LEFM) concepts.

The aim of this study was to address the problems associated with the softening and cracking process in masonry arches; to this end, a discrete model of the Mosca arch bridge over the Dora River in Turin was developed using beam finite elements, and a step-by-step loading process was applied in analogy with Castigliano's [4] analysis. Crack depth is determined according to LEFM concepts: it stabilises at a value that is a function of the axial force and its eccentricity and the geometric and mechanical characteristics of the cross section. Based on crack depth, it is possible to compute the rotational stiffness of an elastic hinge simulating the crack; then, the local stiffness matrix of each individual cracked element is updated and so is for the global stiffness matrix of the entire arch. The procedure takes into consideration the possibility of partial or total crack closure during loading increments subsequent to the one that generated the crack.

The results are then compared with two-dimensional finite element simulation of the arch, where the presence of cracks are accounted for by means of interface elements endowed with cohesive constitutive law.

2 Mechanism of crack opening and closure

The elastic-softening material is replaced by a material with a purely elastic behaviour with the possibility of cracks formation and extension. This hypothesis applies only to structures large enough to allow tension profiles to develop close to the crack tip, as foreseen by linear elastic fracture mechanics. As damage parameter we assume $\xi = \frac{a}{b}$, which correspond to the crack depth a normalised with respect to the arch section height b. As load parameter we consider the stress intensity factor K_I. The stress intensity factor is an amplification factor of the stress field when the loads are symmetrical relative to the crack (e.g., axial force and bending moment) that can be expressed as a function of the axial force F and of the eccentricity e [5]. The normalised axial force (Fig. 1a) can be related to the relative crack depth imposing the critical condition of the stress intensity factor $K_I = K_{IC}$ [6]. In addition, it is possible to obtain the stiffness matrix of a cracked beam finite element (fig.

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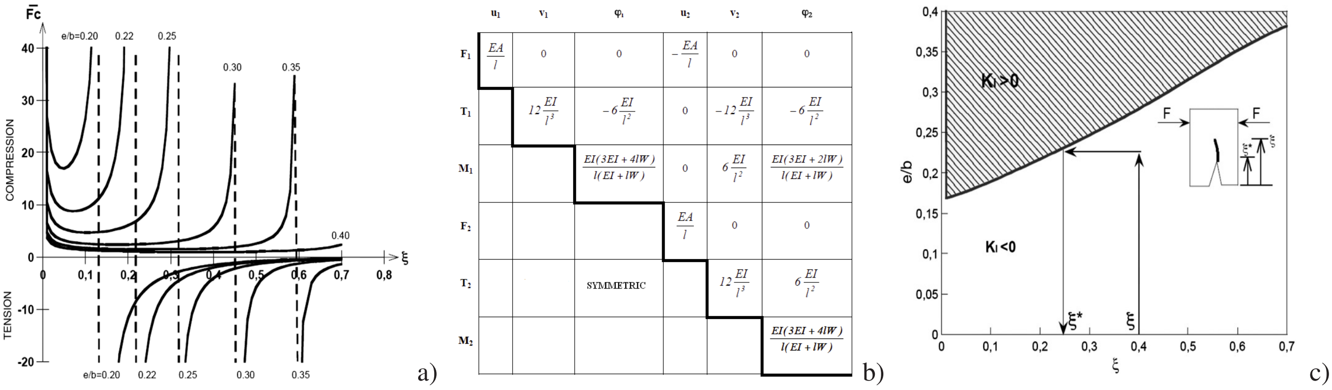

Fig. 1: Normalized eccentric compression as a function of crack depth ξ and relative eccentricity e/b **a**, stiffness matrix of cracked beam element **b** and crack closure curve **c**.

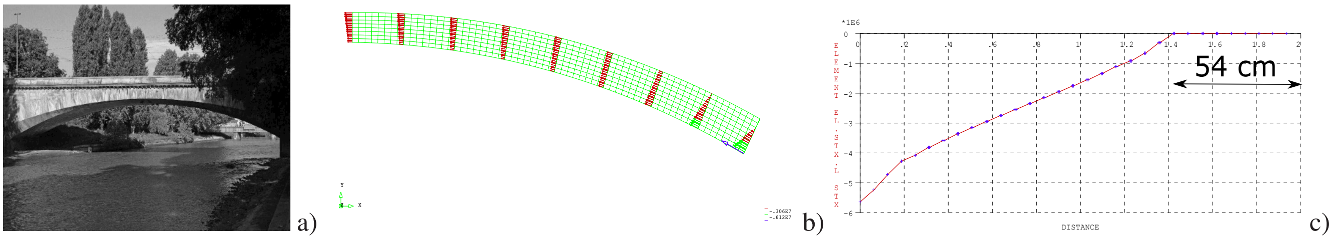

Fig. 2: Mosca Bridge in Turin (Italy) **a**, 2D plane stress mesh exploiting symmetry, with cohesive stresses at the interfaces **b** and cohesive crack stress at the springing **c**.

1b) equating the elastic work and the fracture work [7]. Once that the crack depth is calculated the elastic analysis is repeated, accounting for the stress redistribution. As a consequence, if the crack depth resorts to be in the negative stress intensity factor region, a virtual closure can be determinate through the crack closure curve (Fig. 1c).

3 Numerical results

An application of the aforementioned evolutionary method to a masonry arch bridge (Fig. 2a) designed by Carlo Bernardo Mosca (1792–1867) is described below and compared with a two-dimensional plane stress finite element simulation carried out with the commercial code Diana [8]. The Mosca Bridge, inaugurated in 1830, still spans the Dora River in Turin. This shallow stone arch bridge is unanimously recognised as a pioneering work and, at the same time, as a milestone in the history of stone bridge construction. The arch has been discretised according to the symmetry with the same number of voussoirs adopted for the adaptive analysis [7]. Specialised interface elements were used to represent the interaction of the interfaces. The load has been applied in steps, first considering the segments weight, then the weight of the filling, and last the live load. The 2D simulation confirm that a crack propagate in correspondence of the springing as the filling weight is added, and remain stable during live load application. The proposed finite element simulation confirm the results obtained by the adaptive analysis in terms of crack localisation both in space and in time. In addition, the obtained crack depth is 54 cm, in very good agreement with the results obtained with both the Castigliano analysis [4] and with the adaptive analysis [6].

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