THE PEDESTRIAN SPEED – DENSITY RELATION: MODELLING AND APPLICATION

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Summary

The relation between density and velocity of pedestrian movement has so far mainly been analysed using an empirical approach and fundamental relations found from the fitting of experimental measurements of the main quantities. The present study proposes a phenomenological model that is able to distinguish and take into account various factors that can affect the density-velocity relation by means of the induced microscopic walking phenomena. In particular, three factors are retained: the geographic area; the travel purpose; the effect of the lateral vibrations of the platform on which pedestrians walk, in view of the use of the fundamental diagrams within a crowd-structure interaction model applied to lively footbridges. The main features of the phenomenological model are then applied to a non-linear law found in literature and herein revisited in a more general form. The latter is finally used within a crowd-structure interaction model, which is applied to the T-bridge in Japan, in order to perform a sensitivity study of the deck response to different travel purposes.

Keywords: pedestrian flow; fundamental relations; footbridges.

1. Introduction

Since the Sixties, many studies have been dedicated to the determination of a law that links walking velocity to crowd density. Most of these studies belong to the transportation research field and have the aim of controlling the layout and dimensions of pedestrian walking facilities (e.g. [1][2]). In recent years, research has been directed towards the study of crowd flow patterns under emergency situations (e.g. [3]) and increasing attention has been devoted to the effects of crowd behaviour on the dynamics of structures in the civil engineering field (e.g. [4]). Because of the great number of factors that can affect pedestrian walking behaviour (age, culture, gender, travel purpose, type of infrastructure, walking direction), rather different fundamental diagrams can be found in literature. A complete review of the speed-density relations proposed so far can be found in [2] and [7]: most of them are based on fitting to experimental or observation data.

All the proposed speed-density relations have some common features, concerning the monotonic decreasing trend of the velocity v with increasing density u and the identification of some critical points [2]: the free speed v_M, which corresponds to the mean maximum velocity; the critical density u_c, corresponding to the upper bound for unconstrained free walking; the jam density u_M, that is, the maximum admissible density corresponding to null speed and flow; the capacity speed v_{ca} and density u_{ca}, corresponding to the maximum flow $q_{ca} = u_{ca}\, v_{ca}$. The region of density under u_{ca} is called free flow region, while the congestion region corresponds to a higher density than the capacity. These relations have the advantage of being expressed through very compact formulas; on the other hand, they do not distinguish the effects that each factor has on the walking behaviour. The resulting lack of generality makes them difficult to extend from experimental conditions to different real cases.

One of the above mentioned factors is the motion of the platform on which pedestrians walk. To the authors' knowledge, the sensitivity of the walking speed to the lateral motion of the platform has not be taken into account so far. The influence of platform motion on walking behaviour becomes very important when modelling crow-structure interaction phenomena. This topic has come to public attention in recent years, after a famous footbridge, the London Millennium Bridge [9], was closed the day it was opened because of excessive lateral vibrations induced by the walking crowd. *In situ* observations on actual footbridges allow some modelling assumptions to be introduced: the flow is one-directional and its value never exceeds the capacity flow; the walking surface is smooth and generally free from obstacles. Under such assumptions, a phenomenological relation that links the walking speed to the crowd density and the platform motion can be used within a coupled crowd-structure interaction model, where the crowd is described by a first order hydrodynamic model [4][5].

The present study proposes a speed-density relation based on phenomenological considerations, which takes into account the influence of the platform lateral motion. The general aim of the paper is to outline a modelling open framework that can take into account, in each of its parts, the macroscopic effects of different factors that affect the microscopic properties of pedestrian movement. In particular, each parameter of interest has to be represented by one coefficient, whose value can be determined by means of *ad hoc* experiments. In Section 2 the formulation of the phenomenological model is outlined; in Section 3 the main features of the model are used to revisit a non-linear law proposed by Weidmann [1], the so-called Kladek formula, which is rewritten in a more general form; finally, in Section 4, a sensitivity study on the introduced parameters is performed by applying the revisited Kladek formula within a crowd-structure interaction model [4][6], used to simulate a crowd event on the T-bridge in Japan.

2. Formulation of the model

The fundamental relation that links crowd density to walking velocity is usually expressed in the direct form $v=v(u)$. In this work, a phenomenological relation that links pedestrian velocity to crowd density and deck acceleration $\ddot{z}$ is proposed in the inverse form $u = u(v, \ddot{z})$. Bearing in mind that crowd density u [ped/m^2] is a scarcely intuitive quantity, its reciprocal Pedestrian Area Module (PAM) [10], i.e. the surface S occupied by a pedestrian, is used as a more manageable unit [m^2]. Hence, the proposed relation is based on some phenomenological considerations about $S(v, \ddot{z})$. Obviously, once the latter is obtained, the density can be calculated as the reciprocal of S, and the speed-density relation can be recovered as an inverse relation.

Among the various factors that can affect walking behaviour, two are specifically retained, that is, the geographic area and the travel purpose: the related coefficients are respectively named with the subscripts G and T. These factors are assumed to affect both free speed v_M and PAM S through the coefficients α and β, respectively. Three geographic areas have been considered, according to the biometric data reported in [7], that is, Europe, USA and Asia. The travel purpose classification recovers the one proposed by Oeding [8] and reported by Weidmann [1]: it is based on the evidence that a pedestrian in rush hour traffic walks faster than a pedestrian walking for leisure.

The free speed is expressed in the general form:

$$v_M = \bar{v}_M \alpha_G \alpha_T g(\zeta) \qquad (1)$$

where $\bar{v}_M$ =1.34 m/s is the average free speed [7]; the coefficients α_G and α_T were determined analyzing the data reported in [7] as the ratio between the proposed free speeds and $\bar{v}_M$, and they are reported in Table 1; ζ is the envelope of the deck acceleration time history $|\ddot{z}(t)|$; the corrective factor $g(\zeta)$ takes into account the sensitivity of v to the deck acceleration and has a qualitative trend, based on the following hypothesis:

- the lateral motion of the deck reduces the walking velocity;
- after the pedestrians have stopped because of excessive lateral vibrations at time t_s, a stop-and-go time interval Δt_r should elapse before they start walking again.

It follows:

$$g(\zeta) = \begin{cases} 1 & \zeta \le \ddot{z}_c \cap t \ge t_s + \Delta t_r \\ [\ddot{z}_M - \zeta(x,t)]/(\ddot{z}_M - \ddot{z}_c) & \ddot{z}_c < \zeta < \ddot{z}_M \cap t \ge t_s + \Delta t_r \\ 0 & \zeta \ge \ddot{z}_M \cap t_s < t < t_s + \Delta t_r \end{cases} \qquad (2)$$

where $\ddot{z}_c \cong$ 0.2 m/s^2 corresponds to the threshold of motion perception and $\ddot{z}_M$ =2.1 m/s^2 [11] is the maximum acceptable acceleration above which pedestrians stop walking.

Table 1 Coefficients of geographic area and travel purpose

Geographic area α_G			Travel purpose α_T		
Europe	USA	Asia	Rush-hour Business	Commuters Events	Leisure Shopping
1.05	1.01	0.92	1.20	1.11	0.84

The PAM occupied by a motionless pedestrian can be calculated considering an elliptical or a rectangular form. According to the latter hypothesis, the average surface occupied by a motionless pedestrian is $S_0=w_0d_0$, where w_0 and d_0 are the average lateral width and depth of a human body (Fig. 1). When a pedestrian is walking, a greater surface is required, that is, $S=wd$ (Fig. 1). Both the terms w and d can be expressed as a function of the walking velocity v. In addition, the lateral width could be made sensitive to the deck acceleration, since pedestrians tend to walk with their legs more widespread when the surface is laterally moving, as stressed by various authors (e.g. [9]).

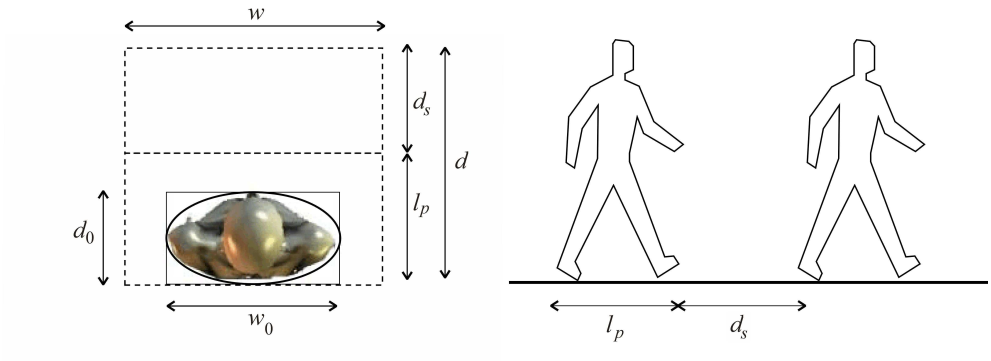


Fig. 1 Human body dimensions: motionless and walking pedestrian

The required forward distance d can be expressed as the sum of two terms: l_p, the step length, which is a function of the walking velocity and step frequency f_p; d_s, the sensory distance, which is defined by Fruin [10] as *the area required by the pedestrian for perception, evaluation and reaction*. While the former can be physically measured, the latter depends to a great extent on cultural and psychological factors and is, therefore, hard to evaluate. As for the lateral width w, even though the same distinction can be made between pacing and sensory zones, a unique width is retained because of the lack of available data. Therefore, S takes the form

$$S = w(\beta_G l_p + \beta_T d_s) \qquad (3)$$

where β_G and β_T are the coefficients of geographic area and travel purpose, respectively. It is worthwhile recalling that the value of the maximum admissible density can be calculated as $u_M=1/S(v=0)$, while the critical density is $u_c=1/S(v=v_M)$.

All the parameters introduced in the modelling framework are characterized in the following on the basis of several experimental data coming from various research fields such as transportation, biomechanics, safety and structural engineering.

According to [7], in a free walking regime a pedestrian requires a lateral additional space equal to about 62% of his average width. Therefore, the simplest expression for the lateral width $w(v)$ is the linear relation:

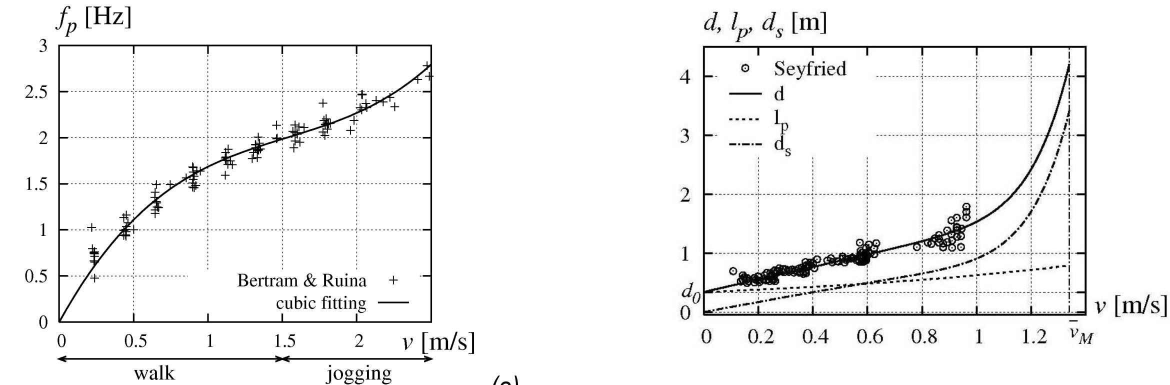
$$w(v) = w_0\left(1 + 0.62\frac{v}{\bar{v}_M}\right) \qquad (4)$$

where w_0=0.45 m [7].

The relation between the step length l_p and the velocity v can be derived from the relation between the step frequency f_p and v, since $l_p = v / f_p$ (Fig. 2b). The frequency-speed relation is derived from a cubic fitting to the experimental data in [12] (Fig. 2a) coming from laboratory tests on a treadmill with fixed velocity:

$$f_p = 0.35v^3 - 1.59v^2 + 2.93v \qquad (5)$$

Fig. 2b shows that, when a pedestrian is motionless (v =0), the body depth d_0, whose value is set equal to d_0=0.36 m according to Seyfried et al. [13], takes the place of the step length.



(a) (b)

Fig. 2 Proposed f_p-v (a) and d, l_p, d_s –v (b) relations

The sensory distance d_s (Fig. 2b) can be obtained as d-l_p, since some experimental data concerning the d-v relation are available in literature [13]. The data have been fitted according to the law

$$d(v) = d_0 + 1.06v + bv^{10} \qquad (6)$$

where $b = (2.08v_M - d_0)/v_M^{10}$, so that $d(v_M)$ takes the value for which $u_c \cong$ 0.3 ped/m², as proposed by Oeding [8].

The geographic area coefficient β_G is derived, considering the dimension occupied by the human body in different countries (Table 1 in [7]), as the ratio between the surface averaged per geographic area and the mean surface S_0. The travel purpose coefficient β_T is determined by fitting the reciprocal of Eq. (3) to the experimental data reported by Oeding [8] and Fruin [10] (Table 2). A comparison between α_T and β_T shows that β_T monotonically decreases for increasing free speeds v_M associated to travel purposes.

Table 2 Coefficients β_T

Geographic area β_G		Travel purpose β_T		
Europe / USA	Asia	Rush-hour Business	Commuters Events	Leisure Shopping
1.075	0.87	0.55	0.93	1.07

3. The Kladek formula revisited

Since the direct form of the fundamental relation is more suitable for practical use, the main features of the proposed phenomenological model are used to revisit a non-linear fundamental law proposed by Weidmann [1], the so-called Kladek formula:

$$v = 1.34\left\{1 - \exp\left[-1.913\left(\frac{1}{u} - \frac{1}{5.4}\right)\right]\right\} \qquad (7)$$

which is rewritten in the more general form:

$$v = v_M\left\{1 - \exp\left[-\gamma\left(\frac{1}{u} - \frac{1}{u_M}\right)\right]\right\} \qquad (8)$$

where v_M is determined according to Eq. (1) and u_M is defined according to the principles explained in the previous section. On one hand, the parameter γ plays the same role as the parameter β_T previously introduced: therefore, it is determined as the fitting free parameter referring to the same experimental data. On the other hand, γ is a synthetic model parameter without a direct phenomenological meaning: hence, it does not show a monotonic trend for increasing free speeds (Table 3).

Table 3 Coefficients γ

	Rush hour/Business	Commuters/Events	Leisure/Shopping
γ	0.273 u_M	0.214 u_M	0.245 u_M
v_M [m/s]	1.69	1.56	1.18

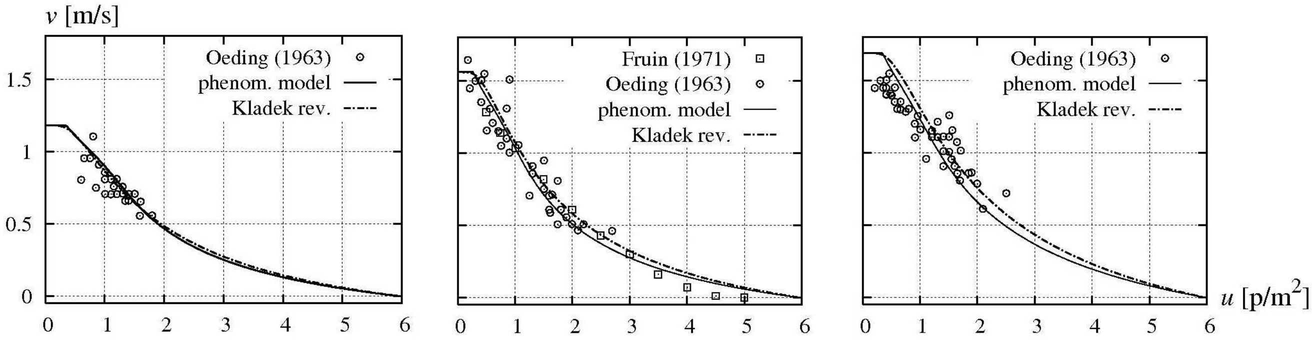


Fig. 3 Fitting to experimental data: leisure/shopping, commuters/events, rush hour/business

Fig. 3 graphs the results of the fitting to experimental data obtained with the phenomenological model and the Kladek formula revisited. As can be noticed, the physical-based model shows a surprising agreement with the revisited Kladek formula, even though the same results are obtained through different approaches, i.e. based on microscopic interpretative modelling and fitting to macroscopic observation data, respectively. The physical-based model represents a complementary tool to shed some light on the role that microscopic walking phenomena play in the macroscopic fundamental relation, while the revisited Kladek formula, because of its continuity and compact direct form, is more suitable for practical use.

4. Application

The revisited Kladek formula is introduced as closure equation within the crowd-structure interaction model described in [4]. In order to test the sensitivity of the structural response to different pedestrian traffic conditions, four computational simulations are performed on the same benchmark, the T-bridge in Japan, by varying the coefficient γ and the value of u_M and v_M for the following combinations: Asia-rush hour (AR), Asia-commuters (AC), Asia-leisure (AL) and USA-leisure (UL), which correspond to a progressive decrease of v for u>0.8 ped/m^2 (Fig. 4a). The substitution of the four fundamental laws in Eq. (5) leads to the f_p-u relations represented in Fig. 4b. The first case (AR) refers to the conditions actually occurred on the T-bridge and described in [4]. The description of the structural model and of the crowd boundary conditions is detailed in [4] and sketched in Fig. 5.

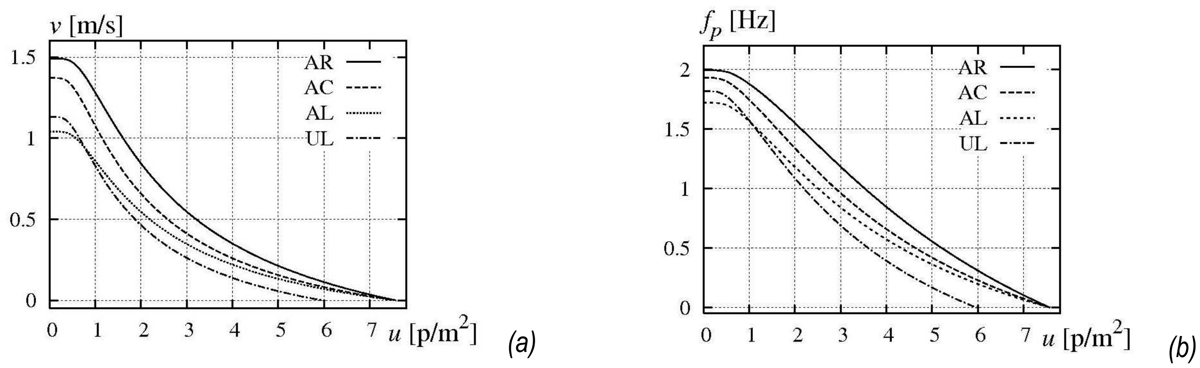


Fig. 4 v-u (a) and f_p-u (b) relations

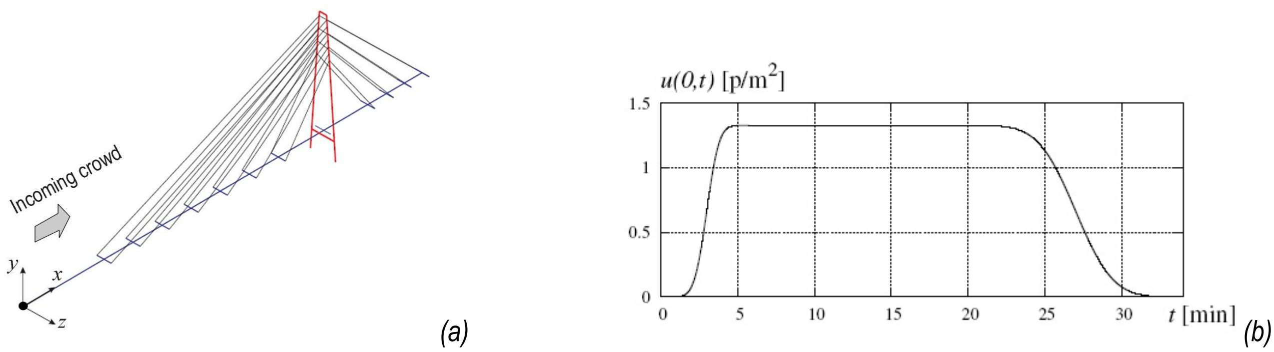


Fig. 5 T-bridge FE model (a) and crowd boundary condition in inlet (b)

The proposed approach allows the evolution in space and time of both Crowd and Structure systems to be described. Fig. 6 reports the time-space distributions of some main variables obtained through the computational simulations: the crowd density u, the deck lateral acceleration ζ and the frequency ratio $f_r = f_{pl} / f_s$, where f_{pl} is the step lateral frequency and f_s is the structural frequency. The overall evolution in time of u is mainly due to the imposed boundary condition at the inlet. In other words, the crowd dynamics is not affected by non-linear traffic phenomena due to a crowd density above the capacity value u_{ca} or to the effects of excessive lateral acceleration of the deck, that is, $\zeta > \ddot{z}_M$ (=2.1 m/s^2 [6]).

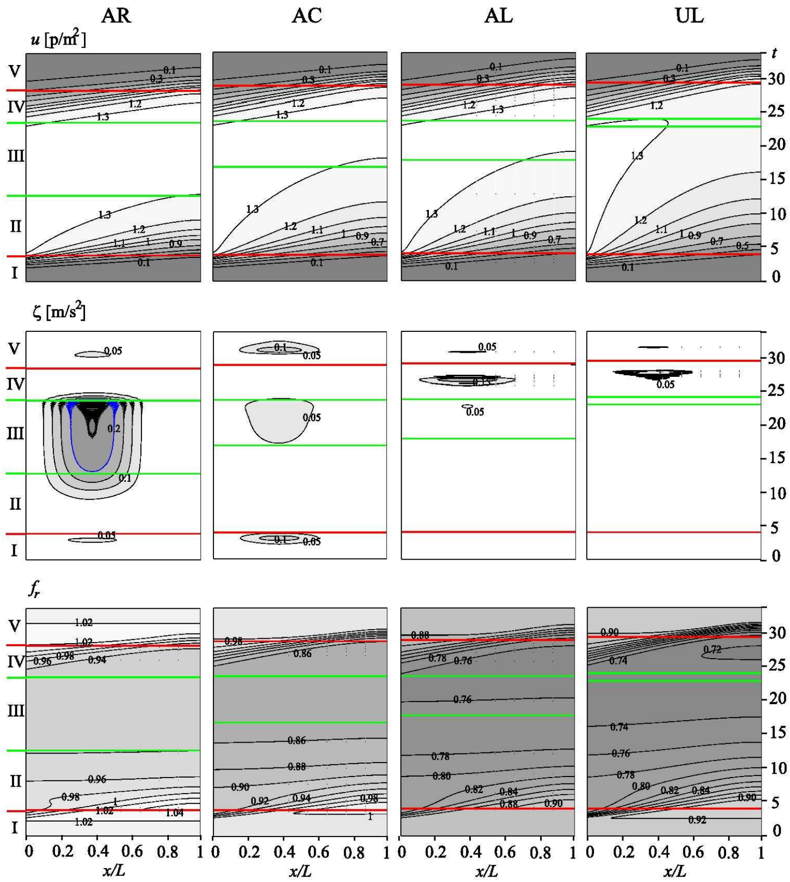


Fig. 6 Evolution in space and time of the main variables

Looking at the time-space distributions of the crowd density (Fig. 6), five crowd regimes have been identified for the four cases (Fig. 7):

- Regime I "advancing front";
- Regime II "filling gradient";
- Regime III "uniform crowd";
- Regime IV "vacating gradient";
- Regime V "leaving front".

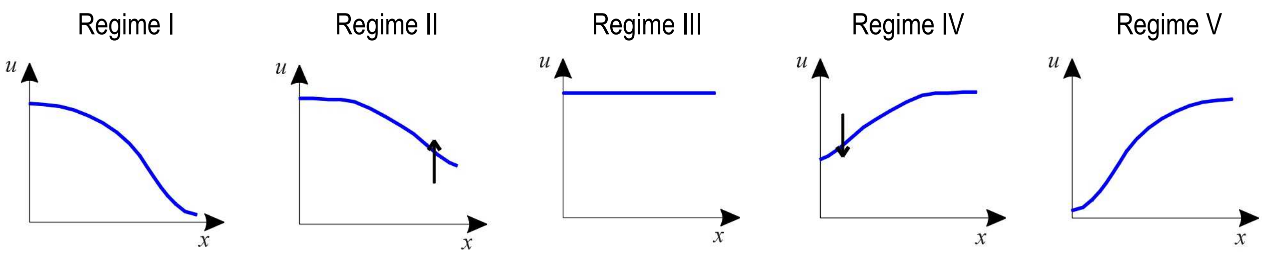


Fig. 7 Scheme of the space distribution of u for the five regimes

The upper limit of regime I and the lower bound of regime V correspond respectively to the maximum and minimum difference between the crowd density at the outlet and at the inlet $\Delta u = u(L,t) - u(0,t)$. The boundaries of regime III have been determined as the time window with a mean value of u along the span equal to 99% of the maximum mean density $\bar{u}$ and a standard deviation less than 0.01 $\bar{u}$.

The five regimes are also highlighted in Fig. 8, which plots the deck acceleration time histories in $x=0.3L$, corresponding to the node monitored by Fujino et al. [14]. First, it can be noticed that the gradual decrease of the pedestrian velocity from AR to UL causes longer regimes I and II and a consequent progressive shortening of regime III. The main consequence can be seen in the deck response, which is gradually shifted in time and decreasing: the case with rush-hour traffic causes a deck vibration amplitude which is almost three times the amplitude obtained in a leisure traffic condition in the same geographic area. The lock-in threshold $\ddot{z}_c$ is exceeded only in the first case AR. In every traffic condition, the regimes I and V are characterized by two local maxima of the structural response that can be related to the travelling load effects. In particular, the highest response in the case AC is due to a value of f_r closer to the unity (Fig. 6), which means that the force is almost resonant with the deck first lateral mode.

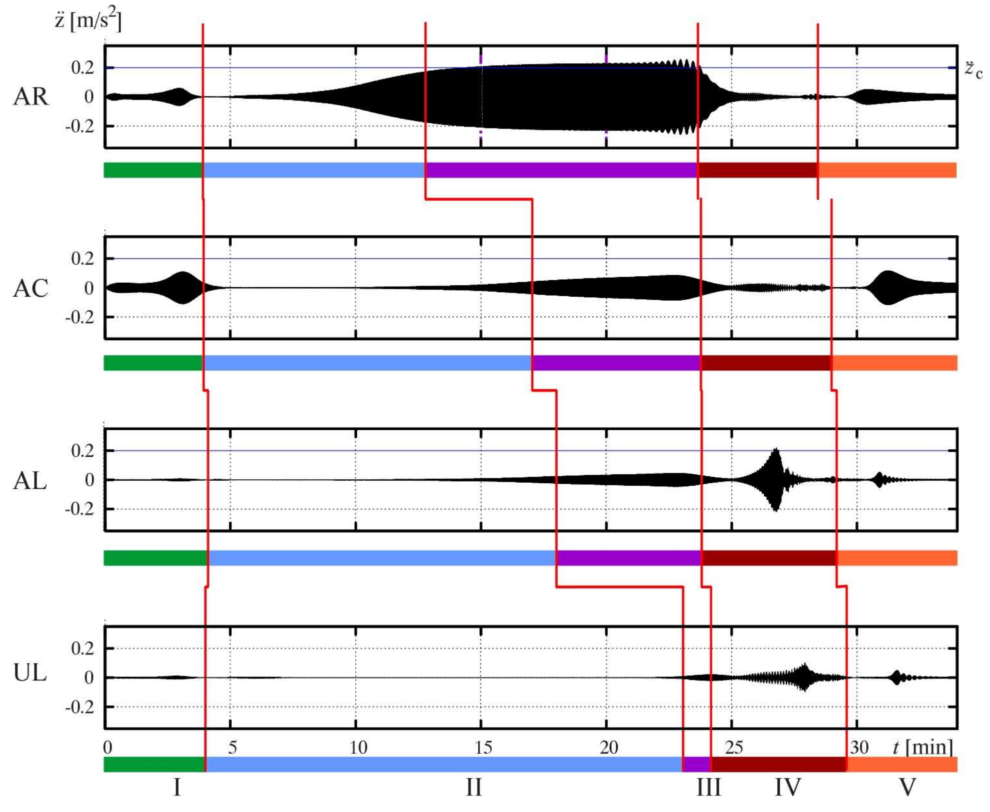


Fig. 8 Time histories of the deck lateral acceleration

5. Conclusion

In this paper a pedestrian fundamental relation based on microscopic modelling of the pedestrian behaviour has been presented. The physical considerations on which the model is based have been used to revisit a non-linear law, the Kladek formula, herein rewritten in a more general form in order to account for the effects of the deck lateral motion, the geographic area and the travel purpose on the walking velocity. The revisited Kladek formula shows an excellent agreement with the physical-based model and, because of its direct compact form, it is more suitable to be used within a crowd-structure interaction model, as shown in the application on a real footbridge. The sensitivity study on the closure equation shows that different crowd travel purposes can lead to quite different structural responses. For this reason footbridges should be designed according to the pedestrian traffic type which is more likely to occur during their lifetime or referring to various possible traffic scenarios.

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