

Marcello Edoardo Delitala

Corso Duca degli Abruzzi 24
10129 Torino

☎ +39 011 0907537

✉ marcello.delitala@polito.it

<http://staff.polito.it/marcello.delitala>



Personal Details

Date of birth December 19, 1975

Place of birth Asti, Italy

Citizenship Italian

Position

November 2005–
current **Assistant Professor**, Mathematical Physics (*disciplinary sector MAT07*), Department of Mathematics - Politecnico di Torino - Italy.

Education

November 2008 **Habilitation à Diriger des Recherches**, *Spécialité: Mathématiques Appliquées*, University of Toulouse - France.

March 2005 **Ph.D. in Mathematics for Engineering Sciences**, Department of Mathematics - Politecnico of Turin - Italy.

October 2000 **Master Degree in Theoretical Physics**, University of Turin - Italy.

Achievements and Awards

2010 **Kepler Prize** of the European Academy of Sciences for the project: "Managing complexity, reducing perplexity: a workshop on Complex Living Systems" involving: G. Ajmone (OCSE, Paris) for the Social Sciences and A. Picco (EMBL - European Molecular Biology Laboratory for biological sciences, Heidelberg). the prize has been granted for a project for the organization of an interdisciplinary international workshop on biological issues which has taken place in Heidelberg, May 16-20, 2011.

<http://www.eurasc.org/kepler/Kepler2010.asp>

2010 **Invited Mathematician**, Interdisciplinary Workshop on Pattern Formation in Morphogenesis, IHES - Paris, January 11-14, 2010).

2007 **ERC Starting Grant 2007**: Proposal classified as "*potentially fundable meeting the threshold of excellence*".
(Less than 5% of the original 9167 proposals has been recommended for funding)

2007 **INdAM-SIMAI Award** (National Institute of High Mathematics and Italian Society of Industrial Mathematics), *Best PhD dissertation* issued in the period 2004-2006.

2007 **Invited Lecturer**, BIOMAT - Mathematics and Life Sciences: Tumor growth and stem cells, University of Granada, Spain, June 11–16, 2007.

2007 **Invited Lecturer**, XVIII UMI (Italian Mathematical Union) Congress, Bari, Italy, September 24–27, 2007.

- 2006 **Thomson ESI December 2006 and January 2007 - Essential Sciences Indicators: Special Topics:** Highly Cited Paper on Mathematics for the article: *Mathematical Methods and Tools of Kinetic Theory towards Modelling Complex Biological Systems*.
Interview available at www.esi-topics.com/fbp/december06-delitala-Bellouq.html
- 2006 **“in-cites”. The top 3 Highly Cited Papers published in the last 2 years for Mathematics.** Article: *Mathematical Topics on the Modelling Complex Multicellular Systems and Tumor Immune Cells Competition*.
Citation available at www.in-cites.com/hotpapers/2006/november06-math.html

Founded Projects

- **FIRB** (Italian Ministry of Research and University) - Project. “Mathematical methods and tools for the modelling and simulation of the onset of cancer, immune competition, and therapies” related to the IDEAS-ERC starting grant, 2009-2013. [Principal Investigator]
- **Health** - Collaborative Large-scale project: FP7 Project. “Resolve chronic inflammation and achieve healthy ageing by understanding non-regenerative repair”, 2008-2012. [Project Manager of the Work-Package 4]
- **SITI** (High Institute on Innovation Territorial Systems) Project: "Security problems in crowded public buildings", 2006 [Co-investigator].
- **Project for Young Researchers** - Politecnico of Torino: “Developments of generalized kinetic (Boltzmann) models in applied sciences", 2002 - 2003 [Coordinator].

Participation in International and National Research Projects

- **PRIN** (Italian Ministry of Research and University): “Modelli della teoria cinetica matematica per particelle attive nello studio di sistemi complessi” (Kinetic Theory models of active particles for modelling complex systems), 2005-2007. [Participant]
- **Research Training Network** (RTN) European Project: Modelling mathematical methods and computer simulations of tumor growth and therapy, 2004–2007. [Participant]
- **INdAM** (National Institute of High Mathematics) Project: "Traffic flow and optimization on complex networks" 2005-2006. [Participant]
- **CNR** (National Council for Research) Strategic Project: “Metodi e modelli matematici nello studio dei fenomeni biologici”, 2003. [Participant]
- **Research Training Network** (RTN) European Project: Using Mathematical Modelling and Computer Simulation to Improve Cancer Therapy, 2000–2003. [Participant]
- **GNFM** (National Group of Mathematical Physics) - **INdAM** (National Institute of High Mathematics) Project: “Metodi e modelli matematici di traffico veicolare”, 2002. [Participant]
- **Compagnia di SanPaolo** (Private Bank) Project: “Contributo della matematica applicata e dell’informatica alla ricerca contro i tumori”, 2001–2004. [Participant]

Training responsibilities

Teaching Teaching of several courses in Applied Mathematics, both at Bachelor's degree level and at Master of Science level.

TEACHING

Mathematical analysis II
Assistant. (2010 - 2011)

BACHELOR DEGREE PROGRAM

Mechanical Engineering
Energetic Engineering
Architectural Engineering

Rational mechanics
(2009 - 2010)

Civil Engineering
Mathematics In Engineering

Rational mechanics
(2011 - current)

Environmental Engineering

Numerical methods and statistical for Engineers
(2011 - current)

Civil Engineering

Mathematical analysis II
Assistant. (2006 - 2009)

Computer and Communication
Networks Engineering
Computer Engineering
Electronic Engineering
Physical Engineering
Telecommunication Engineering

TEACHING

Analytical mechanics
(2010 - current)

MASTER OF SCIENCE PROGRAM

Mechanical Engineering

Mathematical methods for Engineers
(2009 - 2010)

Civil Engineering

Mathematical physics
Assistant. (2005 - 2007)

Computer Engineering
Electronic Engineering
Physical Engineering
Telecommunication Engineering

Mathematical physics laboratory
Assistant. (2005 - 2008)

Computer Engineering
Electronic Engineering
Telecommunication Engineering

Transportation models and Kinetic Theories:
analytical and computational methods
of kinetic theories

Mathematical Modelling in Engineering

Assistant. (2005 - 2007)

Equations of Mathematical Physics
Assistant. (2005 - 2007)

Mathematical Modelling in Engineering

TEACHING

Modelling complex living systems
by kinetic theory methods
(2007 - 2008), (2010 - current)

PH.D. PROGRAM

(Ph.D Course)

Mathematical modelling
in applied sciences
(2009 - 2010)

(Ph.D Course)

Training Ph.D Students	Supervising of the activities of T. Lorenzi, related to the modelling of biological systems (2009-current) - <i>Ph.D Thesis In Mathematics For Engineering Sciences</i> .
	Supervising of the activities of I. Brazzoli, related to the modelling of biological systems (2005-2007) - <i>Ph.D Thesis In Mathematics For Engineering Sciences</i> .
	Supervising of the activities of A. Tosin, related to the modelling of vehicular traffic flow. (2006-2007) - <i>Ph.D Thesis In Mathematics For Engineering Sciences</i> .
Supervision of Master Dissertations	2009 - Genetic mutations (Mondino). 2008 - Bioinformatics in immunology (Peretti), Colonrectal cancer dynamics (Lorenzi). 2007 - Biological Systems (Bombaci), Social dynamics (Calajo). 2006 - Crowd dynamics (Canavesio), Traffic flow modelling (Pollio, Rassa).

Research Books

- [1] A. Bellouquid, M. Delitala, **Mathematical Modeling of Complex Biological Systems. A Kinetic Theory Approach**, (Birkhäuser - Springer, Boston), 2006.

Scientific Papers on Peer Reviewed International Journals

- [1] M. Delitala, Critical analysis and perspectives on the kinetic (cellular) theory of immune competition, *Math. Comp. Model.* 35, 63–75, 2002.
- [2] N. Bellomo, M. Delitala, V. Coscia, On the mathematical theory of vehicular traffic flow I - Fluid dynamic and kinetic modeling, *Math. Mod. Meth. Appl. Sci.* 12, 1801–1843, 2002.
- [3] M. Delitala, Nonlinear models of vehicular traffic flow - New frameworks of the mathematical kinetic theory, *CR Mécanique* 331, 817–822, 2003.
- [4] E. De Angelis, M. Delitala, A. Marasco, A. Romano, Bifurcation analysis for a mean field modeling of tumor and immune system competition, *Math. Comp. Model.* 37, 1131–1142, 2003.
- [5] M.L. Bertotti, M. Delitala, From discrete kinetic and stochastic game theory to modelling complex systems in applied sciences, *Math. Mod. Meth. Appl. Sci.* 14, 1061–1084, 2004.
- [6] A. Bellouquid, M. Delitala, Kinetic (cellular) models of cell progression and competition with the immune system, *Z. angew. Math. Phys.* 55, 295–317, 2004.
- [7] N. Bellomo, A. Bellouquid, M. Delitala, Mathematical topics on the modelling complex multicellular systems and tumor immune cells competition, *Math. Mod. Meth. Appl. Sci.* 14, 1683–1733, 2004.
- [8] M. Delitala, Generalized kinetic theory approach to modeling spread and evolution of epidemics, *Math. Comp. Model.* 39, 1–12, 2004.
- [9] A. Bellouquid, M. Delitala, Mathematical Methods and Tools of Kinetic Theory towards Modelling Complex Biological Systems, *Math. Mod. Meth. Appl. Sci.* 15, 1639–1666, 2005.
- [10] E. De Angelis, M. Delitala, Modelling complex systems in applied sciences methods and tools of the mathematical kinetic theory for active particles, *Math. Comp. Model.* 43, 1310–1328, 2006.
- [11] M.L. Bertotti, M. Delitala, On the qualitative analysis of the solutions of a mathematical model of social dynamics, *Appl. Math. Letters* 19, 1107–1112, 2006.

- [12] V. Coscia, M. Delitala, P. Frasca, On the mathematical theory of vehicular traffic flow II. Discrete velocity kinetic models, *Int. J. Nonlinear Mech.*, 42, 411–421, 2007.
- [13] M. Delitala, A. Tosin, Mathematical modeling of vehicular traffic: a discrete kinetic approach, *Math. Mod. Meth. Appl. Sci.*, 17, 901–932, 2007.
- [14] M.L. Bertotti, M. Delitala, Conservation laws and asymptotic behavior of a model of social dynamics, *Nonlinear Anal. Real. World. Appl.*, 9, 183–196, 2008.
- [15] M.L. Bertotti, M. Delitala, On a discrete generalized kinetic approach for modelling persuaders influence in opinion formation processes, *Math. Comp. Model.*, 48, 1107–1121, 2008.
- [16] F. Berthelin, P. Degond, M. Delitala, M. Rascole, A model for the formation and evolution of traffic jams, *Archives of Rational Mechanics and Analysis*, 187, 2, 185–220, 2008.
- [17] N. Bellomo, M. Delitala, From the mathematical kinetic, and stochastic game theory for active particles to modelling mutations, onset, progression and immune competition of cancer cells, *Physics of Life Reviews*, 5, 183–206, 2008.
- [18] P. Degond, M. Delitala, Modelling and simulation of vehicular traffic jam formation, *Kinetic and Related Models*, 1, 279–293, 2008.
- [19] M.L. Bertotti, M. Delitala, On the existence of limit cycles in opinion formation processes under time-periodic influence of persuaders, *Math. Mod. Meth. Appl. Sci.*, 18, 913–934, 2008.
- [20] N. Bellomo, C. Bianca, M. Delitala, Complexity analysis and mathematical tools towards the modelling of living systems, *Physics of Life Reviews*, 6, 144–175, 2009.
- [21] S. De Lillo, M. Delitala, M.C. Salvatori, Modelling epidemics and virus mutations by methods of the mathematical kinetic theory for active particles, *Math. Mod. Meth. Appl. Sci.*, 19, 1405–1425, 2009.
- [22] N. Bellomo, M. Delitala, On the coupling of higher and lower scales using the mathematical kinetic theory of active particles, *Appl. Math. Lett.*, 22, 646–650, 2009.
- [23] M.L. Bertotti, M. Delitala, Clusters formation in opinion dynamics: A qualitative analysis, *Z. angew. Math. Phys.*, 61, 583–602, 2010.
- [24] A. Bellouquid, M. Delitala, Asymptotic limits of a discrete kinetic theory model of vehicular traffic, *Appl. Math. Letters* 24, 972–978, 2011.
- [25] M. Delitala, T. Lorenzi, A mathematical model for progression and heterogeneity in colorectal cancer dynamics, *Theor. Popul. Biol.*, 79, 130–138, 2011.
- [26] C. Bianca, M. Delitala, On the modelling genetic mutations and immune system competition, *Comput. Math. Appl.*, 61, 2362–2375, 2011.
- [27] M. Delitala, P. Pucci, M.C. Salvatori, From methods of the mathematical kinetic theory for active particles to modelling virus mutations, *Math. Mod. Meth. Appl. Sci.*, 21, 843–870, 2011.

Other Publications: Lecture Notes, Proceedings

- [a] N. Bellomo, A. Bellouquid, M. Delitala, Methods and tools of the mathematical kinetic theory toward modeling complex biological systems (pp. 175–194) in *Transport Phenomena and Kinetic Theory* Eds. C. Cercignani and E. Gabetta, Birkhäuser-Springer (Boston), 2007. [**Chapter of Book**].

- [b] M.L. Bertotti, M. Delitala, N. Bellomo, From the kinetic theory of active particles to the modelling of social behaviors and politics, *Quality and Quantity*, 41, 545–555, 2007. **[Paper on peer reviewed journal]**.
Collection of Lectures of the Conference: "Matematica e società per i quaranta anni della rivista", organized by P. Contucci and S. Graffi, Bologna, 8-9 December 2006.
- [c] A. Bellouquid, M. Delitala, From the Kinetic Theory for active particles to modelling the immune competition, (pp. 31–47) in *Selected Topics on Cancer Modelling Genesis - Evolution - Immune Competition - Therapy*, Eds. N. Bellomo, M. Chaplain and E. De Angelis, Birkhäuser-Springer (Boston), 2008. **[Chapter of book]**.
- [d] N. Bellomo, A. Bellouquid, M. Delitala, From the mathematical kinetic theory of active particles to multiscale modelling of complex biological systems, *Math. Comp. Model.*, 47, 687–698, 2008. **[Paper on peer reviewed journal]**.
Collection of Lectures of the Conference: "Metodos matematicos e modelagem em fenomenos biofisicos", organized by B. Perthame, P. Markowich and J. Zubelli, Angra dos Reis (Rio de Janeiro), Brazil, 5-11 March, 2006.
- [e] N. Bellomo, E. De Angelis, M. Delitala, **Lecture Notes on Mathematical Modelling in Applied Sciences**, SIMAI e-Lecture Notes, ISSN 1970-4429, 1–148, 2008. **[University Lecture Notes]**.
Available on line at: <http://cab.unime.it/journals/index.php/lecture/issue/view/5>
- [f] M. Delitala, **Mathematical modelling of complex living systems** 1–108, 2008. **[Dissertation]**.
Dissertation of the *Habilitation à Diriger des Recherches* obtained at the University of Toulouse, November 2008.
- [g] M. Delitala, On the Mathematical Modelling of Complex Biological Systems. A Kinetic Theory Approach, in *Bollettino UMI* (Italian Mathematical Union), Serie IX, I(3), 603–618, 2008. **[Paper on peer reviewed (national) journal]**.
Proceedings of the XVIII Conference UMI, September 21-27, 2007, Bari, Italy.
- [h] N. Bellomo, E. De Angelis, M. Delitala, **Lecture notes on mathematical modelling from applied sciences to complex systems**, SIMAI e-Lecture Notes, ISBN-13: 978-88-905708-7-2, ISBN-A: 10.978.88905708/72, vol. 8, 1–169, 2010. **[University Lecture Notes]**.
Available on line at: <http://cab.unime.it/journals/index.php/lecture/article/view/576>

Research activity

The development and application of mathematical methods to the modelling, qualitative analysis and simulation of large complex systems in life sciences is a new challenging research field of applied mathematics. The behavior of a complex system, namely a system of several individuals interacting in a non-linear manner, is difficult to understand and model only by the description of the dynamics of a few individual entities.

The interest on the complex systems has seen, in recent years, a remarkable increase, due to a raising awareness that many systems in nature share this kind of *complexity*, and that they cannot be successfully modelled by traditional methods used for systems of the inert matter. Moreover, an increasing number of applications in technology, economy, and social sciences resemble such systems, given the high number of elements and the complex interactions among them. For instance, fields as transportation and communication networks, social and economics interchanges. Generally, these complex systems are such that the collective dynamics is determined by complex individual interactions, while the modelling of individual dynamics does not straightforwardly lead to the mathematical description of the collective dynamics.

The research activity has been mainly focused on the development of a new concept of the “kinetic theory for active particles”, briefly KTAP, to model particles whose microscopic state includes, besides the geometrical and mechanical variables, also an additional variable, called “activity”, that models the

ability to organize their dynamics according to specific purposes and strategies. I believe that this kind of approach is particularly suitable to model complex systems phenomena, and the theoretical work has been coupled with applications, focused on modelling aspects and related mathematical problems, in the following fields:

- Social systems and behavioural economy;
- Vehicular traffic;
- Immune competition and complex biological systems.

The general approach followed in all above fields has been developed along the following guidelines:

- Phenomenological and qualitative analysis of the system under consideration and development of a suitable modelling strategy related to derivation of mathematical tools;
- Derivation of specific models according to afore mentioned mathematical framework and tools;
- Statement of mathematical problems generated by the application of models. The qualitative analysis of these problems is mainly focused on the asymptotic behavior in time;
- Simulations and critical analysis of models.

Systems of real world (both inert and living matter) are generally constituted by several interacting elements. This implies that mathematical models can be designed at various observation and representation scales.

The *microscopic scale* corresponds to modelling, by mathematical equations, the evolution of the variable suitable to describe the physical state of each single object. An alternative to the above approach can be developed if the system is constituted by a large number of elements and it is possible to obtain suitable locally in space averages of their state in an elementary space volume ideally tending to zero. In this case, the modelling can be developed at the *macroscopic scale* which refers to the evolution of locally averaged quantities, called *macroscopic variables*.

Different classes of equations correspond to the above scaling. Generally, models designed at the *microscopic scale* are stated in terms of ordinary differential equations, while models at the *macroscopic scale* are stated in terms of partial differential equations. The modelling is developed within the framework of deterministic causality principles unless some external noise is added. This means that once a cause is given, the effect is deterministically identified.

Motivations to use the *macroscopic scale* instead of the *microscopic one* are also related to practical objective of reducing complexity. For instance, when systems involve a large number of interacting elements, the number of equations of the model may be too large to be computationally tractable. Moreover, only *macroscopic quantities* are often of practical interest, so that it is convenient to deal with equations involving directly these variables.

An alternative approach is the *statistical (kinetic) representation*, where the state of the whole system is described by a suitable probability distribution over the *microscopic state* of the interacting elements, that is the dependent variable in kinetic models. *Macroscopic observable quantities* are computed by moments weighted by the distribution function.

The research activity reviewed in the sequel has been focused at systematic approach to modelling large complex systems belonging to the living matter by suitable developments of the mathematical kinetic theory. Specifically, modelling aspects have been focused on large systems of individuals whose dynamics follows rules determined by their organized, or even intelligent, ability. It is a rather different approach with respect to the traditional methods of the mathematical kinetic theory which refers to elements of the inert matter, where interactions follow rules of classical or quantum mechanics.

It is well understood that none of the above representations is fully appropriate, considering that the number of interacting entities in crowds, swarms, and vehicular traffic is not large enough to justify either the continuum mechanics approximation, namely at the *macroscopic scale*, nor the statistical representation by continuous probability distribution functions over the *microscopic state* according to the methods of the kinetic theory. Moreover, the approach at the *microscopic scale* involves computational complexity problems related to the number of interacting entities (which may even change in time). In many cases this number is very large; therefore, the corresponding number of equations generated to model the individual dynamics becomes so large that fluctuation errors cannot be avoided.

Motivations to use a statistical representation of living systems can be found in various fields of applied sciences. Among others, biology is a science which needs this approach, as well documented in the

interesting paper by Hartwell and *al.* (H.L. Hartwell, J.J. Hopfield, S. Leibner, and A.W. Murray, (1999), From molecular to modular cell biology, *Nature*, **402**, c47-c52).

The following sentence is particularly significant:

Although living systems obey the laws of physics and chemistry, the notion of function or purpose differentiate biology from other natural sciences.

Moreover, in the same paper:

More important, what really distinguish biology from physics are survival and reproduction, and the concomitant notion of function.

Although the above sentences are specifically referred to biological systems, their validity can certainly be extended to a large variety of complex living systems.

The description of the system by methods of the mathematical kinetic theory essentially means defining the microscopic state of the entities interacting within a large system of them, and the distribution function over the above state. The microscopic state always includes geometrical variables suitable to identify their position and shape, and mechanical quantities related to their velocity. However, in the case of living systems, the identification of the microscopic state needs an additional variable, called activity, which is specific of the particular system which is object of modelling. For instance, this variable may be related to biological functions in the case of cell populations, or to the social state in the case of dynamics of populations.

Entities of living systems, called active particles, may be organized into several interacting populations. Each population or functional subsystem, is identified by the function, namely the activity, that is collectively expressed by groups of active particles. Interactions among active particles do not follow rules of classical mechanics due to the fact that they have the ability of expressing specific strategies and, in some cases, generate proliferative and/or destructive events. Rather, stochastic games can be used to model interactions at the microscopic scale, while methods of the mathematical kinetic theory can be used to derive evolution equations for the probability distribution function over the microscopic state of the interacting active particles. Macroscopic quantities are obtained by weighted moments of the afore-mentioned distribution function.

The derivation of mathematical tools, namely with the derivation of a new mathematical theory focused on the modelling of complex phenomena of living system, is the argument of the papers [1] and [10], which show how the mathematical approach can be applied in several fields of applied sciences. Technical developments have been subsequently proposed referring to specific applications, such as social dynamics [5]. [20] reports latest developments of the KTAP's methods and framework and its applications.

As already mentioned, interactions do not follow the laws of classical mechanics, but are conditioned by the different strategies developed by the interacting individuals. Moreover, an additional problem to deal with is the modelling of the heterogeneous behavior (over the activity variable) of active particles related to their self-organizing ability, which is not the same for all of them, and has to be regarded as a random variable linked to a probability distribution that might have a local nature and may be modified by several types of interactions at the microscopic level.

If interaction rules, modelled by stochastic games, are the same for all particles, the term **generalized kinetic theory** is used to identify the mathematical approach, while if heterogeneity of the activity variable is considered, we shall talk about **kinetic theory for active particles** (and the systems will be called "heterogeneous systems"). The various models and mathematical problems related to Vehicular Traffic Flow refers to the first class of equations, while Social Dynamics and Immune Competition deal with heterogeneous systems.

The modelling and related mathematical problems of the Vehicular Traffic Flow have been developed both by methods of mathematical kinetic theory and by hyperbolic conservation laws methods.

The main objectives of traffic flow modelling consists in reproducing the quantitative and qualitative behaviors of macroscopic flow phenomena, as, for instance, bottlenecks and interactions of groups of fast and slow vehicles, or the so called *fundamental diagram* (flow versus density) observed in steady uniform flow. The fundamental diagram shows phase transition phenomena from free to congested flow, while particularly important is the ability of models to reproduce these observed behaviors without artificially introducing into them this information.

After the critical analysis proposed in [2] focused on the understanding of the role of drivers in modelling interactions between vehicles on roads, and the note [3] dealing with modelling heterogeneity phenomena, the subsequent activity has been devoted to modelling traffic flow dynamics by the approach of the generalized kinetic theory with discrete velocity [12], [13].

Discretization of the velocity variable has been proposed to deal with the criticism on the granular nature of traffic flow, that prevents using the assumption of continuity of the distribution function over the

microscopic variable. The car-driver system is considered as an active particle. Microscopic interactions generate slowing down or acceleration of vehicles which may pass (or be passed by) other vehicles by shifting from one velocity to the other. The space variable is an essential feature of the model and plays a relevant role also in the mathematical description of microscopic interactions. This aspect generates modelling and computational problems to be carefully analyzed.

The essential difference between [12] and [13] is that the model proposed in [12] uses a discretization grid depending on the local density technically related to the so called *velocity diagram* (velocity versus density), while in [13] the discretization grid is fixed, and the dynamics at the microscopic level enables the model to reproduce the essential features, including phase transitions, of the fundamental and velocity diagrams.

It is interesting noticing that the macroscopic approach by conservation laws developed in [16] and [18] has the ability to reproduce the above experimental diagrams and several interesting traffic phenomena thanks to an appropriate modelling of the fluid dynamic quantities that are preserved.

Analytic mathematical problems, namely good position of the initial value problem, trend to equilibrium and asymptotic limits, are dealt with in [12], [13], [16], and [24].

The modelling of various phenomena of the Social Competition related to group of interest (or even individuals), is the argument of a set of papers. The individuals are regarded as active particles characterized by a microscopic state referred to their social collocation or other variables, such as social or political opinions.

The number of interacting active particles is supposed to be constant in time, while the distribution over the activity variable is heterogeneous. Microscopic interactions modify such a state, while the overall system is described by a probability density distribution over the microscopic state. Models are analyzed with special attention to their predictive ability. This means that the investigation is addressed to analyze how certain microscopic interactions may generate different types of evolution. Some technical generalizations concerning systems of several interacting populations refers to systems where the total number of interacting particles is still constant in time, but it may change in time within each population. The papers [5], [11], [14], [15], [19] and [23] are focused on models in the case of spatial homogeneity. Interactions modify the activity, while the space dynamics does not play a relevant role. Moreover, the modelling approach refers to active particles with discrete states according to mathematical structures introduced in [5]. Discretization is motivated, similarly to the case of traffic flow, by the need of identifying the state of the particles by ranges of values rather than by a continuous variable.

The approach is definitely new in the literature hand has been devoted first to modelling various aspects of social dynamics [5], [11], [14] and subsequently to opinion formation, [15] and [19], also including the role of external actions. Clustering of opinions is analyzed in [23]. An interesting mathematical problem dealt with in the above papers is the qualitative analysis (followed by simulations) of the asymptotic behavior, that shows trend towards suitable equilibrium configurations that depend not only on the dynamics at the microscopic level, but also on overall quantities such as, for instance in the first class of systems, the total wealth of the social system.

The above cited papers include detailed results on existence, uniqueness and stability of equilibrium solutions, while simulations complete the analysis by computing the shape of these configurations. The qualitative analysis has to tackle the technical difficulty of the identification of appropriate Lyapunov functionals.

The modelling problems, and related analytic and simulation problems related to complex biological systems, specifically the immune competition and spread and evolution of epidemics in the presence of genetic mutations, are the arguments of the another research line.

In the previous models, related to Vehicular Traffic Flow and Social Dynamics, the number of active particles is constant in time. This feature definitely reduces complexity problems. On the other hand, biological systems are such that interactions are accompanied by reproductive and destructive events. This implies severe difficulties in dealing with the qualitative and computational analysis of solutions of mathematical problems generated by the applications of models to describe specific phenomena of the immune competition.

This research line is documented in papers [1], [4], [6], [7], [9], [17], and in the book [I], dealing with models of the immune competition between malignant cells of an aggressive invasive guest and cells of the immune system. In details, papers [1], [6], and [9] deal with modelling issues and qualitative analysis focused on the asymptotic behaviour of the solutions. The main objective consists in assessing the role of the parameters of the model to generate either depletion of malignant cells due to the immune system that remain active, or the opposite behaviour: blow up of malignant cells due to the immune system that remains active. The qualitative analysis (the main results are reported in Chapter 4 of [I]) refers to number density of cells, while simulations ([4] and Chapter 5 of [I]) complete the analysis visualizing the

distribution function over the activity variable, that is related to the heterogeneous behaviour. Paper [7] and the book [I] review various developments. A model for gene mutations in colorectal cancer is dealt with in [25], while [26] deals with genetic mutation in cancer and the competition with immune cells in a more general setting.

It is worth remarking that it is shown in [17] how the modelling of biological systems needs the development of multiscale methods. For instance, referring to [22], coupling the dynamics at the lower molecular (genome) scale to the dynamics at the higher cellular scale. Moreover, asymptotic methods (see Chapter 6 of [I]) can be developed to derive macroscopic models corresponding to biological tissues.

The above research line has been further developed to study modelling and evolution of epidemics [8], and [21], including genetic mutations. This mathematical approach, anticipated in [8] and then developed in [21] and [27], generates a class of interesting mathematical problems due to the fact that the number of equations modelling the evolution of the system is variable in time.