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ORIGINAL PAPER

Fractal analysis and yule statistics for seismic prediction based on 2009 L'Aquila earthquake in Italy

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Abstract Earthquakes represent a five-dimensional set of events: time, three space dimensions, and magnitude. To study the clustering between the space dimensions and the magnitude, a comparative analysis of epicenter and magnitude distributions was performed between the earthquake time series in Italy and around the L'Aquila region. The *b* value analysis reveals a slightly stronger seismic activity in the L'Aquila region compared with the Italian seismicity. Analysis of epicenter distribution shows that earthquakes in the L'Aquila region tend to fill the plane more than the overall distribution. Fractal dimension of hypocenter distribution for the L'Aquila region is approximately twice the *b* value, fulfilling the relation $D=2b$. To investigate the relation between time and magnitude, an original interpretation was given for the variation of the b_Y value based on the Yule statistics. This approach reveals the relationship between earthquakes happened to propagate the existing faults and events creating new faults. The application of b_Y value analysis to the L'Aquila region highlights that clustering of earthquakes may be precursors of large earthquakes, and shows a reliable approach to predict violent earthquakes.

Keywords Fractal analysis · *b* value · Box-counting · Fractal dimension · Yule statistics · b_Y value

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Introduction

Earthquakes represent a five-dimensional set of events: time, three space dimensions, and magnitude (Aitouche et al. 2013) and the crust of the earth has been set up in a highly complex self-organized critical state in which the criticality manifests itself in many different geological phenomena with power law fractal distribution and dynamics (Bak et al. 1987; Kortas 2005). As a specific example, seismicity patterns appear to be complex and chaotic, yet there is order in the complexity (Yong et al. 1998; Telesca et al. 2006). Numerous evidences exist that the clustering of regional seismicity appears to have a fractal-like structure (Smalley et al. 1987; Ouadfeul and Alioaune 2012). Fractal analysis has provided us with a deep insight into the chaotic nature of distributions and geometry associated with earthquake clustering phenomena (Kagan 2007; Sarkheil et al. 2012). The well-known scaling law is the Gutenberg–Richter (GR) frequency–magnitude relationship, which implies a power law relation between the energy release and the frequency of occurrence of earthquakes:

$$\mathrm{Log}\,N = a - bm \tag{1}$$

where N is the cumulative number of earthquakes with magnitude $\geq m$ occurring in a specified area and time window. The constant a takes into account the seismic level of the examined region, and b, or b value, related to the stress state and fracture strength of the crustal medium in the region (Scholz 1968; Wyss 1973), represents the frequency relationship among earthquakes with different magnitudes, and it is also important for estimating the mean recurrence time between consecutive earthquakes (Corral 2006).

The b value is of great importance, because using the concepts of geometrical self-similarity it has been shown that b value can be directly related to the fractal dimension D of the active fault system involved in the seismic activity (Aki 1981), $D=3b/c$, where $c=1.5$ is the coefficient of the seismic moment vs. magnitude relation (Kanamori and Anderson 1975). It is worth noting that an analogous relationship has been established between the b value of acoustic emission data and the fractal dimension of the microcrack network from laboratory fracture tests (Bonnet et al. 2001; Carpinteri et al. 2009a).

Besides the above-described self-similarity approach, based on the statistical characterization of fault and crack patterns, both fractal dimension and b value can be derived or interpreted with different approaches. The fractal dimension can be determined by direct methods, such as box-counting technique (Falconer 1990) and two-point correlation algorithm (Grassberger 1993), whereas changes in b value in acoustic signal due to brittle fracture can be well captured by the Yule model (Carpinteri et al. 2008a), which was originally introduced in 1925 to explain the power law distribution of the number of species in a genus (Yule 1925), family or other taxonomic group, and generalized to explain power laws in several other systems, such as city sizes (Simon 1955), size of firms (Costantini et al. 2005), and spread of sexually transmitted diseases (Jones and Handcock 2003).

The goal of this paper is the collection of the fractal properties to refine the understanding of the earthquakes happened in the L'Aquila region in a different viewpoint. For this purpose, the b value of GR law, 2D and 3D fractal dimensions of epi- and hypocenter earthquake distributions are calculated considering the Italian seismicity from January to November 2009, as well as seismicity of the L'Aquila region. Finally, the Yule statistics is introduced to study the earthquake distributions, highlighting low clustering of earthquakes maybe precursors for large earthquakes.

The b values for entire Italian territory and L'Aquila region

We examined the catalogue of Italian earthquakes from January to November 2009 and, in the same period, the earthquake sequence occurred in the L'Aquila region, culminated in the violent earthquake of 6th April 2009 (ML 5.8, 42.334° N, 13.334° E, and 8.8 km depth). Data analysis was performed discarding earthquakes with magnitude <2. The b values for both time series are calculated using the remaining data (738 events in Italy and 324 in L'Aquila region), shown in Fig. 1.

It can be seen in Figs. 2 and 3, a fit of the GR law to the frequency–magnitude distribution yields $b=0.879$ for Italy and $b=0.845$ for the L'Aquila region. High b value indicates a large number of small earthquakes expected in regions of low stress and strength, whereas lower b value in the L'Aquila region indicates high stress concentration (Scholz 1968; Wyss 1973) and interseismic strain redistribution in the vicinity of the big earthquake rupture (Nakaya 2006).

Fractal dimensions for Italy and L'Aquila region

The fractal dimension D can be understood as a measure of geometric self-similarity. If number N vs. size r distribution of a set of objects has a fractal structure, the following relation will be obtained:

$$N \sim r^{-D} \tag{2}$$

where D is the fractal dimension (Turcotte 1992).

The concept of fractal or scale-invariant clustering will be applied to earthquakes in this section. Earthquake epicenters can be considered to be point events in space and time. To study fractal clustering, we must examine the distribution of events over a wide range of scales. There are several methods for estimating D, and one of the most often used is the box-counting method (Falconer 1990). Here, this technique is used to study the epicenter distribution of earthquakes in the entire Italian region as well as the L' Aquila region. The epicenters in the map were initially superimposed on a square grid with size r_0. The unit square r_0^2 was sequentially divided into small squares of size $r_i=r_0/2$, $r_0/4$, $r_0/8$, …. The number of squares $N(r_i)$ containing at least one epicenter was counted at each step. Fractal dimension D was determined from the slope of the log $N(r_i)$ vs. logr_i line.

The total number of square boxes covering the area, the number of boxes containing epicenters and the box side lengths (with longitude and latitude taken as rectangular coordinates; e.g., the box size $r=0.2°$ corresponds to 22 km) are listed in Tables 1 and 2 for the Italian region and L' Aquila region respectively.

Based on Tables 1 and 2 and Eq. (2), we calculated the fractal dimension D_2 of the epicenter distributions, which are 0.81 for Italy and 1.15 for L'Aquila region as shown in Figs. 4 and 5. Lower D_2 value for Italy means that epicenters are concentrated in some small areas, preferably along fault lines, whereas in the L'Aquila region the epicenters are more homogeneously distributed over the plane.

Geometrical probability of the earthquakes

We next relate the fractal distribution to probability. Covering the two-dimensional region by squares with side length r_i, the probability that a square will include an epicenter can be estimated as follows:

$$P(A) = \mathrm{Sum}(A)/\mathrm{Sum}(\mathrm{total}) \tag{3}$$

where Sum (A) is the summation area of the $N(r_i)$ squares containing epicenters:

$$\mathrm{Sum}(A) = N(r_i)r_i^2 \tag{4}$$

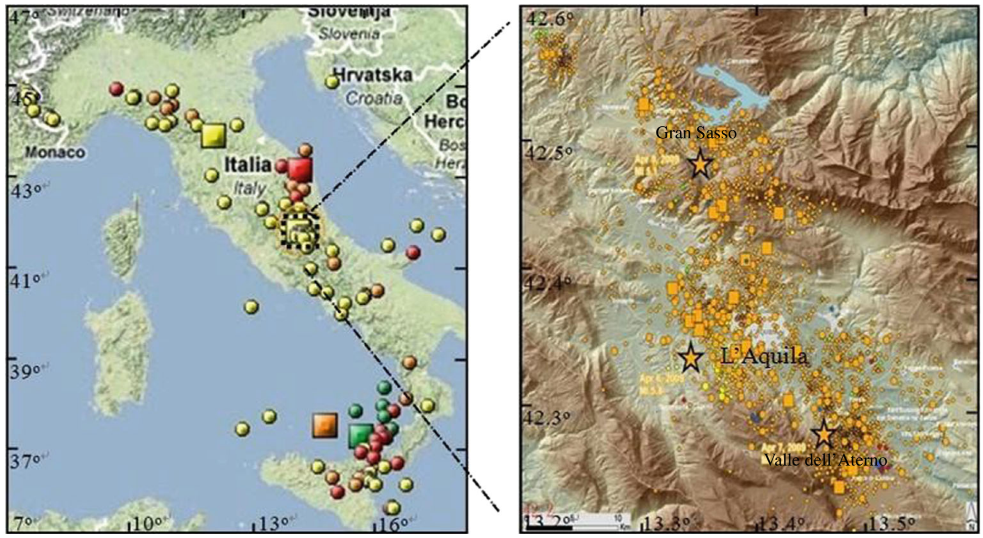

Fig. 1 Distribution of the earthquakes in Italy (*left*) and in L'Aquila region (*right*) from January to November of 2009

Sum(total) is the total area of the region under examination:

$$\mathrm{Sum(total)} = r_0^2 \tag{5}$$

In order to determine D_p, Eq. (2) can be rewritten as:

$$D_p = \frac{\log(N(r_i)/N_0)}{\log(r_0/r_i)} = \frac{\log N(r_i)}{\log(r_0/r_i)} \tag{6}$$

being N_0=1, and:

$$N(r_i) = \left(\frac{r_i}{r_0}\right)^{-D_p} \tag{7}$$

Inserting (4), (5), and (7) into (3) gives:

$$P(r_i) = \left(\frac{r_i}{r_0}\right)^{2-D_p} \tag{8}$$

where D_p is the fractal dimension of probability distribution $P(r_i)$, and would be exactly the exponent D_2 if it was a deterministic and not a statistical factor. Once again D_p value is a measure of epicenter clustering degree, varying from 0 to 2. Values close to zero indicate that epicenters are extremely clustered in small limited areas, whereas values equals to 2 mean that epicenters are densely spread in the whole of the considered area.

The fractal cluster analysis was taken for Italy, as well as for the L'Aquila region, covering the analyzed regions with the grid of boxes used for the box-counting technique. The D_p values obtained for Italy and L'Aquila region (shown in Figs. 6 and 7) are respectively 0.89 and 1.13, indicating that earthquakes are concentrated in some small areas of Italy, whereas locally, in the L'Aquila region, are more densely spread. Moreover, the $2-D_p$ geotechnical probability of faults occurrence confirm these results, indicating that there is a

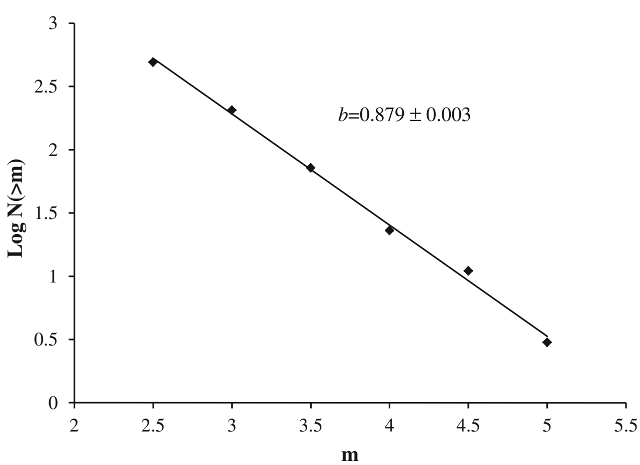

Fig. 2 Log (N>m) vs. m plot to determine the b value of Gutenberg-Richter Law for the whole Italy region

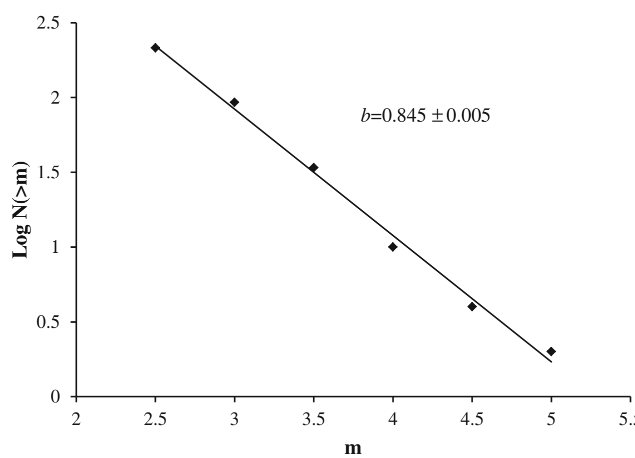

Fig. 3 Log (N>m) vs. m plot to determine the b value of the Gutenberg-Richter Law for L'Aquila region

Table 1 Box size and corresponding box number to determine the fractal dimensions D_2 and D_p for the whole Italy region

Box size (r)	0.2	0.4	0.8	1.6	3.2
Number of boxes $N(r)$	124	94	57	31	13
Total boxes N_i	3,300	810	195	56	16
Seismic cluster probability $P(r)$ (%)	3.76	11.60	29.23	55.36	81.25

greater probability of encountering clusters of faults in the whole of Italy than that in the L'Aquila region itself.

Volume fractal dimension of hypocenters for the L'Aquila region

Here, the box-counting method is used to calculate the fractal dimension D_3 of hypocenters distribution for the L'Aquila region. The cubic volume r_0^3 containing the hypocenters of the L'Aquila region was sequentially subdivided into smaller volumes with size $r_i = r_0/2$, $r_0/4$, $r_0/8$, ….

The fractal dimension D_3 is shown in Fig. 8. A value of D_3 close to 3 implies that earthquakes are distributed densely and homogeneously in the space, while a value close to 1 means that line sources are predominant (Aki 1981). In the analyzed case, $D_3 = 1.55$ indicates that most of the hypocenters tend to fill a two-dimensional domain. In fact, most of the earthquakes are associated with the violent earthquake of the 6th April, and distributed at about 8.8 km in depth.

The value of $D_3 = 1.55$ is about twice the b value=0.845 calculated in the same region. This agrees with the relation given by Aki (1981) and highlights how an increase in the state of stress in a region would be expected to cause clustering of seismicity.

Yule statistics and b_Y value analysis

Understanding and predicting the process of the violent earthquake is an extremely challenging scientific problem. Here, we will introduce the Yule statistics process to give a new point of view to analyze this problem.

The Yule process deals with a population of entities, each having a property, characterized by an integer numeric value. In the original work, "entities" are genera, and their properties are the number of species belonging to each genus. In Simon's (1955) work, entities are single word, and their properties are the number of times each word is used in a text. The Yule process describes a mechanism for generating a population, with successive addition of entities, and with a rule for incrementing the property value of existing entities. The key issue is that if the entity, whose property has to be modified, is chosen with a probability proportional to the size of this property, the resulting property distribution will tend to be a power law.

The Yule process will be adapted in this section to the fault-making process, since geodynamic processes of stress accumulation and interactions between fault segments are the main sources of complexity in the seismicity patterns of active tectonic regions, quantitative analysis of seismicity can help to reveal the nature of the seismotectonic regime of these regions. We are interested in the size distribution of the faults, which results from the processes of crack creation and propagation in time associated with elementary seismic events. Therefore, the faults play the role of genus, whereas the seismic events play the role of species. Each time a new earthquake occurs, this event may either result in the creation of a new elementary crack or in the propagation of an existing fault by a fixed amount. Thus, the genus size (number of species) in the original Yule process is here replaced by the fault size (the number of elementary seismic events that occurs to generate the fault).

In our study, we assume that at each time-step n, new earthquakes create new faults, thereby increasing the total number of faults by n, and m other earthquakes increase the size of the pre-existing faults which are selected in proportion to their sizes (or the number of seismic events that concurred to generate them). It is worth observing that the hypothesis that preferential attachment holds for a population of extending faults is physically sound: whatever the fault mode, the stress distribution is a key factor in determining the next stage of fault process. The stress redistribution due to the presence of a fault creates the conditions for the growth of the fault itself.

We denote by $p_{k,t}$ the fraction of faults composed by k earthquakes at time t, when the total number of faults in the study region is nt. Thus, the number of such faults is $p_{k,t}nt$. Now we compute the probability that the next fault advancement, corresponding to an earthquake, is added to the $i \cdot th$ fault of size k_i (i.e., generated by k_i earthquakes).

Table 2 Box size and corresponding box number to determine the fractal dimensions D_2 and D_p for the L'Aquila region

Box size (r)	0.005	0.01	0.02	0.04	0.08
Number of boxes $N(r)$	256	164	81	30	11
Total boxes N_i	4,200	1,050	263	58	16
Seismic cluster probability $P(r)$ (%)	6.09	15.62	30.80	51.72	68.75

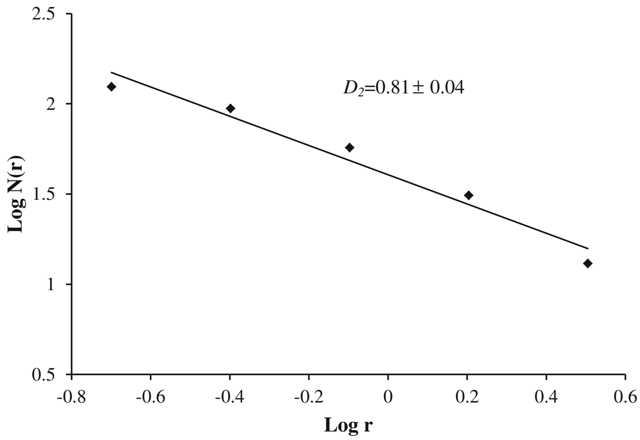

Fig. 4 Log (N(>r)) vs. log(r) plot to determine the fractal dimension D_2 for Italian region

This probability is given by $\frac{k_i}{\sum_i k_i}$, being $\frac{1}{\sum_i k_i}$ as a normalizing factor. The sum $\sum_i k_i$ is simply the total number of earthquakes till the time t, which is equal to $t(m+n)$ in this case. Furthermore, between the time t and the subsequent time instant (t+1), m other new advancements are added to the existing faults, so the probability that size k_i gains a new fault advancement during this interval is $\frac{mk_i}{(m+n)\cdot t}$. The total expected number of faults of size k that gain a new advancement in the same interval is

$$\frac{mk}{t(m+n)}p_{k,t}tn = \frac{mn}{m+1}kp_{k,t} \tag{9}$$

It can be note that the number of faults of size k will decrease on each time step by exactly this number, since by increasing their size by one they become faults with size (k+1). At the same time the number increases because of the faults

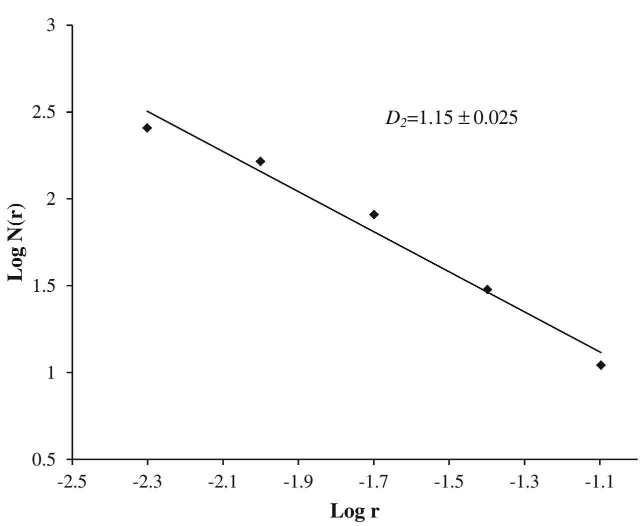

Fig. 5 Log (N(>r)) vs. log(r) plot to determine the fractal dimension D_2 for L'Aquila region

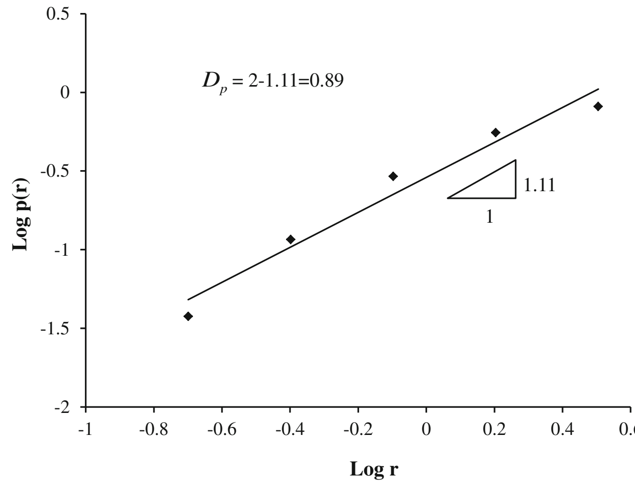

Fig. 6 Geometrical probability dimension D_p for the whole Italy region

of size ($k-1$) also increase their size by one. Thus we can write a master equation for the new number $p_{k,t+1}n(t+1)$ of faults of size k:

$$n(t+1)p_{k,t+1} = ntp_{k,t} + \frac{mn}{m+n}\left[(k-1)p_{k-1,t} - kp_{k,t}\right] \tag{10}$$

the only exception to this equation is for k=1, which instead obeys to:

$$n(t+1)p_{1,t+1} = ntp_{1,t} + n - \frac{mn}{m+n}p_{1,t} \tag{11}$$

since by definition exactly n new such earthquakes appear on each time step. Now we ask what form the distribution of the fault sizes takes in the limit of long time. As $t\rightarrow\infty$, the distribution approaches to a fixed value:

$$p_k = \lim_{t\rightarrow\infty} p_{k,t} \tag{12}$$

independent of time t. So we can get p_1,

$$p_1 = \frac{m+n}{2m+n} \tag{13}$$

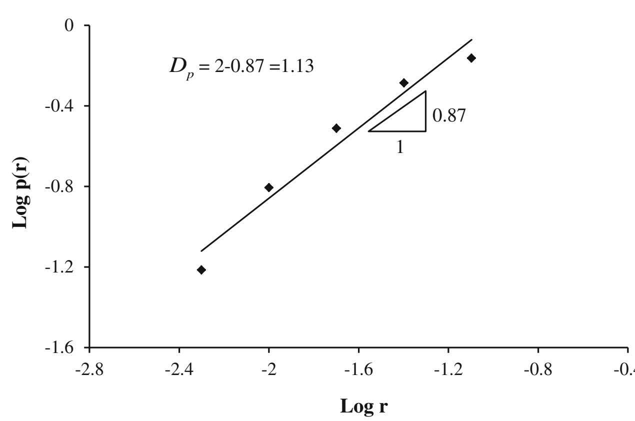

Fig. 7 Geometrical probability dimension D_p for L'Aquila region

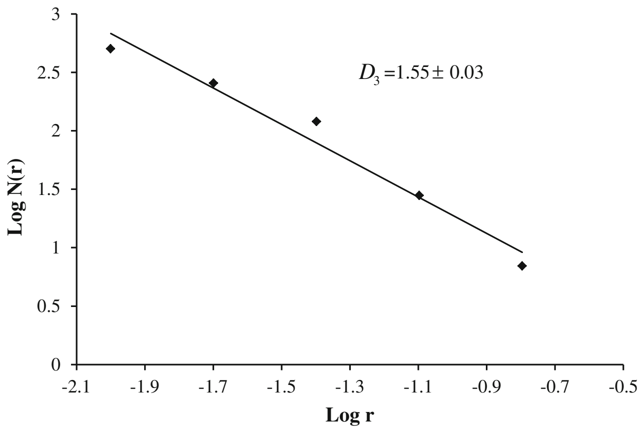

Fig. 8 Volume fractal dimension D_3 for L'Aquila region

Also the solution for p_k,

$$p_k = \frac{k-1}{k+1+n/m} p_{k-1} \tag{14}$$

whose iterating gives:

$$p_k = \frac{(k-1)(k-2)\cdots 1}{(k+1+n/m)(k+n/m)\cdots(3+n/m)} p_1 \tag{15}$$

By substituting Eq. (13) into the above equation, we obtain:

$$p_k = (1+n/m)\frac{(k-1)(k-2)\cdots 1}{(k+1+n/m)(k+n/m)\cdots(2+n/m)} \tag{16}$$

This can be simplified further by making use of a handy property of the Γ function: $\Gamma(x)=(x-1)\times\Gamma(x-1)$, and $\Gamma(1)=1$, thus :

$$p_k = (1+n/m)\frac{\Gamma(k)\Gamma(2+n/m)}{\Gamma(k+2+n/m)} = (1+n/m)B(k,2+n/m) \tag{17}$$

where B (a, b) is again the beta function:

$$B(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b) \tag{18}$$

Since the beta function has a power law tail, for larger values of either of its arguments, it follows a power law. For instance, for large a and fixing b, $B(a,b)\approx a^{-b}$. From this point p_k shows a power law tail with an exponent:

$$\beta = 2 + \frac{n}{m} \tag{19}$$

The ratio n/m can be seen as the ratio of earthquakes to create new faults and to propagate existing faults. It is reasonable to assume that the size of the fault is directly proportional to the number k of earthquakes acting on it. So this can reflect the probability density of the fault sizes.

On the other hand, the probability density function of fault sizes has the following form (Carpinteri et al. 2008b, 2009a, 2009b):

$$p(L) \propto L^{-2b-1} \tag{20}$$

therefore, based on Eqs. (19) and (20), the b_Y value equation is:

$$b_Y = \frac{1}{2}\left(1 + \frac{n}{m}\right) \tag{21}$$

If the ratio n/m decreases, the b_Y value also decreases (Carpinteri et al. 2008a). This means that a process which propagate existing faults (with a large part of earthquakes occurred in the same place to propagate the existing faults, rather than to create new faults) corresponds to lower b_Y values, whereas a process which create new faults (with a large part of earthquakes to create new faults) corresponds to higher b_Y values. Thus, the evolution of b_Y values about the earthquakes can be interpreted as a progressive increase of fault clustering. As an example of application, we consider the earthquakes in the L'Aquila region compared with the catalogue of Italian earthquakes with magnitude>2.0 from the beginning of January to the end of November 2009. Here, (m+n) is the total number of earthquakes occurred in the whole of the Italy. As explained above, m can be seen as the earthquakes happening in the L'Aquila region to propagate the existing faults, and n can be considered as the events occurring in other regions of Italy to play the role of creating new faults.

Figure 9 shows the fluctuations of the b_Y values from January to November 2009. From the variation of b_Y values, and considering the biggest earthquakes shown in the three months (April, July, and August), we can draw some evaluations. From January to April, b_Y value decreases continuously from 2.1 to 0.91, providing a precursory warning of a big event. Starting from January, the b_Y value is equal to 2.1 that corresponds to $n/m\approx 3.0$ in Eq. (21). This means one quarter of the total number of earthquakes in January happened in the L'Aquila region. When b_Y becomes smaller, a greater number of earthquakes occurred there. In particular, the condition b_Y= 1.0 corresponds to n/m=1.0, which means that about one half of the total number of earthquakes happened in Italy were localized in the L'Aquila region, and they were foreshocks of the great event (L'Aquila, ML 5.8). On the other hand, if b_Y value increases sharply (e.g., b_Y=2.5 in August), some large earthquakes occurred in other places, such as the one in Adriatico (ML 5.8, 42.261° N, 16.766° E, 2.0 km depth), see Fig. 9.

In Fig. 9, it is shown that excluding the first b_Y obtained in January, the other b_Y values calculated between February and July are comprised between 1.50 and 0.91. This range of values that is not very different from that obtained by the GR relationship Eq. (1) (see "Discussion and conclusion" section) reflect the criticality of damage process in the L'Aquila region as demonstrated by Carpinteri et al. (2008a, 2009a).

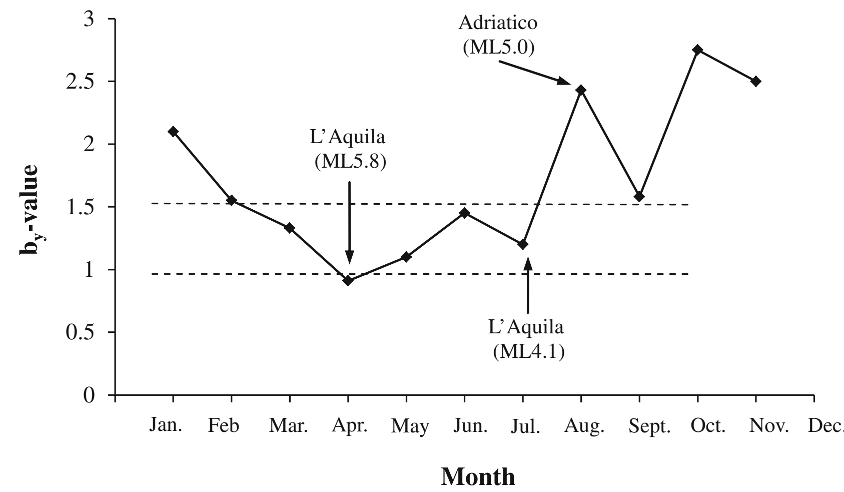

Fig. 9 Yule b_Y values calculated on eleven months from January to November taking into account all earthquakes in L'Aquila region (m) and in other Italian regions (n)

In order to verify this new proposed prediction method, the method was applied to another seismic case in China. We can illustrate this procedure with the sub-catalogue of China (latitude 15~35° and longitude 110~140°) provided by the China Earthquake Data Center (CEDC) for the year 2008, containing 1,829 events with $M \geq 2.5$. As shown in Fig. 10, we can see that the similar conclusion with that in the L'Aquila region can be obtained also for the Chinese earthquake case.

From this point of view, the analysis of b_Y value, during a certain period and in a specific region, represents a powerful method to study the relation between time and earthquakes magnitude, and provide a reasonable method to predict violent events.

Discussions and conclusion

Based on the catalogue of Italian earthquakes from January to November 2009, fractal concepts are delineated to study the earthquakes statistical distribution. The b values calculated by the GR statistics are equal to 0.879 and 0.845 for the Italy and L'Aquila region, respectively. This reveals a high seismic activity in L'Aquila region, compared with the Italian seismicity.

The fractal dimension D_2 is also calculated to analyze the epicenter distribution of the two regions. Notice that the lower surface fractal dimension 0.81 (Italy), corresponds to the irregular epicenter distribution, while the higher surface fractal dimension 1.15 (L'Aquila), indicates that the events are distributed more homogeneously over a two-dimensional surface.

Also, D_p, a measure of clustering degree of the epicenters, is calculated. The value of D_p for the entire Italian region is smaller, indicating a larger clustering of earthquakes in Italy than that in the L'Aquila region. As a matter of fact, more than forty percent of the events are clustered in the L'Aquila region. The distributions of hypocenters in three-dimensional space are shown in Figs. 11 and 12.

Along with the epicenter fractal dimension D_2, fractal dimension D_3 is calculated to study the hypocenter

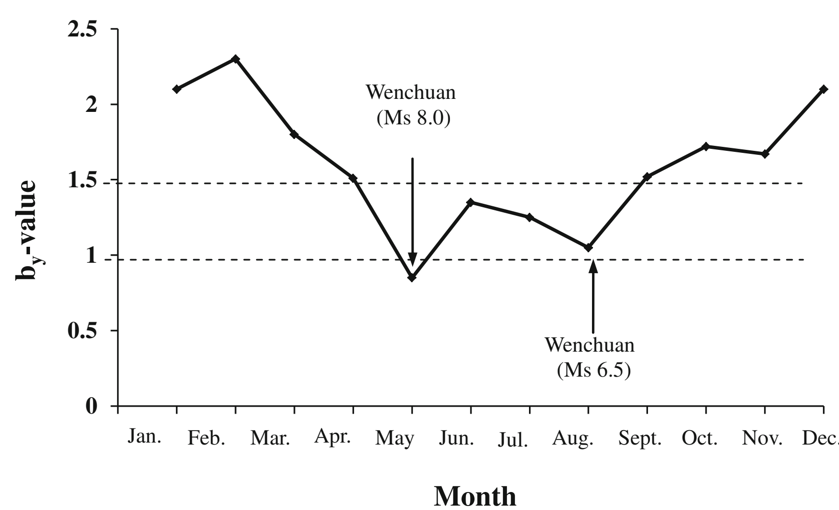

Fig. 10 Application of the Yule b_y values as earthquake precursors based on the seismic data in China

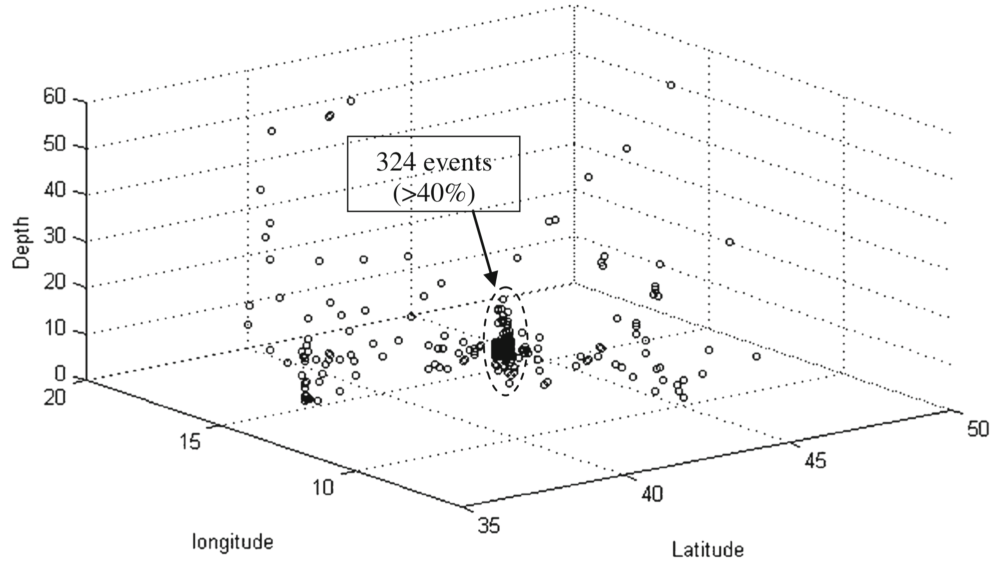

Fig. 11 Distributions of earthquakes hypocenters in the whole Italy region (the *dashed ellipse* comprises the L'Aquila region)

distribution of the L'Aquila region. The value of D_3=1.55 is about twice the b value, 0.845 represented in Fig. 3. This agrees with the relation given by Aki (1981) between the b value and the fractal dimension D, D=3b/c, where c=1.5 was defined by Kanamori (1978) and Hanks and Kanamori (1979). Similar results were obtained by King (1983) by means of a three-dimensional fractal-faulting model. Moreover, the fractal dimension D_3 indicates that most of the earthquakes are concentrated in some special region, shown by dashed ellipse in Fig. 12.

Finally, the Yule statistics has been introduced to study the earthquake distribution. Generally, the $b_Y$value fluctuates with time. In particular, b_Y=1.0 corresponds to the ratio n/m=1.0 from Eq. (21), which means that one half of the earthquakes occurred in the considered area have been foreshocks of a great event. This was well testified by the application of this method for the analysis the L'Aquila region. As it is shown in Fig. 9, the b_Y value decreased continuously from January to April 2009 and the values varied from 2.1 to 0.91, culminating in the violent earthquake of 6th April (ML 5.8).

Considering the possibility of forecasting earthquakes by Yule statistics, some properties can be reported here. Firstly, the variation of b_Y reflects the fracture process of the faults induced by the earthquakes. Secondly, b_Y represents the relation between the newly forming faults and preexisting faults. Finally, b_Y makes use of the appearance of foreshocks to warn of a violent event. Therefore the b_Y value variation could be considered as a reasonable method to forecast the earthquake conditions.

In summary, the fractal analysis provides insight to analyze the epicenter, hypocenter, and cluster degree distributions of earthquakes. Also, b_Y value in Yule statistics, reflecting the

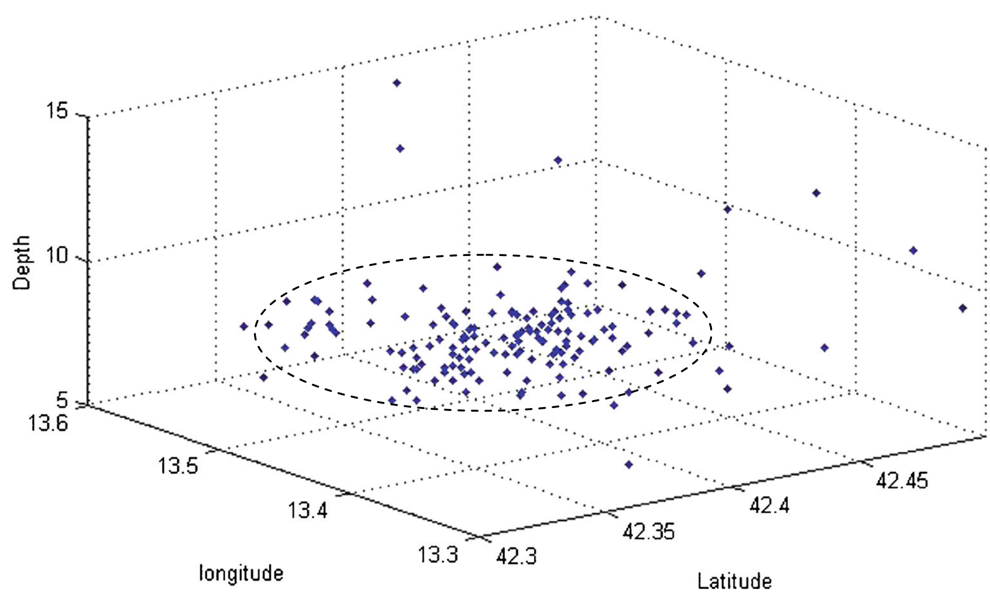

Fig. 12 Distributions of hypocenters of earthquakes in the L'Aquila region (the *dashed ellipse* shows the main area of the hypocenters)

relation between time and magnitude, provides a reasonable method for seismic prediction.

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