Linear algebra and geometry a.y. 2023-2024

Worksheet 4: exercises on chapter 9 from the lecture notes

1. Find parametric equations for the line r through the points $P = (1, 2, -2)$ and $Q = (0, 2, 1)$.

2. Find parametric equations for the line r passing through the point $A = (2, 3, 0)$ and parallel to the line s, whose parametric equations are

$$\begin{cases} x = t \\ y = 1 + t \\ z = 3 - 2t, \end{cases} \qquad t \in \mathbb{R}.$$

3. Find parametric equations for the plane γ through the three points $P_1 = (0, 1, 2)$, $P_2 = (1, 2, 3)$, $P_3 = (1, 3, 5)$.

4. Find parametric equations of the plane π containing the points $P = (1, 3, 1)$ and $Q = (0, 4, 1)$, and parallel to the direction $\vec{\imath} + 3\vec{\jmath}$.

5. Find parametric equations of the line ℓ passing through the point $P = (1, 2, 3)$ and orthogonal to the plane α, whose parametric equations are:

$$\begin{cases} x = 1 - s \\ y = 1 + t \\ z = 2 + s + t, \end{cases} \qquad s, t \in \mathbb{R}.$$

(Hint: if the plane α is parallel to the directions $\vec{v_1}$ and $\vec{v_2}$, then the direction orthogonal to α is given by $\vec{v_1} \times \vec{v_2}$.)

Solutions.

Warning: the same line/plane can be described by different parametric equations, so if you find something different from me it does not necessarily mean that you are wrong!

1. $r: \begin{cases} x = 1 + t \\ y = 2 \\ z = -2 - 3t, \end{cases} \quad t \in \mathbb{R}.$

2. $r: \begin{cases} x = 2 + t \\ y = 3 + t \\ z = -2t, \end{cases} \quad t \in \mathbb{R}.$

3. $\gamma: \begin{cases} x = s \\ y = 1 + s + t \\ z = 2 + s + 2t, \end{cases} \quad s, t \in \mathbb{R}.$

4. $\pi: \begin{cases} x = 1 + s + t \\ y = 3 - s + 3t \\ z = 1, \end{cases} \quad s, t \in \mathbb{R}.$

5. $\ell: \begin{cases} x = 1 - t \\ y = 2 + t \\ z = 3 - t, \end{cases} \quad t \in \mathbb{R}.$

Please note. Remember that in general there might be more than one technique to solve the same exercise. If you find a typo, or something that you do not understand, let me know!