SSiEG – Summer School in Enumerative Geometry
SISSA, Trieste, July 3–7 2017

	Monday 7/3	**Tuesday 7/4**	**Wednesday 7/5**	**Thursday 7/6**	**Friday 7/7**
9:00-9:30	Registration				
9:30-11:30	*Manolache I*	*Manolache II*	*Manolache III*	*Manolache IV*	*Manolache V*
	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
11:30-13:00	*Liu I*	*Liu II*	*Liu III*	*Liu IV*	*Liu V*
	Lunch	Lunch	Lunch	Lunch	Lunch
15:00-15:30	*Barbieri*	Exercise session	*Sauvaget*	Exercise session	
15:30-16:00	*Roquefeuil*	Exercise session	*Van Zelm*	Exercise session	
	Coffee break	Coffee break	Coffee break	Coffee break	Coffee break
16:30-17:00	*Bott*	Exercise session	*Sertoz*	Exercise session	
17:00-17:30	*Bychkov*		*Mukherjee*		
20:00			Conference dinner		

Abstracts - Courses

- **Cristina Manolache**

  Boundary contributions to enumerative invariants

  Abstract. Enumerative questions have a very long history in Mathematics and have been revolutionised in the nineties with the construction of the moduli space of stable maps and the machinery allowing us to integrate on these very singular spaces. By now we have several moduli spaces on which we can integrate, but the invariants we obtain are very often not enumerative. My goal is to investigate how different compactifications of moduli spaces of curves on a given variety give rise to different invariants. I will first give several examples of compactifications such as stable maps, reduced maps, and quasi-maps. Then, I will explain virtual classes and why it is difficult to see how components of a moduli space contribute to virtual classes. In the end, I will give examples of boundary contributions to enumerative invariants. More precisely, I will discuss the relationship between Gromov–Witten invariants and reduced invariants (or Gopakumar–Vafa invariants) and the relationship between Gromov–Witten invariants and quasi-map invariants.

- **Chiu-Chu Melissa Liu**

  Gromov-Witten invariants, Fan-Jarvis-Ruan-Witten invariants, and Mixed-Spin-P fields

  Lecture 1: Moduli of stable maps and Gromov-Witten invariants
  We will introduce stable maps and their moduli, and define Gromov-Witten (GW) invariants of non-singular complex projective varieties in terms of the fundamental class (in nice cases) or the Behrend-Fantechi virtual fundamental class (in the general case). We will give explicit examples when the target is a projective space or a hypersurface in a projective space.

  Lecture 2: Stable maps with fields
  We will introduce moduli of stable maps to a projective space with fields, and describe an alternative definition of GW invariants of Calabi-Yau hypersurfaces in projective spaces in terms of the Kiem-Li cosection localized virtual cycle. This is an exposition of "Gromov-Witten invariants of stable maps with fields" by Huai-Liang Chang and Jun Li.

  Lecture 3: Witten's top Chern class and Fan-Jarvis-Ruan-Witten invariants
  The Fan-Jarvis-Ruan-Witten (FJRW) invariants of a quasi-homogeneous polynomial are virtual counts of solutions to the Witten's equation associated to the quasi-homogeneous polynomial. We will describe an algebraic definition of FJRW invariants (in the "narrow" sector) of a Fermat polynomial $x_1^n + ... + x_n^n$ in terms of the Kiem-Li cosection localized virtual cycle. This is a special case of the general construction in "Witten's top Chern class via cosection localization" by Huai-Liang Chang, Jun Li, and Wei-Ping Li.

  Lecture 4: Landau-Ginzburg/Calabi-Yau correspondence
  We will describe the Landau-Ginzburg/Calabi-Yau correspondence relating the FJRW invariants the Fermat polynomial $W_n = x_1^n + ... + x_n^n$ and GW invariants of the Calabi-Yau hypersurface $\{W_n = 0\}$ in P^n.

  Lecture 5: Mixed-Spin-P Fields
  We will describe the theory of Mixed-Spin-P (MSP) fields which interpolates the FJRW theory of the Fermat polynomial W_n and GW theory of the Calabi-Yau hypersurface $\{W_n = 0\}$ in P^n. We will present some computations of GW invariants and FJRW invariants using localization on MSP moduli spaces. This is based on joint work with H.-L. Chang, J. Li, and W.-P. Li.

Abstracts - Contributed talks

- **Anna Barbieri**

  A convergence property for a deformation of Joyce generating functions

  Abstract. A generating function f for the generalized Donaldson-Thomas invariants on a (abelian) category was introduced by Joyce in 2006. It is a continuous and holomorphic formal sum whose coefficients satisfy recursive laws, and it is known to be well-posed and convergent only in the case of a finite abelian category. One of the open questions in Joyce's article is to study the convergence of this function and to extend the result to a generic triangulated category. In a joint work with J.Stoppa, we deform f into a formal power series $f_{\mathbf{s}}$ which is well-defined also in the case of triangulated categories. We study and prove the convergence of the graded (with respect to the underlying lattice $K_0(\mathcal{C})$ components of a deformation of $f_{\mathbf{s}}$.

- **C.J. Bott**

  Mirror Symmetry for K3 Surfaces With Non-symplectic Automorphism

  Abstract. Mirror symmetry is the phenomenon originally discovered by physicists that Calabi-Yau manifolds come in dual pairs, each of which produces the same physics. Mathematicians studying enumerative geometry became interested in mirror symmetry around 1990, and since then, mirror symmetry has become a major research topic in pure mathematics. There are several constructions in different situations for constructing the mirror dual of a Calabi-Yau manifold. It is a natural question to ask: when two different mirror symmetry constructions apply, so they agree? We consider two mirror symmetry constructions for K3 surfaces known as BHK and LPK3 mirror symmetry, the first inspired by the Landau-Ginzburg/Calabi-Yau correspondence, and the second more classical. In particular, for certain K3 surfaces with a purely non-symplectic automorphism of order n, we ask if these two constructions agree. Results of Artebani-Boissière-Sarti and Comparin-Lyon-Priddis-Suggs show that they agree when n is prime. We will discuss new techniques needed to solve the problem when n is composite.

- **Boris Bychkov**

  Degrees of the strata of Hurwitz spaces

  Abstract. Let $\mathcal{H}_{0;k_1,\dots,k_m}$ be the space of meromorphic functions of degree $k_1 + \ldots + k_m$ on genus 0 algebraic curve with the numbered multiplicities of the preimages $k_1, \dots, k_m$ of the point ∞ and the zero?s sum of the finite critical values. The closure in $P\overline{\mathcal{H}}_{0;k_1,\dots,k_m}$ of the set of functions having prescribed ramifications forms the discriminant stratum. The degree of the stratum is the intersection index of its Puancare dual class with the complementary degree of the first Chern class of the tautological line bundle. I will talk about the certain method of computation of the degrees of the strata of small codimension. As a consequence we will have a closed formulae for some series of so called double Hurwitz numbers and some new relations on the generating series for integrals of ψ-classes over the moduli space of stable genus 0 curves with marked points. My talk will follow the paper arXiv:1611.00504v1.

- **Ritwik Mukherjee**

  Counting curves in a linear system with up to eight singular points
  Abstract. Consider a sufficiently ample line bundle $L \to X$ over a compact complex surface X. We obtain an explicit formula for the number of curves in the linear system $H^0(X, L)$ that pass through the appropriate number of generic points, having δ nodes and one singularity of codimension k, provided $\delta + k \leq 8$.

- **Alexis Roquefeuil**

  Lagrangian cone in Gromov–Witten theory
  Abstract. We will introduce the notion of Lagrangian cone associated to the potential function of genus 0 Gromov–Witten invariants, as an enrichment of the Frobenius structure associated to the quantum cohomology. We will describe its geometry while relating it to properties of Gromov–Witten theory. We will then compare the cone to the quantum connection/D–module.

- **Audrien Sauvaget**

  Tautological rings of spaces of r-spin structures with effective cycles
  Abstract. The recent developments in the study of moduli spaces of holomorphic differentials have allowed to compute the Poincaré-dual classes of loci of differentials with prescribed singularities. This classes can be expressed using the standard tautological classes of the moduli space of curves. An open conjecture by Pandharipande and Farkas gives closed formulas for these classes in terms of Chiodo's classes of moduli of r-spin structures. We will explain how this result would allow to describe a family of subrings of the cohomology rings of the moduli spaces of r-spin structures obtained by enriching the classical tautological rings with classes of loci of effective spin structures defined by Polischuk

- **Emre Sertöz**

  Enumerative geometry of theta characteristics
  Abstract. Deformations of individual theta characteristics have been studied extensively. But geometric questions regarding the relationship of multiple theta characteristics require a different take on the existing moduli spaces. We define the appropriate compactification of multiple spin curves which then allow us to study problems of classical nature pertaining to pairs of theta hyperplanes. We answer the following questions via divisor computations and degeneration arguments on these compactified moduli spaces: how many fibers of a given one-parameter family of curves admit pairs of theta hyperplanes sharing a common point of contact? If a curve admits a pair of theta hyperplanes sharing a common point of contact, does it admit others? How many points of common contact are there on this curve?

- **Jason van Zelm**

  Nontautological bielliptic cycles
  Abstract. Tautological classes are geometrically defined classes in the Chow ring of the moduli space of curves which are particularly well understood. The classes of many known geometrically defined loci were proven to be tautological. A bielliptic curve is a curve with a 2-to-1 map to an elliptic curve. In this talk we will build on an idea of Graber and Pandharipande to show that the closure of the locus of bielliptic curves in the moduli space of stable curves of genus g is non-tautological when g is at least 12.