Linear algebra and geometry a.y. 2023-2024

Mixed quizzes on vectors

1. Let $\vec{\imath}, \vec{\jmath}, \vec{k}$ be the unit vectors of a coordinate systems in S_3, and let $s \in \mathbb{R}$ be a parameter. Consider the vectors

$$\vec{u}_s = s\vec{\imath} + 3\vec{\jmath} - \vec{k}, \qquad \vec{v} = \vec{\imath} - \vec{\jmath} + \vec{k}, \qquad \vec{w} = 2\vec{\imath} + \vec{\jmath} - \vec{k}.$$

Which of the following statements is true?

(a) There exists $s \in \mathbb{R}$ such that $\vec{u}_s$, $\vec{v}$, $\vec{w}$ are coplanar.
(b) The vectors $\vec{u}_s$ and $\vec{v}$ are orthogonal for all $s \in \mathbb{R}$.
(c) The module of $\vec{u}_s \cdot (\vec{v} \times \vec{w})$ is 6 for at least one value of $s \in \mathbb{R}$.
(d) The module of $\vec{u}_s \cdot (\vec{v} \times \vec{w})$ is 1 for at least one value of $s \in \mathbb{R}$.

2. Consider the points

$$A = (1, 1, 1), \qquad B = (3, 3, 7), \qquad C = (0, 3, -2), \qquad D = (2, 3, 3).$$

Which of the following statements is true?

(a) None of the other statements is true.
(b) The points A, B, C, D are coplanar.
(c) The points A, B, D are collinear.
(d) The area of the triangle whose vertices are B, C, D is $\frac{1}{2}\sqrt{91}$.

3. Let $\vec{\imath}, \vec{\jmath}, \vec{k}$ be the unit vectors of a coordinate systems in S_3. Consider the vectors

$$\vec{v} = \vec{\imath} + \vec{\jmath}, \qquad \vec{w} = \vec{\imath} - \vec{\jmath} + 2\vec{k}.$$

Which of the following statements is true?

(a) The equation $\vec{x} \times \vec{v} = \vec{w}$ has a unique solution.
(b) There exist infinitely many vectors $\vec{x}$ such that $\vec{x} \times \vec{v} = \vec{w}$.
(c) There do not exist vectors $\vec{x}$ such that $\vec{x} \times \vec{v} = \vec{w}$.
(d) None of the other statements is true.

4. Let $\vec{\imath}, \vec{\jmath}, \vec{k}$ be the unit vectors of a coordinate systems in S_3. Consider the vectors

$$\vec{v}_\alpha = \alpha\vec{\imath} + \vec{\jmath} + \alpha\vec{k}, \qquad \vec{w} = \vec{\imath} - \vec{\jmath} + 2\vec{k},$$

where $\alpha \in \mathbb{R}$ is a real parameter. Which of the following statements is true?

(a) None of the other statements is true.
(b) There exists $\alpha \in \mathbb{R}$ such that $\vec{v}_\alpha$ and $\vec{w}$ are orthogonal.
(c) For all $\alpha \in \mathbb{R}$ the angle formed by $\vec{v}_\alpha$ and $\vec{w}$ is acute.
(d) For all $\alpha \in \mathbb{R}$ the angle formed by $\vec{v}_\alpha$ and $\vec{w}$ is obtuse.

5. In the space of applied vectors in S_3 consider

$$\vec{u} = \vec{\imath} + 2\vec{\jmath} - \vec{k} \qquad \text{and} \qquad \vec{v} = 2\vec{\imath} - \vec{\jmath} + \vec{k}.$$

Find the correct statement.

(a) $\vec{u} \times \vec{v}$ is orthogonal to the vector $-\vec{\imath} + 3\vec{\jmath} + 5\vec{k}$.
(b) $\vec{u} \times \vec{v}$ is parallel to the vector $2\vec{\imath} - \vec{\jmath} + \vec{k}$.
(c) $\vec{u} \times \vec{v}$ is orthogonal to the vector $2\vec{\imath} + 4\vec{\jmath} - 2\vec{k}$.
(d) $\vec{u} \times \vec{v}$ forms an obtuse angle with the vector $\vec{\jmath} - \vec{k}$.

6. In the space of applied vectors, consider a nonzero vector $\vec{v}$.
Which of the following statements is true?

(a) For all $\vec{w} \neq \vec{0}$ the vector $\vec{w} \times \vec{v}$ is not parallel to $\vec{v}$.
(b) The equation $\vec{x} \times \vec{v} = \vec{w}$ has solutions for all $\vec{w}$.
(c) There exists a vector $\vec{w}$ not parallel to $\vec{v}$ such that $(\vec{w} \times \vec{v}) \times \vec{v} = \vec{0}$.
(d) There exists a vector $\vec{w}$ such that the equation $\vec{x} \times \vec{v} = \vec{w}$ has solutions.

7. In the space of applied vectors consider the vectors

$$\vec{u} = 2\vec{\imath}, \qquad \vec{v} = 6\vec{\imath} - \vec{\jmath}, \qquad \vec{w} = \vec{\jmath} - 3\vec{k}.$$

Find the correct statement.

(a) $\vec{u}$, $\vec{v}$ and $\vec{w}$ are not coplanar.
(b) $\vec{w}$ is orthogonal to $\vec{u} \times \vec{v}$.
(c) $\vec{w}$ is orthogonal to $\vec{u} + \vec{v}$.
(d) None of the other statements is true.

8. For all values of the parameter $t \in \mathbb{R}$, consider the points in S_3:

$$P = (1, 2, 3), \quad Q = (0, 0, t), \quad R = (1, -1, t), \quad S = (1, 1, t).$$

Which of the following statements is true?

(a) P, Q, R, S are the vertices of a square whose area is $|t - 3|$ for infinitely many values of t.

(b) P, Q, R, S are the vertices of a tetrahedron whose volume is $|2t - 6|$ for infinitely many values of t.

(c) P, Q, R, S are the vertices of a tetrahedron whose volume is $\left|\frac{t}{3} - 1\right|$ for infinitely many values of t.

(d) P, Q, R, S are coplanar for infinitely many values of the parameter t.

9. Let $\vec{\imath}, \vec{\jmath}, \vec{k}$ be the unit vectors of a coordinate systems in S_3, and consider

$$\vec{u} = \vec{\imath} - 2\vec{\jmath} + 3\vec{k} \quad \text{and} \quad \vec{v} = \vec{\jmath} - \vec{k}.$$

Which of the following statements is true?

(a) The length of the cross product of $\vec{u}$ and $\vec{v}$ equals $\sqrt{3}$.

(b) The area of the triangle defined by the vectors $2\vec{u}$ and $2\vec{v}$ is $\sqrt{3}$.

(c) The vectors $\vec{u}$ and $\vec{u} + \vec{v}$ are orthogonal.

(d) The cosine of the angle formed by $2\vec{u}$ and $3\vec{v}$ is $\sqrt{3}/2$.

10. Let $\vec{\imath}, \vec{\jmath}, \vec{k}$ be the unit vectors of a coordinate systems in S_3, and consider the three orthogonal unit vectors

$$\vec{u} = \frac{\sqrt{2}}{2}\vec{\imath} + \frac{\sqrt{2}}{2}\vec{\jmath}, \qquad \vec{v} = \frac{\sqrt{2}}{2}\vec{\imath} - \frac{\sqrt{2}}{2}\vec{\jmath}, \qquad \vec{w} = -\vec{k}$$

and the vector $\vec{r} = 2\vec{\imath} + 2\vec{\jmath} + 4\vec{k}$.

Which of the following statements is true?

(a) The vector $\vec{r}$ is orthogonal to the vector $\vec{u}$.

(b) The length of the cross product $\vec{u} \times \vec{v}$ is $1/2$.

(c) None of the other statements is true.

(d) $\vec{r} = 2\sqrt{2}\vec{u} - 4\vec{w}$.

Solutions

1. (c)
2. (a)
3. (b)
4. (b)
5. (c)
6. (d)
7. (a)
8. (c)
9. (a)
10. (d)