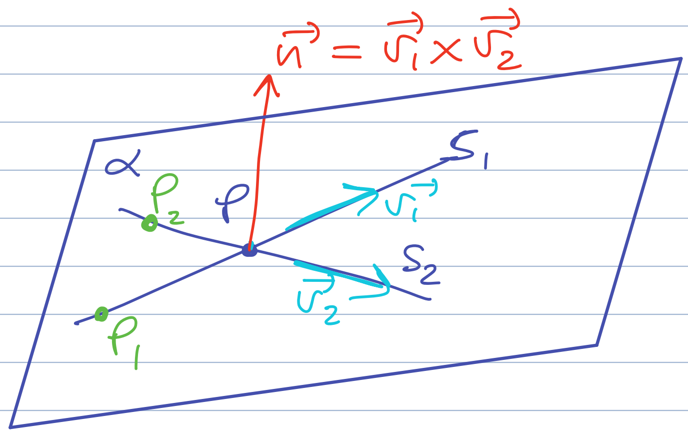
ex: $S_1: \begin{cases} x=-1 \\ z=2 \end{cases}$ $S_2: \begin{cases} 2x+y-2z=-6 \\ y+z=2 \end{cases}$

2 coplanar lines meeting at one point P.
We need to find equations for the plane α containing them:



To find the equations for α we can:

i) Find P explicitly, as well as $\vec{v_1}$ and $\vec{v_2}$, and write param. eq'ns.

ii) Find P explicitly, and $\vec{n} = \vec{v_1} \times \vec{v_2}$, and write cartesian equation.

iii) Find P, and other 2 points, $P_1 \in r_1$ but $P_1 \notin r_2$, and $P_2 \in r_2$ but $P_2 \notin r_1$, and find the equation of the plane thru the 3 points P, P_1, P_2.

HOMEWORK: try all 3 methods!

i) $P = s_1 \cap s_2$

from the equations of s_1: $P = (-1, y, 2)$

and from the equations of s_2: $y = 0$

$\Longrightarrow P = s_1 \cap s_2 = (-1, 0, 2)$

$\vec{v}_1$ = direction vector of s_1

$$\vec{v}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$\vec{v}_2$ = direction vector of s_2

$$\vec{v}_2 = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

Parametric equations for α are:

$$\begin{cases} x = -1 + 0 \cdot s + 3 \cdot t \\ y = 0 + 1 \cdot s - 2 \cdot t \\ z = 2 + 0 \cdot s + 2t \end{cases} \qquad s, t \in \mathbb{R}$$

$$\begin{cases} x = -1 + 3t \\ y = s - 2t \\ z = 2 + 2t \end{cases} \qquad s, t \in \mathbb{R}$$

ii) $P = (-1, 0, 2)$

$\vec{n}$ normal vector to α: $\vec{n} = \vec{v}_1 \times \vec{v}_2$, with

$\vec{v}_i$ = direction vector of s_i:

$$\vec{v}_1 \times \vec{v}_2 \text{ "=" } \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$$

$\Longrightarrow \alpha$ has equation $2x + 0y - 3z = d$

$2x - 3z = d$

$P \in \alpha \Longrightarrow 2\cdot(-1) - 3\cdot 2 = d$
$d = -8$

$\Longrightarrow \alpha$ has equation $2x - 3z = -8$

iii) $P = (-1, 0, 2)$

All points of s_1 are of the form $(-1, t, 2)$, $t \in \mathbb{R}$
$\Longrightarrow$ to find a point $P_1 \neq P$ it is enough to choose $t \neq 0$, for example $P_1 = (-1, 1, 2)$.
Note that $P_1 \in s_1$ but $P_1 \notin s_2$.

Now to find a point $P_2 \in s_2$ with $P_2 \notin s_1$, choose for example $z = 0 \rightsquigarrow y = 2$
$\rightsquigarrow 2x + 2 - 0 = -6$
$x = -4$

$P_2 = (-4, 2, 0)$

The plane α thru P, P_1, P_2 is of the form
$ax + by + cz = d$.

$P \in \alpha \Longrightarrow -a + 0\cdot b + 2\cdot c = d$
$P_1 \in \alpha \Longrightarrow -a + b + 2c = d$
$P_2 \in \alpha \Longrightarrow -4a + 2b + 0\cdot c = d$

and we get the system:
$$\begin{cases} -a + 2c = d \\ -a + b + 2c = d \\ -4a + 2b = d \end{cases}$$

Solving the system we get again $\alpha: -2x + 3z = 8$ ✓