Linear algebra and geometry a.y. 2023-2024

Mixed quizzes on affine geometry: lines and planes, distances

1. Let A, B and C be the points $(0,0,1)$, $(0,1,0)$ and $(1,0,0)$ in $\mathbb{R}^3$.
   Find the correct statement.
   (a) The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel.
   (b) The points A, B and C are collinear.
   (c) There is no plane containing the points A, B and C.
   (d) The triangle with vertices A, B and C is equilateral.

2. Let $A = (1,2,3)$, $B = (2,3,3)$, $C = (1,3,4)$ and $D = (2,4,4)$ be points in $\mathbb{R}^3$.
   Find the correct statement.
   (a) The quadrilateral $ABDC$ is a parallelogram.
   (b) The points A, B, C and D are not coplanar.
   (c) The quadrilateral $ABCD$ is a rectangle.
   (d) The points A, B, C and D are collinear.

3. Consider the line $\ell : \{x = y = z\}$ and the plane $\pi : \{x + y + z = 3\}$.
   Find the correct statement.
   (a) $\ell \parallel \pi$.
   (b) $\ell \perp \pi$.
   (c) $\ell \cap \pi = \emptyset$.
   (d) $\ell \subset \pi$.

4. Let $r : \{x = y + 1 = z + 2\}$ and $s : \{2x = 2y + 2 = 3z + 2\}$ be two lines in S_3.
   Find the correct statement.
   (a) The line s is orthogonal to the plane $x - y = 0$.
   (b) The two lines are coplanar.
   (c) The two lines are skew.
   (d) The line r is orthogonal to the plane $y + 1 - x = 0$.

5. Consider the line $r : \{x - y = z + 2y + 3 = 0\}$ and the family of planes $\pi_h : \{x + y + hz = 0\}$, where $h \in \mathbb{R}$ is a real parameter.

   Find the correct statement.

   (a) The line r and the plane π_h gave non-empty intersection for all values of h.
   (b) The line r and the plane π_h have non-empty intersection if $h \neq 1$.
   (c) The line r is contained in the plane π_h if $h = 1$.
   (d) The line r and the plane π_h are parallel for all values of h.

6. Consider the line

$$r : \begin{cases} x = 2 + t \\ y = 2 - t \\ z = t \end{cases}$$

   Which of the following statements is true?

   (a) There are infinitely many planes containing r and the point $(2, 2, 2)$.
   (b) There are infinitely many planes containing the point $(2, 2, 2)$ and perpendicular to r.
   (c) There are infinitely many planes containing r and the two points $(2, 2, 2)$ and $(2, 2, 0)$.
   (d) There are infinitely many planes containing r and the point $(2, 2, 0)$.

7. Consider the lines

$$r : \begin{cases} x = 2 + t \\ y = t \\ z = 4t - 3 \end{cases} \quad \text{and} \quad s_h : \begin{cases} x - y = 0 \\ 3x + y - z = h, \end{cases} \quad \text{with } h \in \mathbb{R}.$$

   Find the correct statement.

   (a) The lines r and s_h are parallel for infinitely many values of $h \in \mathbb{R}$.
   (b) When $h = -1$, the two lines meet in the point $P = (3, 1, 1)$.
   (c) The lines r and s_h are skew for infinitely many values of $h \in \mathbb{R}$.
   (d) The lines r and s_h are perpendicular for infinitely many values of $h \in \mathbb{R}$.

8. Consider the lines $r : \{z - 1 = x + y - 2 = 0\}$ and $s : \{x - 2 = y + z - 4 = 0\}$, and let $d = d(r, s)$ be the distance between them.

   Find the correct statement.

   (a) $0 < d \leq \sqrt{3}$.
   (b) $\sqrt{3} < d \leq 2\sqrt{3}$.
   (c) $2\sqrt{3} < d \leq 3\sqrt{3}$.
   (d) $3\sqrt{3} < d \leq 4\sqrt{3}$.

9. Consider the lines

$$r : \begin{cases} x + y = 1 \\ y = 2 \end{cases} \qquad \text{ed} \qquad s : \begin{cases} x = -3 \\ y = 3z \end{cases}$$

   Which of the following statements is true?

   (a) There exists a plane that contains r and s.
   (b) There exists a plane that contains r and is orthogonal to s.
   (c) There exists a plane that contains r and is parallel to s.
   (d) There exists a plane that contains s and is orthogonal to r.

10. Consider the three planes

$$\alpha : z = 0\,, \quad \beta : y - z = 0\,, \quad \gamma : y + z = 0\,.$$

   Which one of the following statements is true?

   (a) α, β and γ only share a unique common point.
   (b) α, β and γ have empty intersection.
   (c) α, β and γ share a common line.
   (d) α, β and γ are parallel to each other.

11. Consider the lines

$$r : \begin{cases} x + z = 0 \\ 2x + y + z = 0 \end{cases} \qquad \text{and} \qquad s : \begin{cases} x = 1 - t \\ y = -1 + t \\ z = 1 - t \end{cases}$$

Which of the following statements is true?

(a) The distance from r to s is $\sqrt{2}$.
(b) The distance from the origin to both lines is 0.
(c) The distance from r to s is $\sqrt{3}$.
(d) The distance from r to s is $\sqrt{6}$.

12. Let $\vec{\imath}, \vec{\jmath}, \vec{k}$ be the unit vectors of the coordinate axes in $\mathbb{R}^3$, and consider the planes

$$\pi_1 : 2x - y - 3z - 6 = 0 \quad \text{and} \quad \pi_2 : x + y + 2z - 4 = 0\,.$$

Find the correct statement.

(a) $\pi_1 \cap \pi_2$ is a line parallel to the vector $\vec{\imath} - 7\vec{\jmath} + 3\vec{k}$.
(b) $\pi_1 \cap \pi_2 = \emptyset$.
(c) The point $P = (1, 1, 1)$ belongs to the line $\pi_1 \cap \pi_2$.
(d) $\pi_1 \cap \pi_2$ is orthogonal to the vector $\vec{\imath} - 7\vec{\jmath} + 3\vec{k}$.

13. Given the line $r : x - y = x + y - z = 0$, find the correct statement.

(a) r passes through the point $(1, 1, 0)$.
(b) r is contained in the plane $z = 0$.
(c) r is contained in the plane $2x + z = 0$.
(d) r has direction vector parallel to the vector having coordinates $(1, 1, 2)$.

14. The points of the line ℓ given by parametric equations $\ell : P_0 + t\,\vec{v}$ satisfy Cartesian equations

$$\begin{cases} x + z = 1 \\ x + y = 0 \end{cases}$$

Which of the following statements is true?

(a) $P_0 = (1, 0, 0)$ and $\vec{v} = \vec{\imath} + \vec{k}$.
(b) $P_0 = (0, 0, 1)$ and $\vec{v} = \vec{\imath} - \vec{\jmath} - \vec{k}$.
(c) $P_0 = (0, 0, 0)$ and $\vec{v} = \vec{\imath} + \vec{\jmath} + \vec{k}$.
(d) $P_0 = (1, 1, 0)$ and $\vec{v} = \vec{0}$.