Robust and Randomized Control Design of Mini-UAV: The MH1000 Platform

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Overview

- UAV Application: The MH1000 Platform
- Flight Test Video and Computer Graphics Animation
- Randomized Algorithms: A Success Story
- Conclusions

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Italian National Project for Fire Prevention

- This activity is supported by the Italian Ministry for Research within the National Project

Study and development of a real-time land control and monitoring system for fire prevention

- Five research groups are involved together with a government agency for fire surveillance and patrol located in Sicily
- The aerial platform is based on the MicroHawk configuration, developed at the Aerospace Engineering Department, Politecnico di Torino, Italy

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MH1000 Platform - 1

- Platform features
  - wingspan 3.28 ft (1 m)
  - total weight 3.3 lb (1.5 kg)

MH1000 Platform - 2

- Main on-board equipment
  - various sensors and two cameras (color and infrared)
- DC motor
- Remote piloting and autonomous flight
- Flight endurance of about 40 min
- Flight envelope
  - min/max velocity: 33 ft/s (10 m/s) – 66 ft/s (17 m/s)
  - average velocity: 43 ft/s (14 m/s)

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Flight Envelope (Limits)

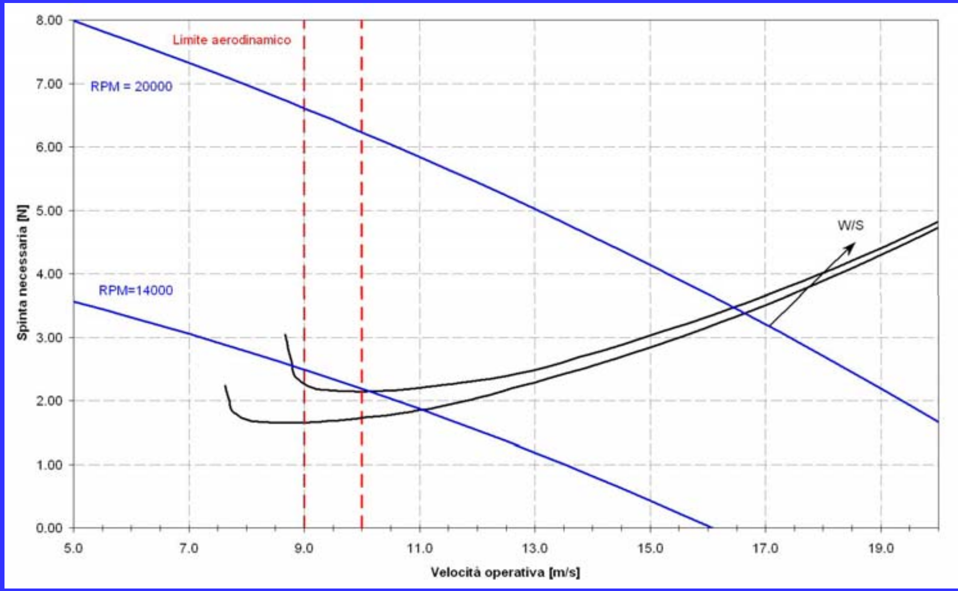
Wing loading effect → total weight

Aerodynamic constraint (red) → minimum flight speed (stall effect)

Propeller sizing effect

Propulsive constraint (blu) → maximum flight speed

velocity: 33 ft/s (10 m/s) – 66 ft/s (17 m/s)



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Basic on-board Systems

DC motor: Hacker B20-15L (4:1)

- weight: 58 g
- dimensions: Ø 20 x 40 mm
- Kv: 3700 rpm/volt

controller: Hacker Master Series 18-B-Flight

- weight: 21 g
- dimensions: 33 X 23 X 7 mm
- current drain: 18 A

battery: Kokam 2000HD (3x)

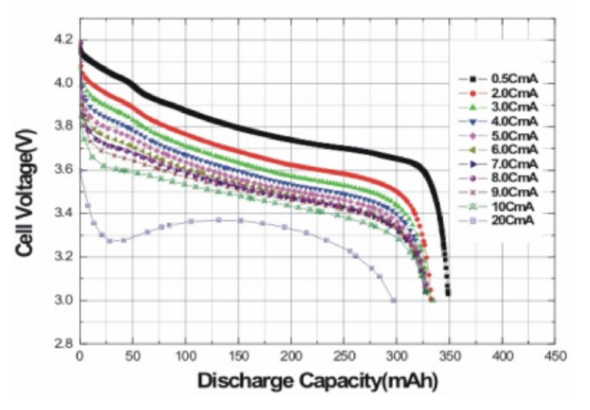
- weight: 160 g
- dimensions: 79 X 42 X 25 mm
- capacity: 2000 mAh

receiver: Schulze Alpha840W

- weight: 13.5 g
- dimensions: 52 X 21 X 13 mm
- 8 channels

servo: Graupner C1081 (2x)

- weight: 13 g
- dimensions: 23 X 9 X 21 mm
- torque: 12 Ncm



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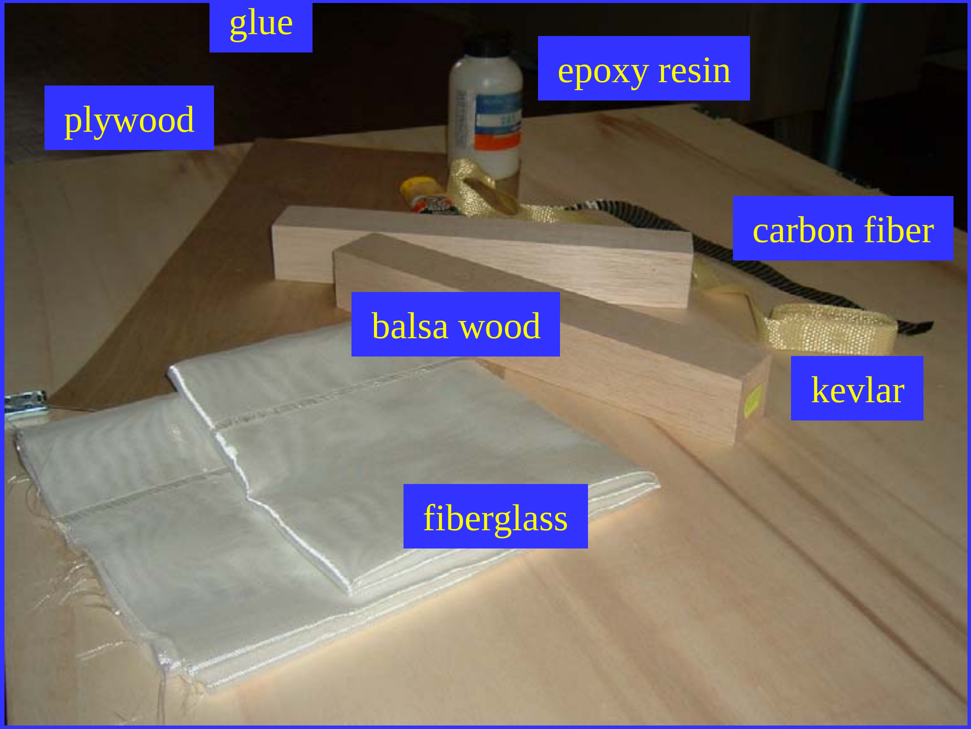
Prototype Manufacturing - 1

raw material

polistyrene



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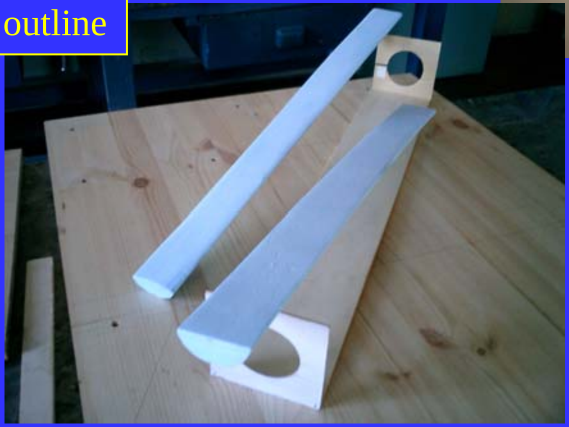
Prototype Manufacturing - 2

hot wire foam cutting machine

working instruments

lifting surfaces outline

slide outline

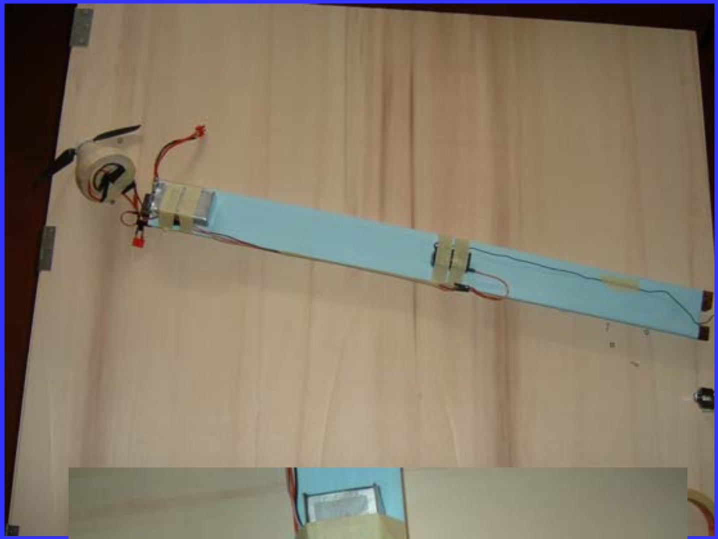


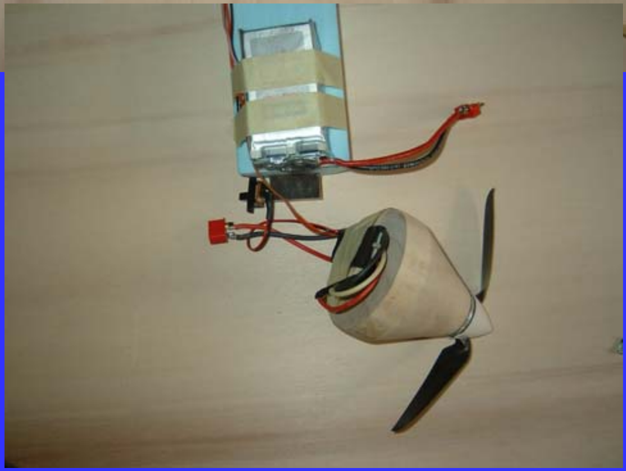
fuselage reference

Prototype Manufacturing - 3

prototype

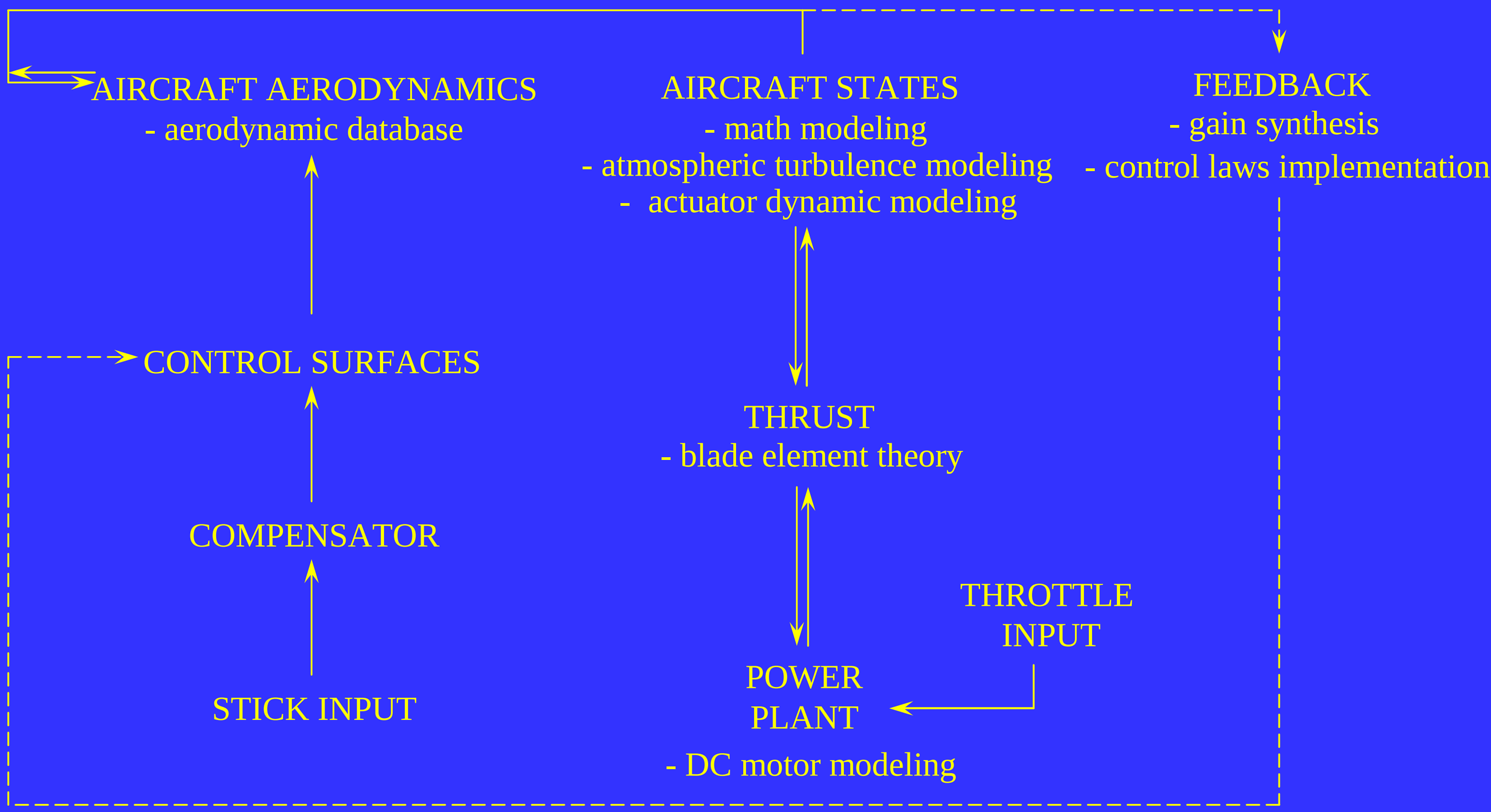
easy construction
rapid manufacturing
bad model reproducibility
inaccurate geometry





Aircraft Dynamics - 1

Aircraft math model implementation within an off-line flight simulator



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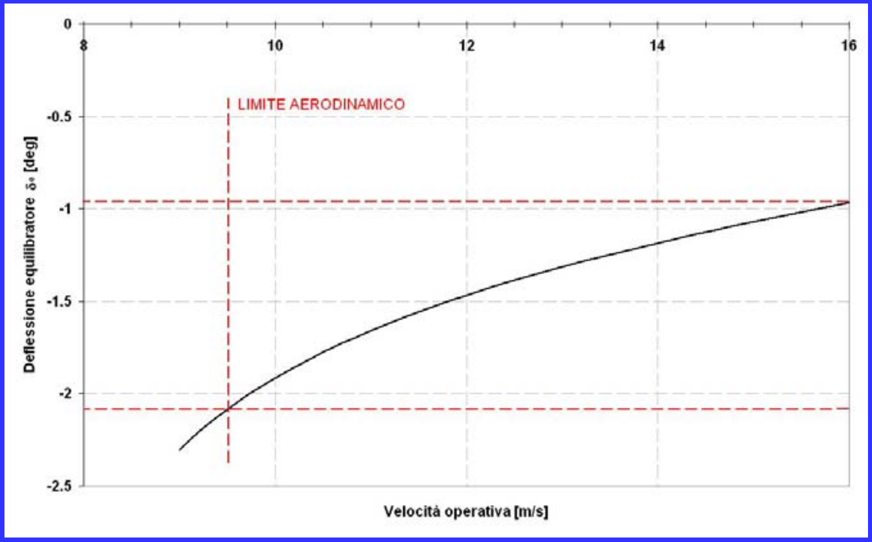
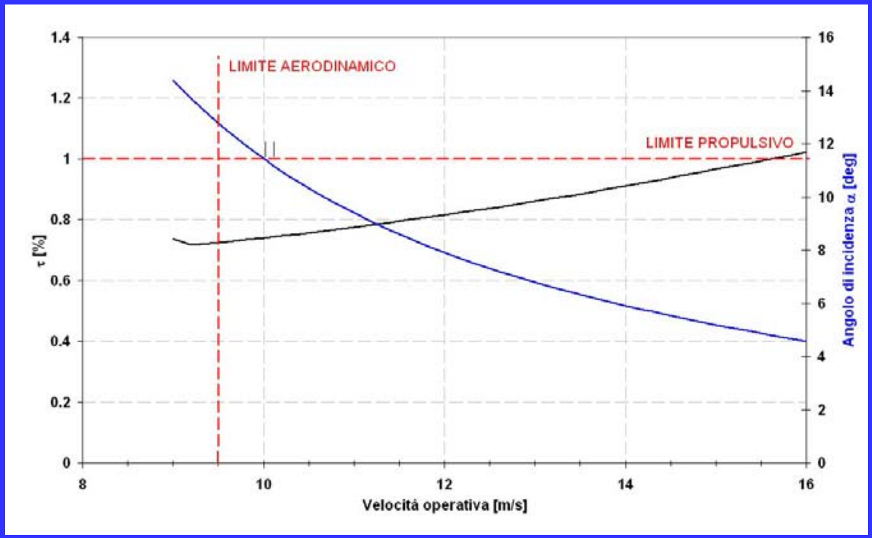
Aircraft Dynamics - 2

Aircraft math model implementation with an off-line flight simulator

- trimmability analysis (search for stationary equilibrium condition)





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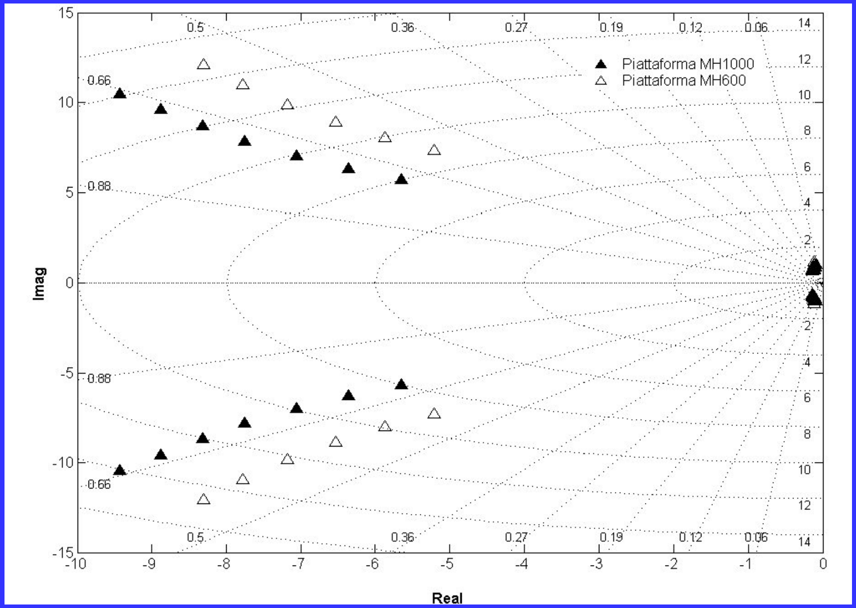
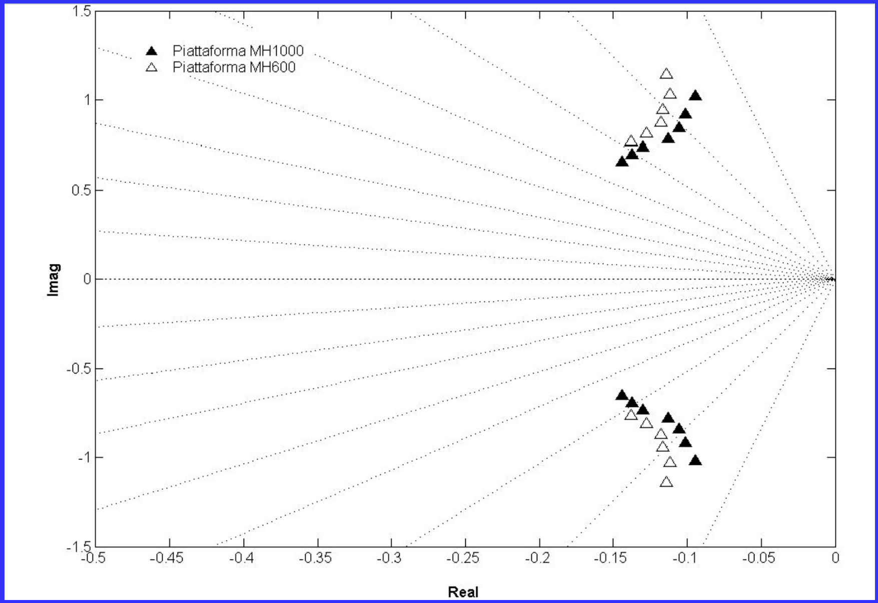
Aircraft Dynamics - 3

Aircraft math model implementation with an off-line flight simulator

- open loop dynamics characterization



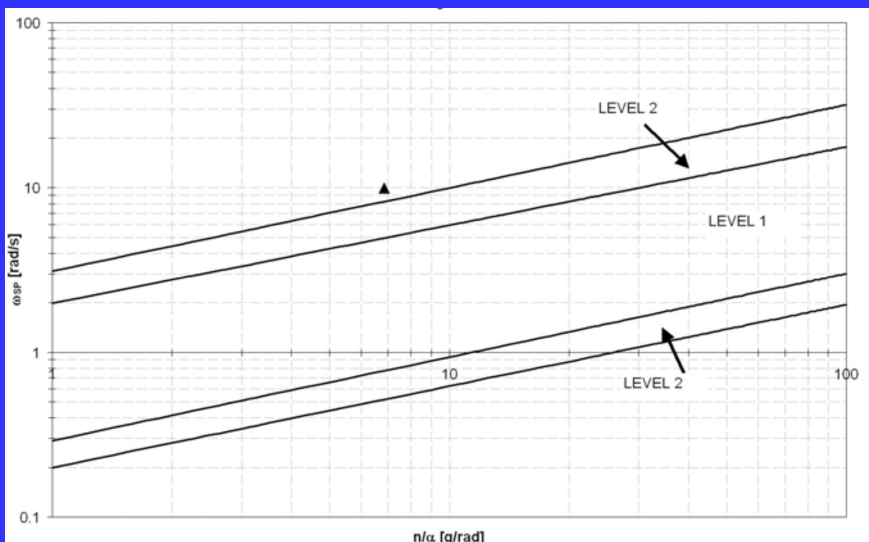


Aircraft Dynamics - 4

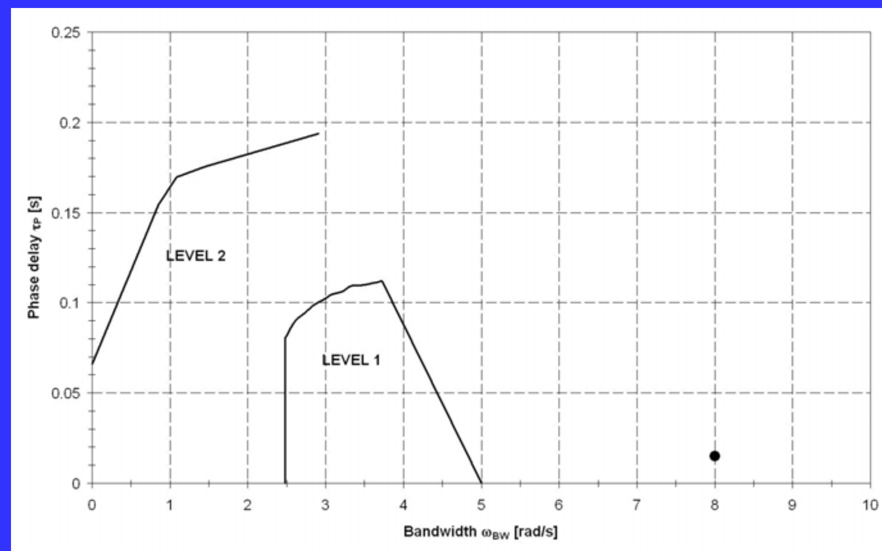
Aircraft math model implementation with an off-line flight simulator

- compliance to standard requirements MIL 8785C/MIL 1797A standards

CAP criterion



Bandwidth criterion



State Space Model

- State space formulation obtained by linearization of the full (12 coupled nonlinear ODE) model

$$\dot{x}(t) = A(\Delta)\, x(t) + B(\Delta)\, u(t)$$

$$u(t) = -\, K\, x(t)$$

where $x = [V,\ \alpha,\ q,\ \theta]^T$ (V flight speed, α angle of attack, q and θ pitch rate and angle), Δ uncertainty

- Consider longitudinal plane dynamics stabilization
- Control u is the symmetrical elevon deflection

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Uncertainty Description - 1

- We consider structured parameter uncertainties affecting plant and flight conditions, and aerodynamic database
- Uncertainty vector $\Delta = [\delta_1, \ldots, \delta_{16}]$ where $\delta_i \in [\delta_i^-, \delta_i^+]$
- Key point: There is no explicit relation between state space matrices A and B and uncertainty Δ
- This is due to the fact that state space system is obtained through linearization and off-line flight simulator
- The only techniques which could be used in this case are simulation-based which lead to randomized algorithms

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Uncertainty Description - 2

- We consider random uncertainty $\Delta = [\delta_1, \ldots, \delta_{16}]^T$
- The pdf is either uniform (for plant and flight conditions) or truncated Gaussian (for aerodynamic database uncertainties)
- Flight conditions uncertainties need to take into account large variations on physical parameters
- Uncertainties for aerodynamic data are related to experimental measurement or round-off errors

Plant and Flight Condition Uncertainties

parameter	pdf	$\bar{\delta}_i$	%	δ_i^-	δ_i^+	#
flight speed [ft/s]	*U*	42.65	± 15	36.25	49.05	1
altitude [ft]	*U*	164.04	± 100	0	328.08	2
mass [lb]	*U*	3.31	± 10	2.98	3.64	3
wingspan [ft]	*U*	3.28	± 5	3.12	3.44	4
mean aero chord [ft]	*U*	1.75	± 5	1.67	1.85	5
wing surface [ft^2]	*U*	5.61	± 10	5.06	6.18	6
moment of inertia [lb ft^2]	*U*	1.34	± 10	1.21	1.48	7

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Aerodynamic Database Uncertainties

parameter	pdf	$\overline{\delta_i}$	σ_i	#
C_X [-]	*G*	-0.01215	0.00040	8
C_Z [-]	*G*	-0.30651	0.00500	9
C_m [-]	*G*	-0.02401	0.00040	10
C_{Xq} [rad^{-1}]	*G*	-0.20435	0.00650	11
C_{Zq} [rad^{-1}]	*G*	-1.49462	0.05000	12
C_{mq} [rad^{-1}]	*G*	-0.76882	0.01000	13
C_X [rad^{-1}]	*G*	-0.17072	0.00540	14
C_Z [rad^{-1}]	*G*	-1.41136	0.02200	15
C_m [rad^{-1}]	*G*	-0.94853	0.01500	16

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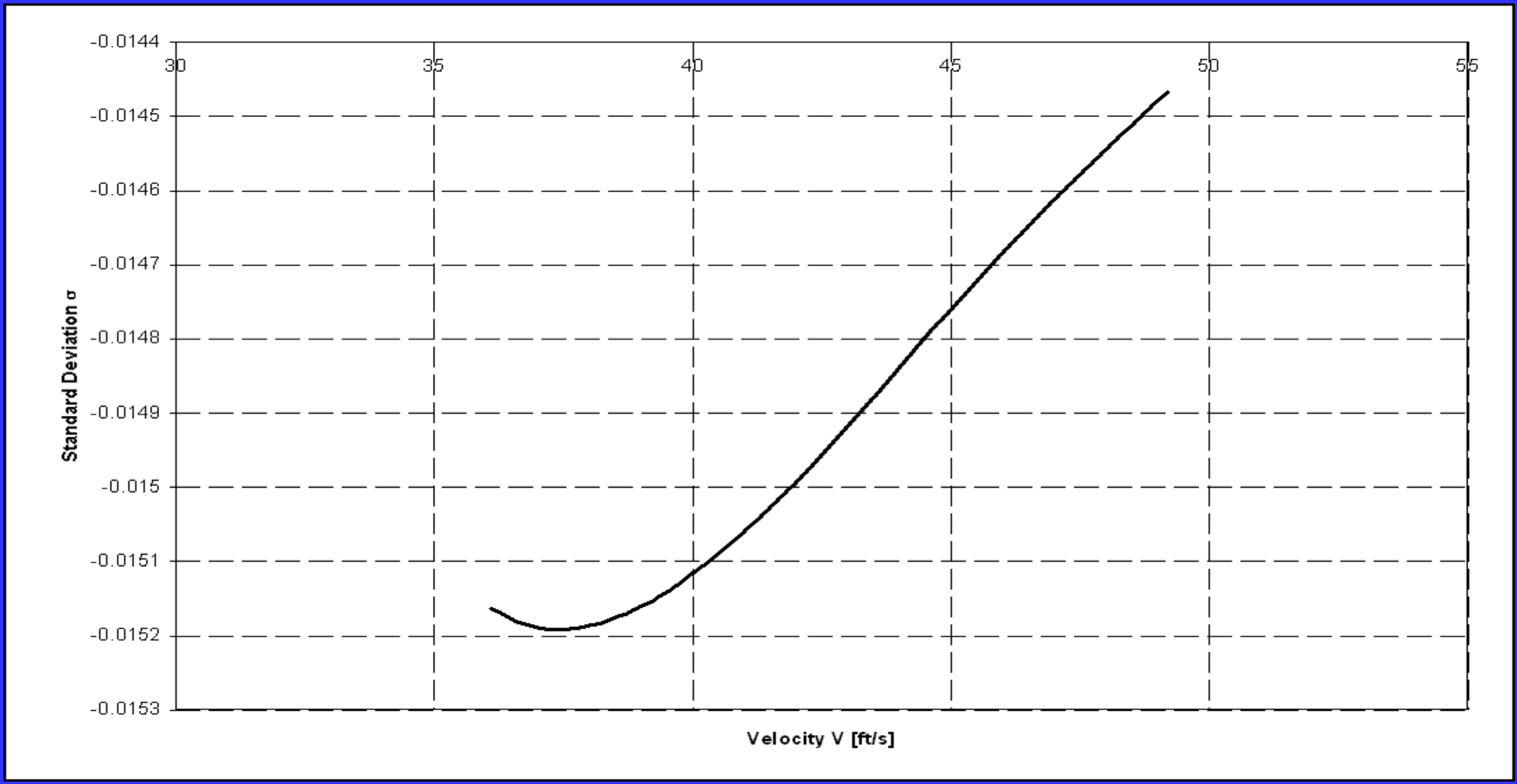
Standard Deviation and Velocity



Standard deviation is experimentally computed from the velocity

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Critical Parameters and Matrices

- We select flight speed (δ_1) and take off mass (δ_3) as critical parameters
- Flight speed is taken as critical parameter to optimize gain scheduling issues
- Take off mass is a key parameter in mission profile definition
- We define critical matrices

$$A_c^1 \quad A_c^2 \quad A_c^3 \quad A_c^4 \qquad B_c^1 \quad B_c^2 \quad B_c^3 \quad B_c^4$$

- They are constructed setting δ_1, δ_3 to the extreme values δ_1^-, δ_1^+, δ_3^-, δ_3^+ and all the remaining δ_i are equal to $\overline{\delta_i}$

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Phase 1: Random Gain Synthesis (RGS)

- Critical parameters are flight speed and take off mass
- Specification property

$$S_1 = \{K: A_c - B_c K \text{ satisfies the specs below}\}$$

$\omega_{SP} \in [4.0,6.0]$ rad/s $\zeta_{SP} \in [0.5,0.9]$ $\omega_{PH} \in [1.0,1.5]$ rad/s

$\zeta_{PH} \in [0.1,0.3]$ $\Delta\omega_{SP} < \pm 45\%$ $\Delta\omega_{PH} < \pm 20\%$

where ω and ζ are undamped natural frequency and damping ratio of the characteristic modes; $_{SP}$ and $_{PH}$ denote short period and phugoid mode

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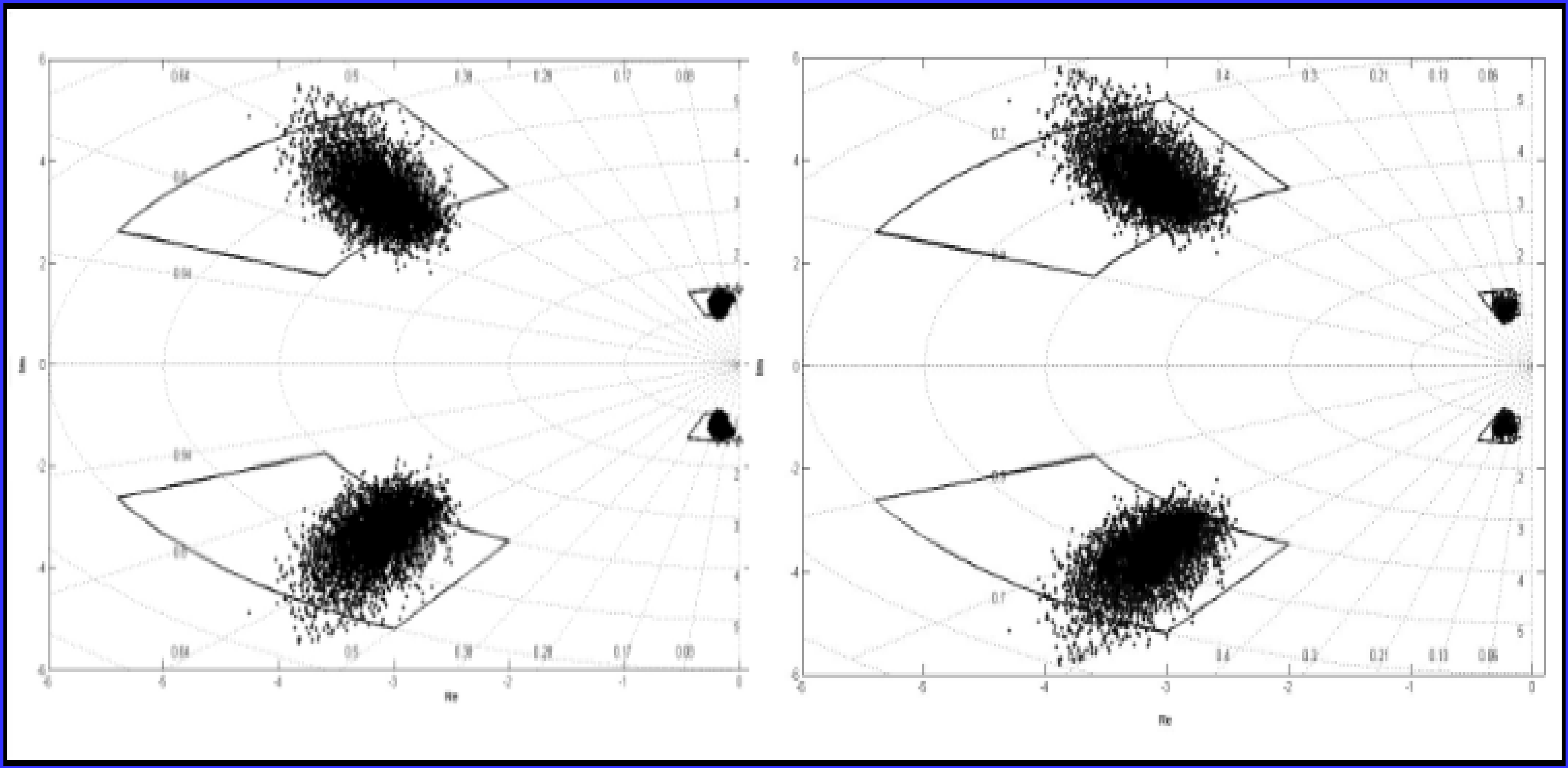
Specs in the Complex Plane



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Volume of the Good Set

- Define a bounding set B of gains K

$$B = \{K: k_i \in [k_i^-, k_i^+],\ i = 1,\ldots,4\}$$

- Define the volume of the *good set*

$$\mathrm{Vol}_{good} = \int_A \mathrm{d}\, K$$

where $A = \{K \in B \cap S_1\}$

- Vol_B is simply the volume of the hyperrectangle B

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Log-Over-Log Bound

- For any accuracy $\varepsilon \in (0,1)$ and confidence $\eta \in (0,1)$, let

$$N \geqslant (\log(1/\eta)) \,/\, (\log(1/(1-\varepsilon)))$$

- Then, if $\mathrm{Vol}_{good} > \varepsilon \mathrm{Vol}_B$, the probability that there is no gain K^j, $j=1,\ldots,N$, in the set $\{B \cap S_1\}$ is less than η

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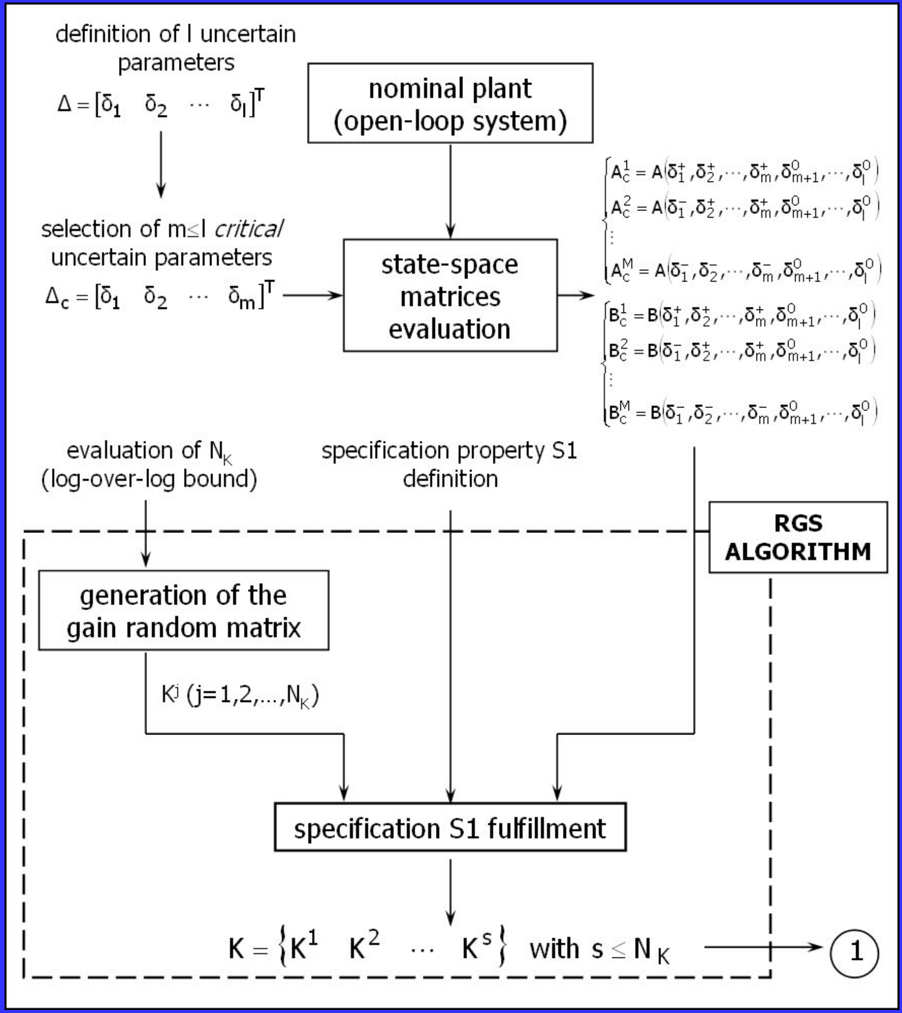
Randomized Algorithm 1 (RGS)

- Uniform pdf for controller gains K in given intervals
- Accuracy and confidence

  $\varepsilon = 4 \cdot 10^{-5}$ and $\eta = 3 \cdot 10^{-4}$
- Number of random samples is computed with “Log-over-Log” Bound obtaining $N = 200{,}000$
- We obtained $s = 5$ gains K^i satisfying specification property S_1



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Randomized Algorithm 1 (RGS)

Given ε, η ∈ (0,1), RGS *returns the set of gains* $\{K^1,\ldots,K^s\}$ *satisfying* S_1

1. Compute N using the Log-over-log Bound;
2. For fixed j=1,2,…,N, generate uniformly the gain random matrix $K^j \in B$;
3. Set C=0;
4. For fixed i=1,2,3,4, compute the closed-loop matrix

$$A_{cl}^i(K^j) = A_c^i - B_c^i K^j;$$

   - if $K^j \in S_1$, set $C = C+1$;
   - otherwise, set $C = C$;
5. End;
6. If $C = 4$, return the gain K^j;
7. Set $j = j+1$ and return to Step 2;
8. End

Random Gain Set

gain set	K_V	K_α	K_q	K_θ
K^1	0.00044023	0.09465000	0.01577400	-0.00473510
K^2	0.00021450	0.09581200	0.01555500	-0.00323510
K^3	0.00054999	0.09430800	0.01548200	-0.00486340
K^4	0.00010855	0.09183200	0.01530000	-0.00404380
K^5	0.00039238	0.09482700	0.01609300	-0.00417340

Phase 2: Random Stability Robustness Analysis (RSRA)

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- Take $K_{rand} = K^i$ obtained in Phase 1
- Randomize Δ according to the given pdf and take N random samples Δ^i
- Specification property

$$S_2 = \{\Delta: A(\Delta) - B(\Delta) \ \ K_{rand} \textit{ satisfies the specs of } S_1\}$$

- Computation of the empirical probability of stability p_N

Empirical Probability p_N

- Consider fixed gain K_{rand}
- Define the probability

$$p_{true} = \int_C p(\Delta)\, d\Delta$$

where $C = \{\Delta \in B \cap S_2\}$ and $p(\Delta)$ is the given pdf

- Then, we introduce a "success" indicator function

$$I(\Delta^j) = 1 \text{ if } \Delta^j \in S_2$$

or $I(\Delta^j) = 0$ otherwise

- The empirical probability for S_2 is given by

$$p_N = S/N$$

where S is equal to the number of successes

Chernoff Bound

- For any accuracy $\varepsilon \in (0,1)$ and confidence $\eta \in (0,1)$, let

$$N \geqslant 1/(2\,\varepsilon^2)\,\log(2/\eta)$$

- Then, the probability that

$$|\, p_N - p_{true} \,| > \varepsilon$$

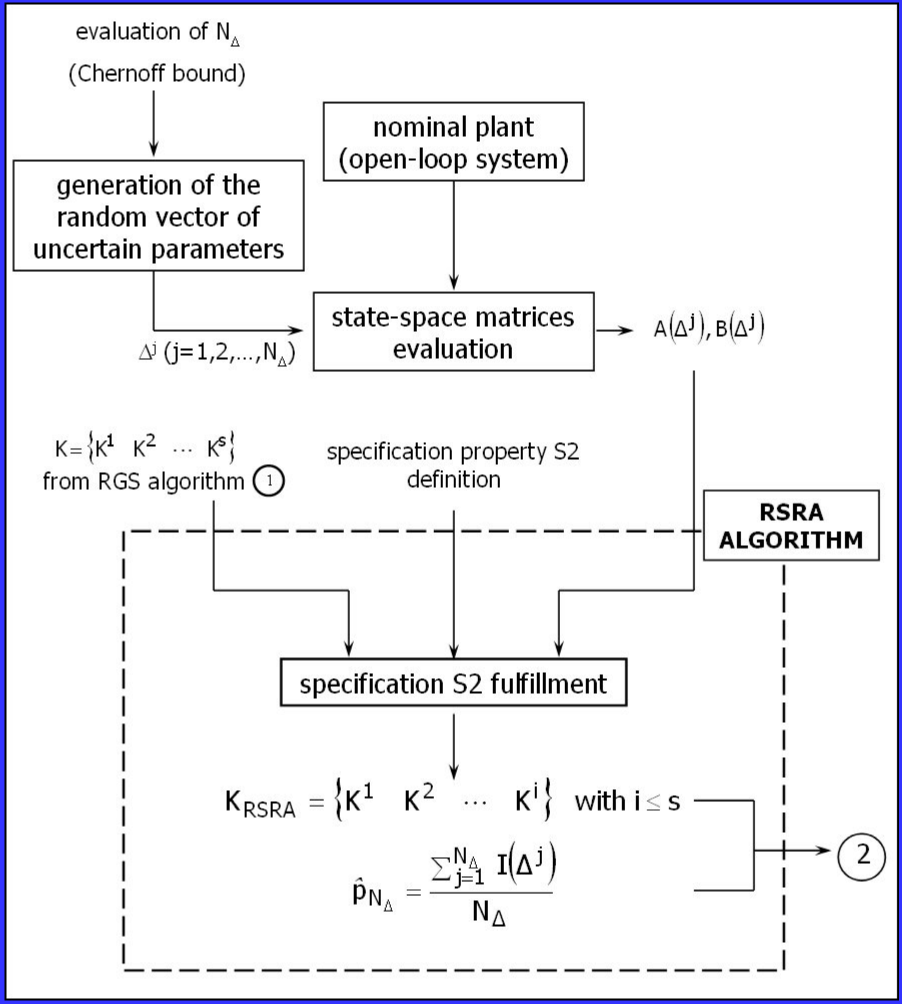
holds is less than η

Randomized Algorithm 2 (RSRA)

- Take K_{rand} from Phase 1
- Accuracy and confidence

$$\varepsilon = \eta = 0.0145$$

- Number of random samples is computed with Chernoff Bound obtaining $N = 5{,}000$
- Empirical probability is defined using an indicator function



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Randomized Algorithm 2 (RSRA)

Given $\varepsilon, \eta \in (0,1)$, RSRA *returns the empirical probability* p_N *that* S_2 *is satisfied for a gain* K_{rand} *provided by Algorithm 1*

1. Compute N using the Chernoff Bound;
2. Generate N random vectors $\Delta^j \in B$ according to the given pdf;
3. For fixed $j=1,2,\ldots,N$, compute the closed-loop matrix

$$A_{cl}(\Delta^j) = A(\Delta^j) - B(\Delta^j)K_{rand};$$

   - if $A_{cl}(\Delta^j) \in S_2$, set $I(\Delta^j) = 1$;
   - otherwise, set $I(\Delta^j) = 0$;
4. End;
5. Return the empirical probability p_N

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Empirical Probability of Stability for Phase 2

gain set	empirical probability
K^1	88.56%
K^2	90.60%
K^3	89.31%
K^4	93.86%
K^5	85.14%

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Probability Degradation Function

- Flight condition uncertainties are multiplied by the *amplification factor* ρ > 0 keeping the nominal value constant

$$\delta_i \in \rho\,[\delta_i^-, \delta_i^+] \qquad \text{for } i = 1, 2, \ldots, 7$$

- No uncertainty affects the aerodynamic database, i.e.

$$\delta_i = \bar{\delta}_i \qquad \text{for } i = 8, 9, \ldots, 16$$

- For fixed ρ∈[0,1.5] we compute the empirical probability for different gain sets K^i
- The plot empirical probability vs ρ is the probability degradation function

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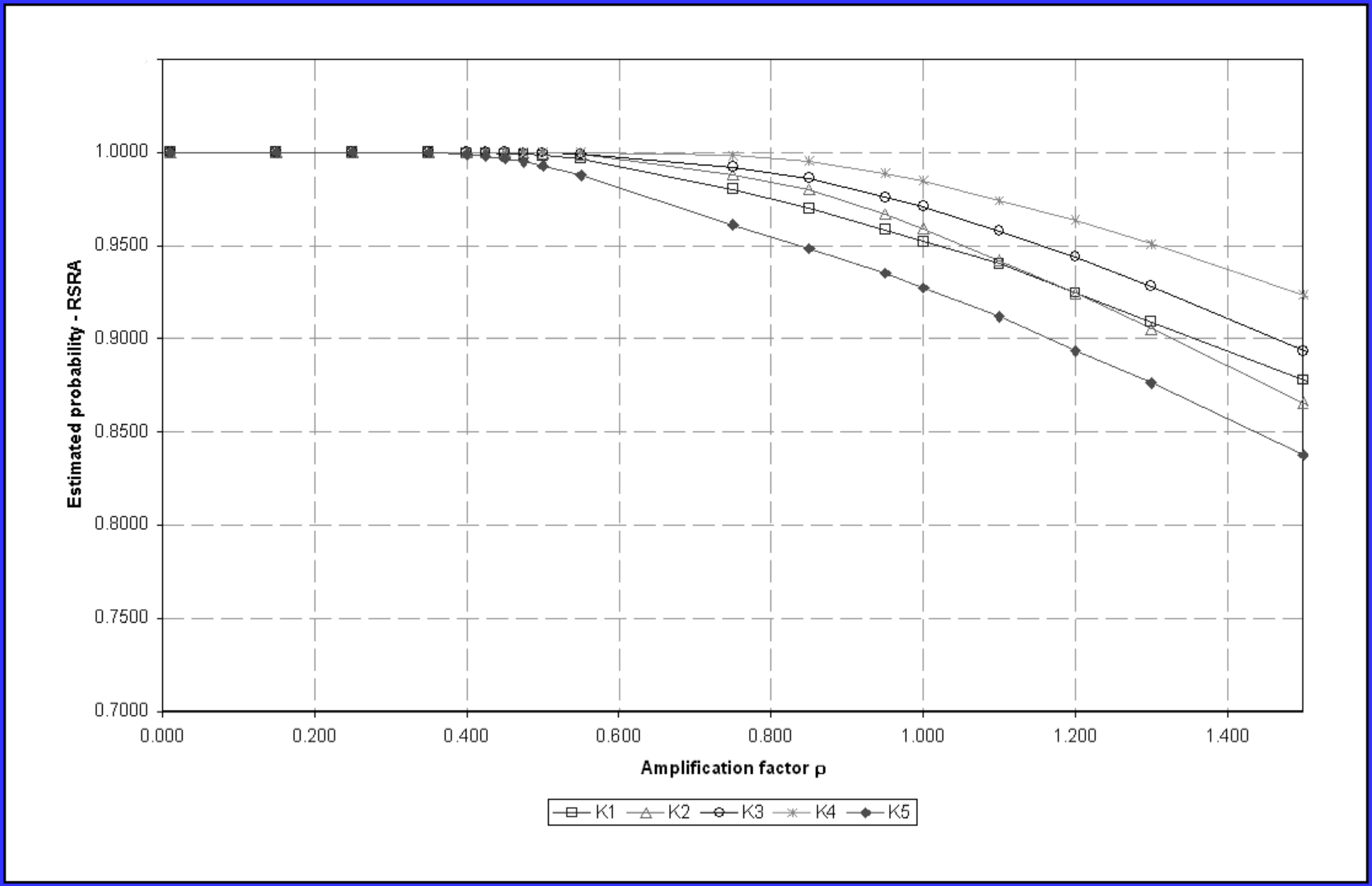
Probability Degradation Function for Phase 2



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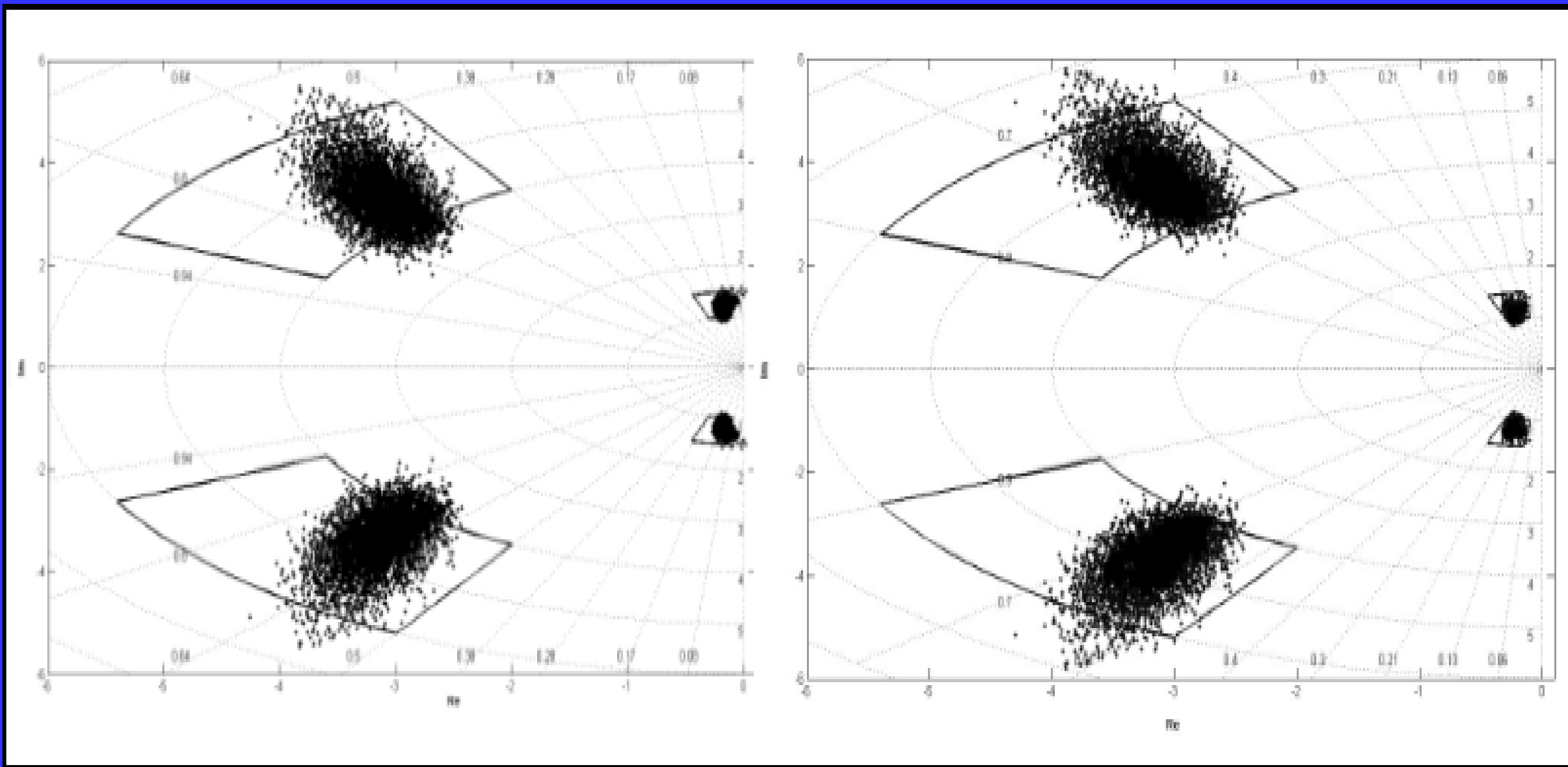
Root Locus Plot for Phase 2



Root locus for K^2 (left) and K^4 (right)

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Phase 3: Random Performance Robustness Analysis (RPRA)

- This phase is similar to Phase 2, but military specs are considered (bandwidth criterion)
- Specification property

$$S_3 = \{\Delta : A(\Delta) - B(\Delta)\ K_{rand} \textit{ satisfies the specs below}\}$$

$$\omega_{BW} \in [2.5, 5.0] \text{ rad/s} \qquad \tau_P \in [0.0, 0.5] \text{ s}$$

where ω_{BW} and τ_P are bandwidth and phase delay of the frequency response

- Computation of the empirical probability that S_3 is satisfied

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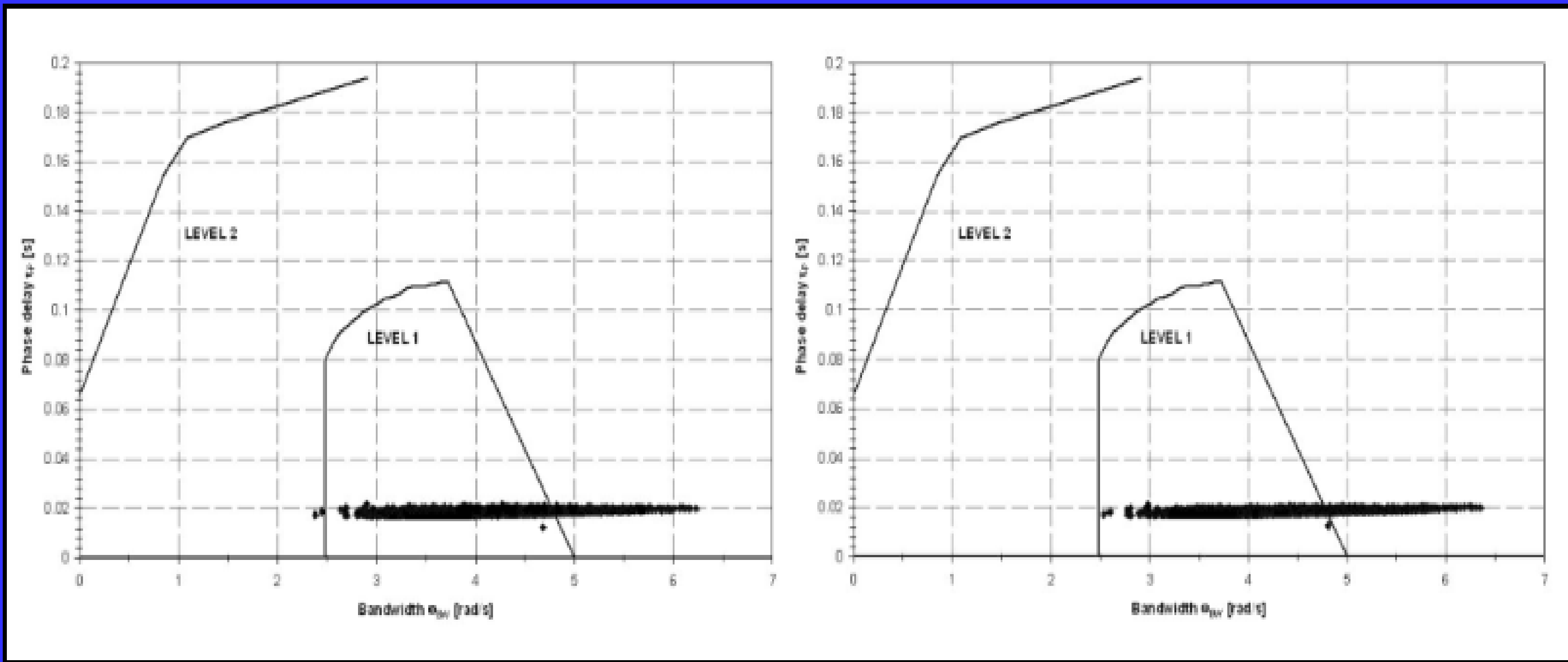
Bandwidth Criterion



Randomized Algorithm 3 (RPRA)

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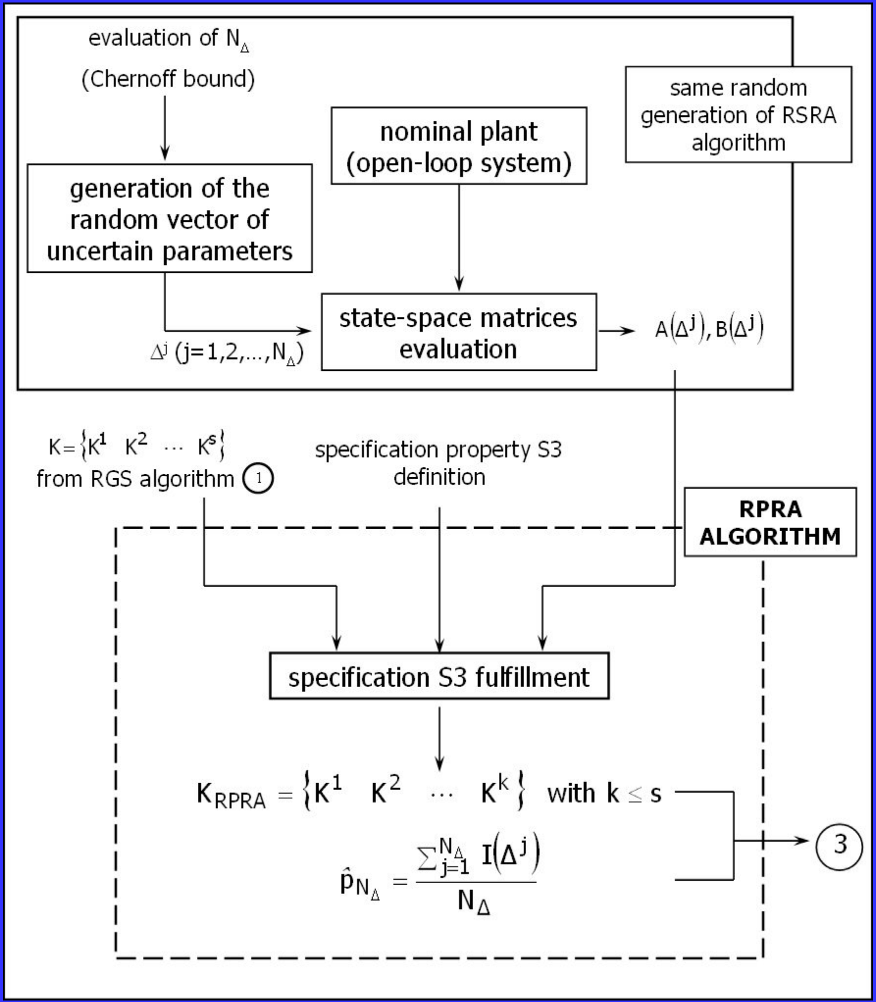
- Take K_{rand} from Phase 1
- Numer of random samples is computed with the Chernoff Bound obtaining N =5,000
- Empirical probability is defined using an indicator function



Randomized Algorithm 3 (RPRA)

Given N and $A_{cl}(\Delta^j)$, $j=1,2,\ldots,N$, *provided by Algorithm 2*, RPRA *returns the empirical probability* p_N *that* S_3 *is satisfied for a gain* K_{rand} *provided by Algorithm 1*

1. For fixed $j=1,2,\ldots,N$
   - if $A_{cl}(\Delta^j) \in S_3$, set $I(\Delta^j) = 1$;
   - otherwise, set $I(\Delta^j) = 0$;
2. End;
3. Return the empirical probability p_N

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Empirical Probability of Performance for Phase 3

gain set	empirical probability
K^1	93.58%
K^2	95.16%
K^3	90.80%
K^4	84.78%
K^5	96.06%

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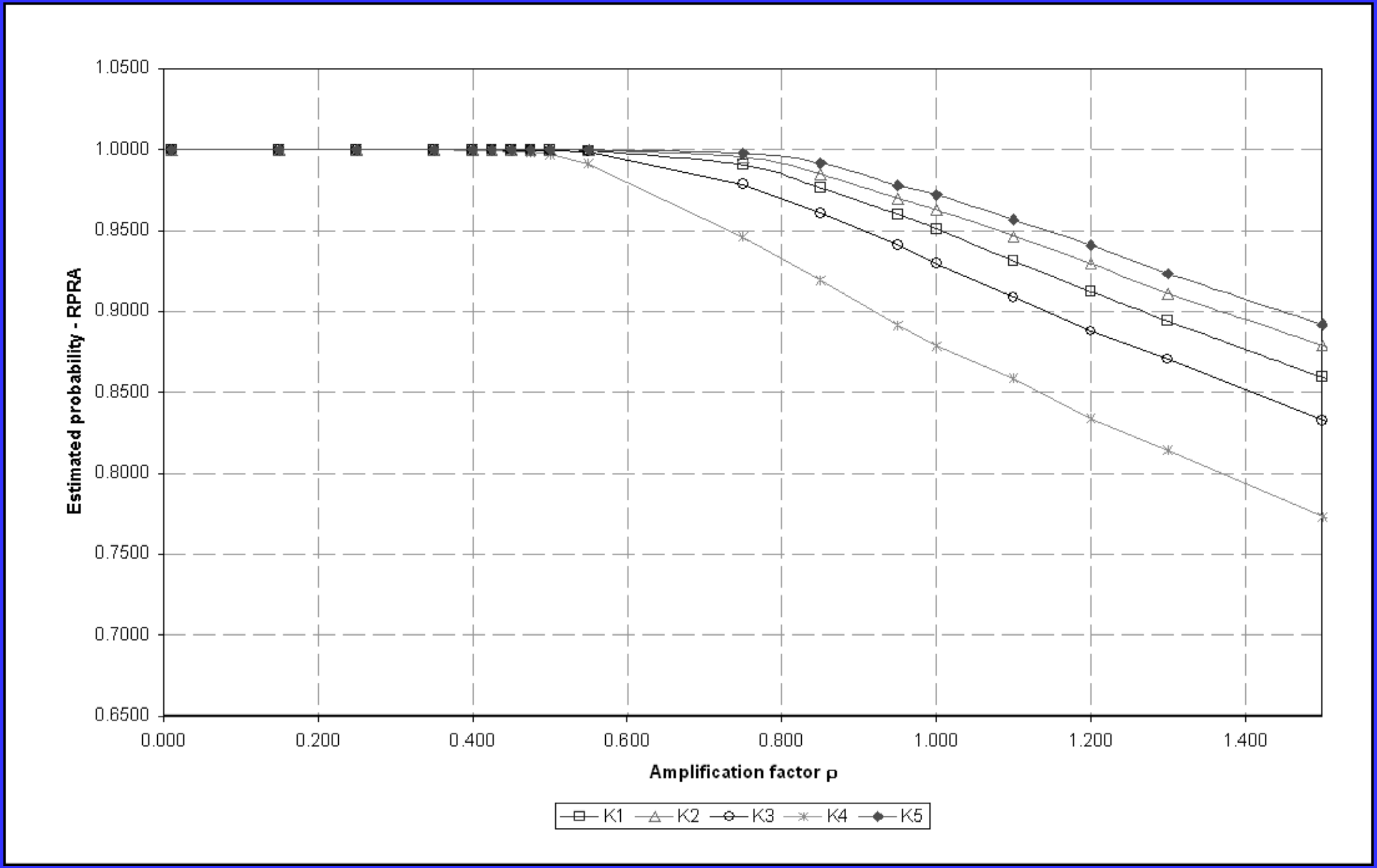
Probability Degradation Function for Phase 3



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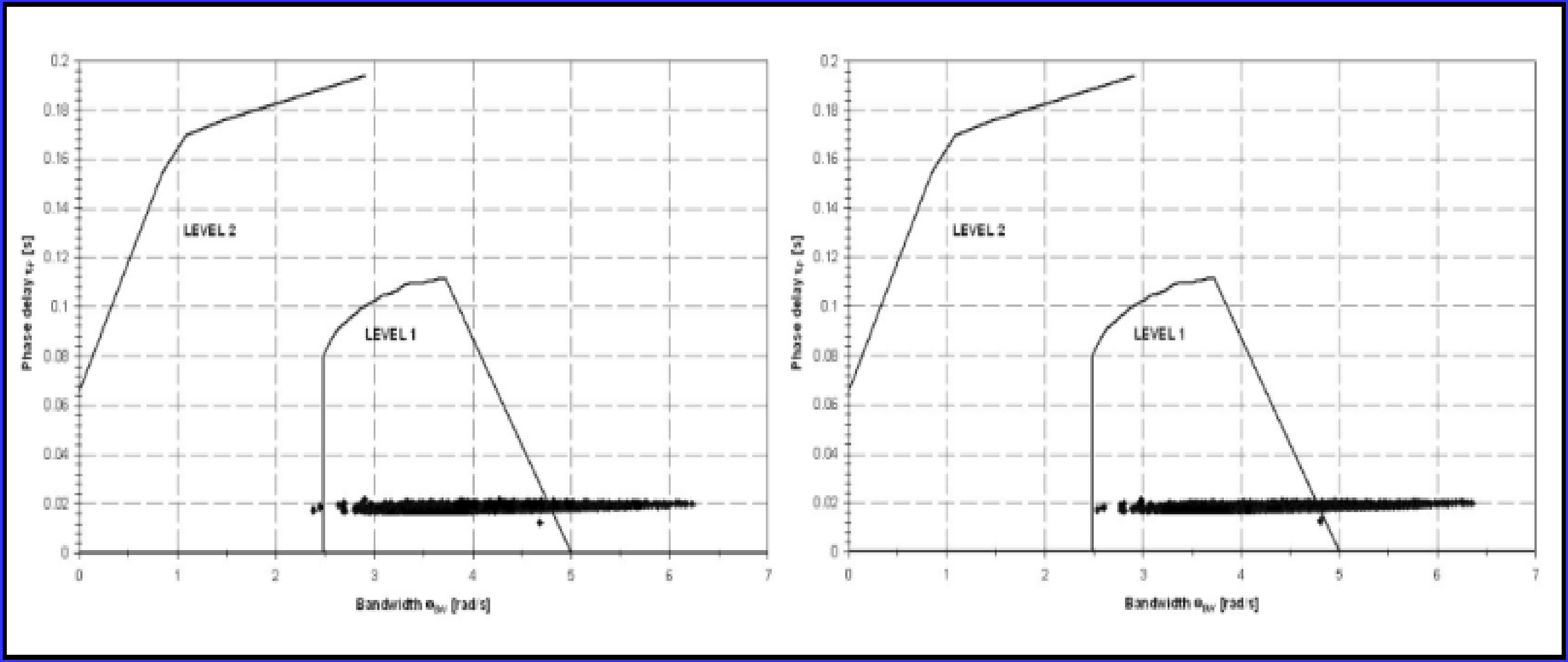
Bandwidth Criterion for Phase 3



Bandwidth criterion for K^1 (left) and K^3 (right)

Gain Selection

- Multi-objective criterion as a compromise between different specifications

Finally we selected gain K^1 as the best compromise between all the specs and criteria!

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Conclusions: Flight Tests in Sicily - 1

- Evaluation of the payload carrying capabilities and autonomous flight performance
- Mission test involving altitude, velocity and heading changing was performed in Sicily
- Checking effectiveness of the control laws for longitudinal and lateral-directional dynamics
- Flight control design based on RAs for stabilization and guidance

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Conclusions: Flight Tests in Sicily - 2

- Satisfactory response of MH1000
- Possible improvements by iterative design procedure
- Stability of the platform is crucial for the video quality and in the effectiveness of the surveillance and monitoring tasks

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Color Camera: Right Turn

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Color Camera: Landing Phase

Infrared Camera - 1

Infrared Camera - 1

road

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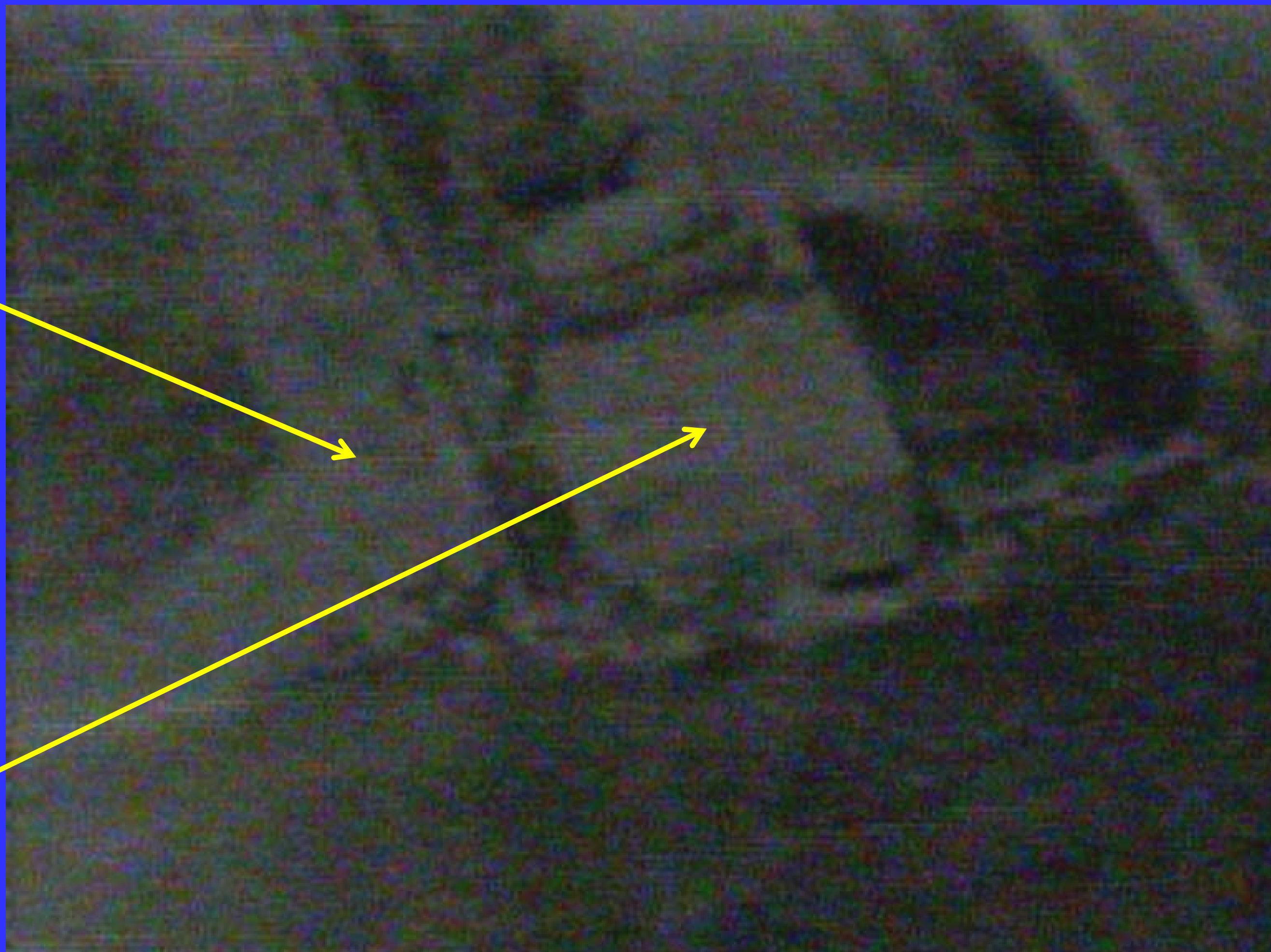
Infrared Camera - 1



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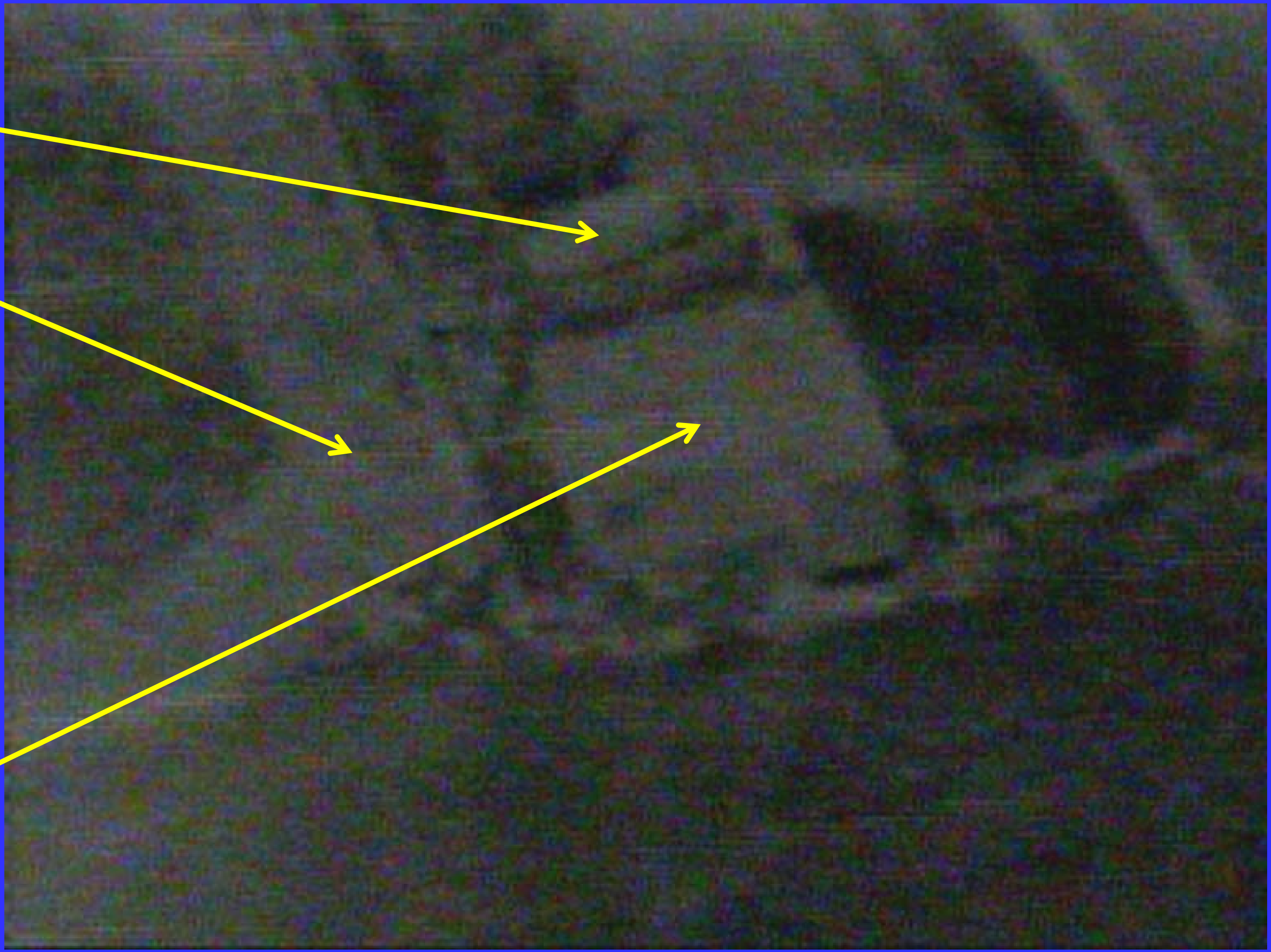
Infrared Camera - 1



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Infrared Camera - 1

Infrared Camera - 2

Infrared Camera - 3

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Acknowledgment

- Thanks to Laura Lorefice and Barbara Pralio. This talk is based on our joint work[1]

- Thanks to Andrea Sanna and his students for the flight test video and the computer graphics animation

[1] L. Lorefice, B. Pralio and R. Tempo (2006)

Randomized Algorithms: A Success Story

A Success Story

- Randomized Algorithms (RAs) are successfully used in various areas

1. Sorting problems (e.g., QuickSort algorithm)
2. Data structuring, search trees, graph algorithms,
3. Mathematics of finance: Computation of integrals
4. Genomics: String matching and classification
5. Motion and path planning problems

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Powerful Tool

- Randomized Algorithms provide a powerful tool for problems which are computationally intractable (in a classical sense)
- We introduce a different notion of problem tractability
- Objective: Breaking the curse of dimensionality[1]

[1] R. Bellman (1957)

Computationally Intractable Problems

- Many control problems are computationally intractable
- This is generally meant for *NP*-hard problems

- Examples:
  - fixed order controller design
  - static output feedback
  - stability of interval matrices
  - μ computation

Randomized Algorithms and Control

- Remarkably, randomized methods and algorithms are not used *systematically* in systems and control
- Objective of this research is the development of mathematically rigorous methods, not straightforward use of Monte Carlo simulations

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References

- "*Randomized Algorithms*" by R. Motwani and P. Raghavan, Cambridge University Press, 1995
- "*Randomized Algorithms for Analysis and Control of Uncertain Systems*" by R. Tempo, G. Calafiore and F. Dabbene, Springer-Verlag, 2005

- Additional documents, papers, MATLAB™ codes, etc, please consult

http://staff.polito.it/roberto.tempo

Conclusions

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PAC Algorithms

- Randomized algorithms are Probably Approximately Correct (PAC)
- We give up a guaranteed deterministic solution
- This implies accepting a “small” risk of giving a wrong solution
- The risk can be made arbitrarily small (but not zero) taking suitable values of so-called confidence and accuracy