Design of passive systems for control of inelastic structures

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SUMMARY

A design strategy for control of buildings experiencing inelastic deformations during seismic response is formulated. The strategy is using weakened, and/or softened, elements in a structural system while adding passive energy dissipation devices (e.g. viscous fluid devices, etc.) in order to control simultaneously accelerations and deformations response during seismic events. A design methodology is developed to determine the locations and the magnitude of weakening and/or softening of structural elements and the added damping while insuring structural stability. A two-stage design procedure is suggested: (i) first using a nonlinear active control algorithm, to determine the new structural parameters while insuring stability, then (ii) determine the properties of equivalent structural parameters of passive system, which can be implemented by removing or weakening some structural elements, or connections, and by addition of energy dissipation systems. Passive dampers and weakened elements are designed using an optimization algorithm to obtain a response as close as possible to an actively controlled system. A case study of a five-story building subjected to El Centro ground motion, as well as to an ensemble of simulated ground motions, is presented to illustrate the procedure. The results show that following the design strategy, a control of both peak inter-story drifts and total accelerations can be obtained. Copyright © 2008 John Wiley & Sons, Ltd.
INTRODUCTION

Traditional methods for seismic retrofitting of buildings often rely on stiffening, strengthening and adding damping to structures to reduce drift and deformations during the seismic response. Recently, retrofit techniques include often addition of passive control devices [1, 2]. These devices usually dissipate energy by means of viscosity, friction, or yielding, and may also stiffen the building, hence reduce the inter-story drifts. Many researchers attempted to design optimally the passive control devices in structures (see an extensive presentation in [3]). Gluck et al. [4] followed by Agrawal and Yang [5] introduced an optimized design for the distribution of added stiffness and damping in linear structures using a linear quadratic regulator (LQR), or using other concepts of active control. Passive control, however, cannot insure by itself a reduction of accelerations of buildings’ floors, in particular in those undergoing inelastic deformations. Moreover, this control often may lead to an increase in these accelerations due to the additional forces created by stiffening and strengthening. It should also be noted that most of the above-mentioned methodologies assume a linear behavior of the damped structure and only a few consider yielding structures (e.g. [6]).

Reinhorn et al. [7] and Viti et al. [8] introduced the concept of weakening structures (reducing strength), while introducing added viscous damping to reduce simultaneously total accelerations and inter-story drifts. Design methodologies for softening the structure (reducing stiffness) and adding damping devices have been proposed by Cimellaro et al. [9–12] and others. Although some of these methodologies can address structures undergoing inelastic deformations, those are mostly limited to softening of structural elements (or connections). Loh et al. [13] presented a procedure to design of visco-elastic dampers, using an LQR algorithm to determine the control forces that are subsequently transformed to forces, which can be provided by visco-elastic dampers, similarly to the idea introduced by Gluck et al. [4]. However, this method assumes also that the structure is linear and does not reach the strength limit.

Inelastic structures have their maximum acceleration response limited by their strength capacity but they may become unstable plastic mechanisms with, or without, deteriorating properties and geometric nonlinearities. Weakening or softening may aggravate the instability problem. A solution using structural control principles may allow design of stable and efficient systems.

Recently Lavan et al. [14] proposed a design methodology for weakening the structures and adding damping devices, which assumes proportional modifications in strength and stiffness. This assumption, however, becomes questionable, when certain types of weakening and/or softening techniques are used (e.g. rocking columns [15]). The method is non-iterative and the solution is based on the assumption that same displacement history can be achieved with either an active, or a passive controlled structural system.

The approach presented in this paper is an extension of the method introduced by Lavan et al. [14]. The method proposed herein, however, is more general and it allows an uncoupled change of stiffness and strength, suitable for implementation in inelastic structures (Figure 1). Moreover, the method is using an iterative procedure unrestricted by the displacement performance, offering a broader and better performance with the structural changes.

DESIGN METHODOLOGY

The proposed design method is based on two steps. In the first step, an appropriate nonlinear control law is selected and used to determine a control ‘force’ vector, which contains the required changes.
in the structural system. The control requirements usually involve also reduction of stiffness, strength, and damping, which may lead to an unstable inelastic system. However, by using in the first step a suitable nonlinear control algorithm, such as the ‘sliding mode control (SMC)’ [16, 17], the stability of the system is automatically assured. In the second step, the ‘active control force’ is redesigned and converted into an equivalent ‘passive force’, whose components can be implemented by addition or subtraction of stiffness, strength, and damping [4, 14]. The conversion is using an optimization algorithm designed to minimize the difference in the response of the ‘active’ system obtained in the first step and in the equivalent redesigned system. While passively controlled elastic structures are inherently stable, inelastic structures may become unstable in the presence of plastic hinges with, or without, deterioration. Moreover, weakened structures may be prone to such instability. The main reason for which an active control law is used during the design procedure of the ‘passive system’ is to guarantee stability of the controlled structure in case of weakening. In fact, the ‘SMC’, used here for exemplification, has been designed using the Lyapunov direct method [17] and this guarantees that the controlled weakened inelastic structure will remain stable. Although the design strategy does not provide optimal solution, it achieves feasible controlled reduction of accelerations and deformations while assuring stability.

**PROBLEM FORMULATION**

Consider a multi-degree-of-freedom (DOF) nonlinear inelastic structure subjected to base acceleration. The general equation of motion of the nonlinear system with active control forces is given by

\[
M\ddot{x}(t) + C\dot{x}(t) + Kx(t) + T_s f_s[x(t)] = Hu(t) + \eta w(t)
\]

where \(x(t)\) is the displacement vector, over-dots indicate time derivatives, \(M, C\) are mass and inherent damping matrices, respectively, \(K\) is the stiffness matrix of all linear elements, \(f_s[x(t)]\) is a vector of restoring forces in all nonlinear structural elements, \(T_s\) is a location matrix of the nonlinear elements, \(u(t)\) is a vector of active control forces, \(H\) is a location matrix for the active control forces, \(w(t)\) is a scalar denoting the base acceleration, \(\eta\) is the base excitation directivity.
matrix. The component $i$ of the nonlinear restoring force vector $f_s[x(t)]$ can be modeled using Sivaselvan and Reinhorn’s continuous evolutionary model [18]

$$
\dot{f}_i = k_i \dot{x}_i(t) \left( x_i + (1 - x_i) \left\{ 1 - \left| \frac{f_{si}}{f_{yi}} \right| \right\} \left[ \eta_{li} + \eta_{2i} \text{sgn}(f_{si} \dot{x}_i) \right] \right)
$$

(2)

in which $k_i$ is the elastic stiffness, $x_i$ is the ratio of post-yielding to pre-yielding stiffness, $n_i$ is the power controlling the transition from elastic to inelastic range, $\eta_{li}$ and $\eta_{2i}$ are parameters controlling the shape of the unloading curve ($\eta_{2i} = 1 - \eta_{li}$ for compatibility with the plasticity theory), $f_{si}^*$ is the hysteretic share of the restoring force, $f_{yi}^* = (1 - x_i) f_{yi}$ is the yield force of the hysteretic component. This model is similar to the Bouc–Wen model [19], with the evolution of the restoring force expressed in terms of velocity [18]. Note that the $i$th component is linear elastic, if $x_i = 1.0$. In a state space notation, Equation (1) can be written as follows:

$$
\dot{z}(t) = Az(t) - \bar{B}f_s(z) + Bu(t) + e(t)
$$

(3)

where $z(t)$ is the state vector, $A$ is the linear elastic system matrix, $B$ is the control forces location matrix, $\bar{B}$ is the location matrix of restoring forces, and $e(t)$ is the excitation vector given by

$$
z(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix}
$$

(4)

The state-space formulation in Equation (3) is the base for further description of the design procedure.

**Step 1: Design of active control law.** There are many active control strategies, which can be used for this design step; however, there are only few, which can be used for inelastic yielding systems, which develop hysteretic behavior. One of the methods, capable to consider the nonlinear behavior of the system and accommodate uncertainties involved in structural analysis, is the SMC [16, 17, 20]. The SMC includes the definition of a surface that ‘traps’ the response vector in the Nyquist plane and drives it to the origin, and a rule that brings the response vector into the sliding surface. The main advantages of SMC are: (i) the possibility of stabilizing nonlinear systems, which cannot be stabilized by using continuous state feedback laws; (ii) its robustness against a large class of model uncertainties; (iii) the need for a reduced amount of information in comparison to classical control techniques.

Yang et al. [20] present the details of the SMC procedure to design a controller based on SMC for a nonlinear shear-type building. A possible law for continuous SMC in the case of saturated controller can be formulated as

$$
u_i(t) = \begin{cases} 
\dot{x}_i^* G_i - \delta_i \dot{x}_i & \text{if } |\dot{x}_i^* G_i - \delta_i \dot{x}_i| \leq \bar{u}_i \max, \\
\bar{u}_i \max \text{sgn}[\dot{x}_i^* G_i - \delta_i \dot{x}_i] & \text{otherwise}
\end{cases}
$$

(5)

where $\bar{u}_i$ is the control force, $\bar{u}_i \max$ represents the upper bound of the $i$th control force, $\delta_1, \delta_2, \ldots, \delta_r (\delta_r \geq 0)$ define the sliding margin, and $\dot{x}_i^*(0 \leq \dot{x}_i^* \leq 1)$. $G_i$, and $\dot{x}_i$ are the control coefficients explicitly derived in Appendix A. In this proposed procedure, the upper bound $\bar{u}_i \max$
is derived from the yielding strength of members making the method useful for inelastic structures. To determine the adequate upper bound, the algorithm is applied iteratively and the control force is set to stay just above the lowest acceptable strength of the member. This process is illustrated in the example below. While the theory and implementation of SMC for active systems are not new [20], the formulation presented above is setting the stage for the design of structural elements in step 2. Note that the SMC was selected here for explanation of the proposed strategy; however, different algorithms for inelastic structures can be used alternately without loss of generality.

**Step 2: Design of an equivalent passive control system.** The continuous time changing control force obtained in step 1, \( u(t) \), cannot be delivered by a passive system. However, a passive system, which can deliver almost the same performance, can be designed, if structural properties can be increased or decreased. The control force \( u(t) \) obtained in step 1 is resolved into a vector of passive forces including change in stiffness, damping, and strength at the locations of the assumed control components. Through an optimization procedure, the components and sizes of the passive forces are established. An error function is defined and expressed as the difference between the total forces acting upon each DOF included in the control forces obtained in step 1 and the total forces acting in the modified passively controlled (strength modified, stiffness modified, and damped) structure. The passive system is designed minimizing this error function. The error vector is first defined at a given time \( t \)

\[
E(t) = E(f_p, f_d, t) = \left\{ \begin{array}{l}
-Hu(t) + T_s f_s a(x_a(t), \dot{x}_a(t)) + C_0 \dot{x}_a(t) \\-T_s f_s p(x_p(t), \dot{x}_p(t)) + (C_0 + C_d) \dot{x}_p(t) \end{array} \right\}
\tag{6}
\]

where \( u \) are the active control forces obtained in the first step, the superscript ‘a’ and ‘p’ indicate that the term is evaluated from the actively or passively controlled configuration, respectively, \( f_s a \) and \( f_s p \) are the restoring forces, \( C_0 \) and \( C_d \) are the damping matrices of inherent damping and of the added devices, respectively, \( x_a \) and \( x_p \) are the displacement histories of the controlled structure. It is important to emphasize that all the restoring forces are considered in the error formulation, because these restoring forces can be altered due to weakening (or strengthening). Integrating numerically the square of the error vector of each component over time (for all DOFs) and expressing all time dependencies with the subscript ‘\( t \)’ the following error functions are obtained:

\[
e_i(c_i, \beta_i, k_i) = \int_t W_i(t) E_i^2(t) \, dt, \quad i = 1, \ldots, n \tag{7}
\]

where \( W_i \) are time-dependent weighting factors for the different DOFs and \( E_i \) is the entry \( i \) of the vector \( E \) of Equation (6). The objective function to be minimized is defined as the sum of the error functions described in Equation (7) given by the following expression:

\[
F(X) = \|e\|_1 = e_1 + e_2 + \cdots + e_n \tag{8}
\]

where \( X \) is the vector of design variables defined as follows:

\[
X = [\beta \ c \ k] \quad \text{with} \quad \beta = [\beta_1, \ldots, \beta_n], \quad c = [c_1, \ldots, c_n], \quad k = [k_1, \ldots, k_n] \tag{9}
\]

where \( c \) is the vector of damping coefficients of the added dampers, \( k \) is the vector of stiffness changes, and \( \beta \) is a vector of weakening or strengthening factors, such that the yielding force in
the element \(i\) is \(\beta_i\) multiplied by its original yielding force. With \(\beta_i < 1\), weakening is obtained. A nonlinear optimization problem is derived as follows:

Minimize: \(F(\mathbf{X})\) — as objective function

subjected to: \(X_i^L \leq X_i \leq X_i^U\), \(i = 1, \ldots, n\) as side constraints

The solution of this problem can be determined following an analytical procedure once some mathematical manipulations are made \([11]\) to reformulate the objective function \(F(\mathbf{X})\) as a quadratic form. Assuming that the history of \(f_{si}\) in Equation (4) is known (or taken from previous iterations), then upon substitution of Equation (4) into Equation (3) the hysteretic force in the modified structure can be then written as follows:

\[
f_{si}(t) = Q_i(t)k_i + D_i(t)\beta_i, \quad i = 1, \ldots, n
\]

where

\[
Q_i(t) = a_i x_i(t + \tau) + (1 - a_i) \dot{x}_i(t) \left(1 - \frac{|f_{si}^*|}{|f_{yi}^*|} (\eta_1 + \eta_2 \text{sgn}(f_{si}^*(t)\dot{x}_i(t)))\right)
\]

\[
D_i(t) = f_{si}^*(t)
\]

In addition, let us define

\[
A_i(t) = -h_i u_i(t) + t_{si} f_{si}^a(t) + c_i \dot{x}_i^a(t), \quad i = 1, \ldots, n
\]

where \(h_i\) and \(t_{si}\) are unit terms of \(\mathbf{H}\) and \(\mathbf{T}_s\) matrices. Substituting Equation (7) into Equation (8), using the terms in Equations (11)–(13), and assuming unitary weights, the following quadratic error function is obtained:

\[
e_i(c_i, \beta_i, k_i) = \sum_t (R_{ti} - x_{ti}^P c_i - D_{ti} \beta_i - Q_{ti} k_i)^2, \quad i = 1, \ldots, n
\]

where \(R_{ti}\) contains all known quantities as follows:

\[
R_{ti} = A_{ti} = x_{ti}^P c_{i0}, \quad i = 1, \ldots, n
\]

Equation (14), containing the quadratic error function, can be rewritten in a compact form

\[
e_i(c_i, \beta_i, k_i) = F_i + G_i c_i^2 + H_i \beta_i^2 + I_i k_i^2 - 2L_i c_i - 2M_i \beta_i
\]

\[
-2N_i k_i + 2O_i c_i \beta_i + 2P_i c_i k_i + 2S_i k_i \beta_i, \quad i = 1, \ldots, n
\]

where

\[
F_i = \sum_t (R_{ti})^2, \quad G_i = \sum_t (\dot{x}_{ti}^P)^2, \quad H_i = \sum_t (D_{ti})^2
\]

\[
I_i = \sum_t (Q_{ti})^2, \quad L_i = \sum_t (R_{ti} \dot{x}_{ti}^P), \quad M_i = \sum_t (R_{ti} D_{ti})
\]

\[
N_i = \sum_t (R_{ti} Q_{ti}), \quad O_i = \sum_t (D_{ti} \dot{x}_{ti}^P), \quad P_i = \sum_t (Q_{ti} \dot{x}_{ti}^P), \quad S_i = \sum_t (D_{ti} Q_{ti})
\]
All of the variables defined in Equation (17) are evaluated in an iterative manner, as shown in Figure 2. Their values are based on the history of $x_p$, $\dot{x}_p$ and $v_p$ from the previous iterations.

Hence, the objective function becomes a simple function of the design variables. Imposing the first-order condition

$$\frac{\partial e_i(c_i, \beta_i, k_i)}{\partial c_i} = 0, \quad \frac{\partial e_i(c_i, \beta_i, k_i)}{\partial k_i} = 0, \quad \frac{\partial e_i(c_i, \beta_i, k_i)}{\partial \beta_i} = 0, \quad i = 1, \ldots, n$$

The solution of the unconstrained optimization problem is determined as

$$c_i = \frac{-I_i L_i H_i + L_i S_i^2 + N_i P_i H_i - N_i O_i S_i - M_i P_i S_i + I_i M_i O_i}{-2O_i P_i S_i + I_i O_i^2 + P_i^2 H_i - I_i G_i H_i + G_i S_i^2}$$

$$k_i = \frac{L_i P_i H_i - L_i O_i S_i + N_i O_i^2 - N_i G_i H_i + M_i G_i S_i - M_i O_i P_i}{-2O_i P_i S_i + I_i O_i^2 + P_i^2 H_i - I_i G_i H_i + G_i S_i^2}, \quad i = 1, \ldots, n$$

$$\beta_i = \frac{-L_i P_i S_i + I_i L_i O_i + N_i G_i S_i - N_i O_i P_i - I_i M_i G_i + M_i P_i^2}{-2O_i P_i S_i + I_i O_i^2 + P_i^2 H_i - I_i G_i H_i + G_i S_i^2}$$

The Hessian matrix in the problem is given by

$$H_i = \begin{bmatrix}
\frac{\partial^2 e_i}{\partial c_i^2} & \frac{\partial^2 e_i}{\partial c_i \partial k_i} & \frac{\partial^2 e_i}{\partial c_i \partial \beta_i} \\
\frac{\partial^2 e_i}{\partial k_i \partial c_i} & \frac{\partial^2 e_i}{\partial k_i^2} & \frac{\partial^2 e_i}{\partial k_i \partial \beta_i} \\
\frac{\partial^2 e_i}{\partial \beta_i \partial c_i} & \frac{\partial^2 e_i}{\partial \beta_i \partial k_i} & \frac{\partial^2 e_i}{\partial \beta_i^2}
\end{bmatrix} = 2 \begin{bmatrix}
G_i & P_i & O_i \\
P_i & I_i & S_i \\
O_i & S_i & H_i
\end{bmatrix}$$

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In order to obtain a global optimum solution, the Hessian must be positive definite for all values of $\beta_i$, $k_i$, and $c_i$, which requires that the eigenvalues must be positive in the entire feasible region.

Imposing the Kuhn–Tucker conditions, the boundary conditions dictated by the side constraints are also considered. For each boundary condition, an optimal solution is found and then the optimal solution among all the optimal boundary conditions is chosen as a global optimum solution.

For the first iteration, the displacement history of the actively controlled structure ($x_a$) is used as a guess for the displacement history of the weakened and damped structure ($x_p$). Then, the initial optimal coefficients are substituted in Equations (1) and the updated displacement history ($x'_p$) of weakened and damped structure is obtained. Updated values of the restoring forces and velocity vectors for passive case are determined. The procedure is repeated iteratively substituting the updated coefficient $c_j$, $k_i$, and $\beta_i$ in the objective function as shown in Figure 2 until convergence is obtained. The algorithm converges usually within five iterations or less for all practical purposes.

Since a time history analysis is not required for solving the optimization problem, but only to evaluate the coefficients in Equation (17), the number of time history analyses required is small. Hence, the computational effort required to achieve the final design is relatively small. Note that alternative constraint optimization techniques can also be applied to the optimization problem described in Equation (10). At the end of step 2, the algorithm provides values at locations determined in step 1 for the added dampers and for the magnitude of the structural modifications, i.e. weakening and softening.

The proposed design methodology will assist the professional engineer in the selection of the location of the weakening elements in the structure without compromising stability. The use of the active control solution, which is determined in step 1, results in aggregated force vector at all possible locations that can be implemented by passive techniques (i.e. possible changes in stiffness, strength, damping, and active components). The second step provides the designer with types, sizes, as well as corrected locations of supplemental dampers and structural modifications, by optimizing the implementation of the ‘active control’ force components calculated in step 1.

For example, each entry of the design vector $c$ represents a value for a damper of unknown size at a given possible location. The matrix $C_d$, which is the supplemental damping matrix, is a function of the size of the dampers, as well as their locations. Since all possible locations for dampers are considered, the procedure in step 1 would assign a zero or a small negligible size for locations where dampers are not actually needed. Such dampers will be excluded from implementation. Hence, sizes and locations of dampers are attained. The same arguments hold for the locations of weakening and softening structural components.

The methodology described here does not seek a global optimal response, but is searching for a feasible response, which addresses multiple conflicting objectives, such as controlling accelerations and drifts in inelastic systems.

**NUMERICAL EXAMPLE: FIVE STORY**

To illustrate the procedure outlined previously, and the performance of its final designed passively controlled structure, a five-story shear-type nonlinear structure is considered. The masses on each floor are identical and equal to $m = 1000$ kg and the period of the structure is 0.5 s the yield displacement $D_y$ of each story was fixed equal to 0.01 m. A bilinear hysteretic model with 10% kinematic hardening was adopted. The elastic stiffness and yielding forces of the story units are shown in Figure 3. The typical story height considered is $h = 3.0$ m.
The inelastic parameters of Sivaselvan–Reinhorn hysteretic model [18] for each story are: \( a_i = 0.1, \eta_{1i} = \eta_{2i} = 0.5, \) and \( n_i = 95. \) The structure has been tested using El Centro earthquake and a series of 100 synthetic accelerograms developed at MCEER [21]. The yielding occurs throughout the whole uncontrolled building with large ductility demands, so the structure would fail without passive energy dissipation devices installed. For the design of the sliding surface using the LQR method, a diagonal weighting matrix \( Q \) with diagonal elements \( Q_{ii} = [10^6, 10^6, 10^6, 10^6, 10^6, 1, 1, 1, 1, 1, 1] \) is considered. A sliding margin of \( \delta_{ii} = 10^6 \text{kN ton cm/s} \) for \( i = 1–6, \) is used.

**Evaluation of upper limit of control force:** Theoretically, it has been shown that a complete compensation for the earthquake excitation can be achieved when each story is equipped with a controller that has no limit on the maximum force. However, such solution is unachievable with realistic limited resources. In the case studied here, control is assumed to act at all stories with a maximum horizontal control force restricted to the saturation limit of \( U_{\text{max}} \) (defined in Equation (5)). The behavior of the final solution for different values of \( U_{\text{max}} \) will be now examined. A sensitivity analysis is carried out to supply the reader with some sense to the effect of this parameter on the final design. It should be noted that such a sensitivity analysis is not required for design purposes. Three different distributions of force saturation limits throughout the height of the building are used in the analysis: (i) rectangular, uniform; (ii) triangular, with the vertex at the top; and (iii) convex parabolic, with zero at the top. The norm infinity of absolute accelerations \( \|a\|_{\infty} \) and inter-story drift \( \|d\|_{\infty} \) over all stories are determined, as follows:

\[
\|x\|_{\infty} = \max(|x_1|, \ldots, |x_n|) \quad (21)
\]

The two norms for accelerations, \( a, \) and displacements, \( d, \) are evaluated and plotted as function of maximum saturation control force \( U_{\text{max}} \) for the three distributions (see Figure 4). It should be noted that the sensitivity analysis shows that the maximum drift is not affected by the distribution of \( U_{\text{max}} \) and it tends to 0 monotonically when the ratio \( U_{\text{max}}/F_y \) increases.
Another important step in the proposed method is the selection of the weight factors $Q_i$ and the sliding margins $\delta_{si}$ of the SMC. A sensitivity analysis of the response of the active-controlled structure to the weighting factors $Q_i$ and the sliding margins $\delta_{si}$ is performed in order to supply the reader with understanding of their effect on the behavior of the final design. The effect of the weight factor $Q$ (assumed the same at all story levels) on the response when the building is actively controlled as shown in Figure 5.

It is shown that increasing the weighting factor (i.e. $Q = 10^6$–$10^9$), a better response is obtained in terms of acceleration and drift, for a wider range of values of $U_{\text{max}}/F_y$. As the weighting factor reduces, the acceleration tends to increase monotonically with the increment of $U_{\text{max}}/F_y$, while the drift reaches a minimum for values of $U_{\text{max}}/F_y$ between 0.2 and 0.4. The sliding margin $\delta_{si}$ does not have any effect on the response for values of $U_{\text{max}}/F_y < 0.1$ (see Figure 6). When $U_{\text{max}}/F_y > 0.1$, then the response improves both in terms of the acceleration and displacement with
a decrease in the values of the sliding margin. This behavior is expectable because the smaller the sliding margin, the smaller the oscillations around the sliding surface are [20].

Once the general effect of $Q$ and $\delta_{si}$ on the active-controlled structure is established, it is interesting to determine how these parameters affect the final design procedure described in this paper. As can be observed in Figure 7, the sliding margin $\delta_{si}$ does not affect the acceleration response for values of $U_{max}/F_y > 0.2$. On the other hand, for values of $U_{max}/F_y < 0.2$ both acceleration and drift responses improve, when the sliding margin decreases; however, for values of $U_{max}/F_y > 0.2$ the sliding margin $\delta_{si}$ (Figure 7) has an opposite effect in respect to the active-controlled structure shown in Figure 6.

In fact, while the acceleration is not affected, the drift increases when the sliding margin reduces (see Figure 7). More specifically, small values of the sliding margin generate drift of about 0.7%, while increasing the sliding margin it is possible to reduce the drift response about 35%, as shown

Figure 6. Sensitivity of the response actively controlled for different sliding margin factors $\delta$.

Figure 7. Sensitivity of the response to the sliding margin factor $\delta$. 
in Figure 7. Finally, it is shown in Figure 8 that the weighting factor $Q$ neither affects the drifts nor the accelerations for values of $U_{\text{max}}/F_y > 0.2$.

This leads to the conclusion that the weighting factor $Q$ affects the method proposed herein for values of $U_{\text{max}}/F_y < 0.2$, while more attention should be given to the selection of the sliding margin $\delta_{si}$ for $U_{\text{max}}/F_y > 0.2$.

Results of the design methodology: For the case study presented herein, a saturation limit is chosen as $U_{\text{max}} = 1500\text{kN}$ (for $U_{\text{max}}/F_y = 0.06$—where $F_y$ is the first-story shear yield force), in order to obtain a reduction in term of absolute accelerations and drifts without using high values of control forces (Figure 4). A lower bound of 0.5 and 0.0$kNs/m$ is assigned to the weakening parameters $\beta_j$ and to the damping coefficients $c_i$, respectively, while an upper bound of 1.0 and $138kNs/m$ is chosen for the same parameters to keep a limitation on the damping demands. For simplicity, softening is not considered in this example.

The results of the design methodology proposed above are shown in columns 2 and 3 of Table I, where it is observed that weakening and damping are required at all stories. The procedure converges after few iterations ($\leq 5$).

For comparison purposes, the distribution at the first iteration is shown in columns 4 and 5 of Table I, where it is assumed that the response history of the passively controlled structure is equal to

<table>
<thead>
<tr>
<th>Story level</th>
<th>Proposed method</th>
<th>Iteration 1</th>
<th>Lavan et al. [14]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(c_{\text{opt}})\text{ (kNs/m)}$</td>
<td>$(\beta(D_y/D_{y0}))$</td>
<td>$(c_{\text{opt}})\text{ (kNs/m)}$</td>
</tr>
<tr>
<td>No.</td>
<td>$(\beta)$</td>
<td>$\beta(D_y/D_{y0})$</td>
<td>$c_{\text{opt}}$</td>
</tr>
<tr>
<td>5</td>
<td>2.100</td>
<td>0.85</td>
<td>5.800</td>
</tr>
<tr>
<td>4</td>
<td>5.800</td>
<td>0.83</td>
<td>10.600</td>
</tr>
<tr>
<td>3</td>
<td>7.600</td>
<td>0.83</td>
<td>14.000</td>
</tr>
<tr>
<td>2</td>
<td>20.400</td>
<td>0.84</td>
<td>16.300</td>
</tr>
<tr>
<td>1</td>
<td>33.600</td>
<td>0.85</td>
<td>17.700</td>
</tr>
</tbody>
</table>

$(c_{\text{opt}})\text{ (kNs/m)}$ and $(\beta(D_y/D_{y0}))$ are the optimal weakening and damping parameters, respectively.
Figure 9. Weakening and damping distribution for the proposed iterative method (a,b), at the first step of iteration (c,d) and with method proposed by Lavan et al. (e,f).
Table II. Maximum accelerations and drift response for El Centro earthquake \((U_{\text{max}}/F_y = 0.06)\).

<table>
<thead>
<tr>
<th>Story level</th>
<th>Uncontrolled</th>
<th>Proposed method</th>
<th>Iteration 1</th>
<th>Lavan et al. [14]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Drift (x_i) (%)</td>
<td>(\ddot{x}_i) (m/s(^2))</td>
<td>Drift (x_i) (%)</td>
<td>(\ddot{x}_i) (m/s(^2))</td>
</tr>
<tr>
<td>5</td>
<td>0.84</td>
<td>9.12</td>
<td>0.48</td>
<td>7.28</td>
</tr>
<tr>
<td>4</td>
<td>0.64</td>
<td>7.51</td>
<td>0.42</td>
<td>6.07</td>
</tr>
<tr>
<td>3</td>
<td>0.37</td>
<td>6.62</td>
<td>0.36</td>
<td>5.31</td>
</tr>
<tr>
<td>2</td>
<td>0.50</td>
<td>6.85</td>
<td>0.52</td>
<td>4.33</td>
</tr>
<tr>
<td>1</td>
<td>0.90</td>
<td>4.88</td>
<td>0.57</td>
<td>3.57</td>
</tr>
</tbody>
</table>

Figure 10. Peak response distributions for different weakening and damping distributions.

It should be noted, however, that the original weakening distribution [14] considers also softening that is proportional to the weakening coefficient.

The maximum inter-story drift \(x_i\) and the maximum absolute acceleration \(a_i\) of each floor are presented in columns (2)–(9) of Table II and in Figure 10.

As can be seen from the above results, the response of the weakened and damped structure shows a reduction of 23\% (in average), in terms of drift at every story level with respect to the uncontrolled structure and 25\% reduction (in average), in terms of acceleration.

The hysteretic loops at the first and the fifth-story level of the building for the uncontrolled (a,d), active (b,e), and passive (c,f) are shown in Figure 11, which indicate that the controlled building still experiences inelastic deformation, but it is able to reduce both drifts and accelerations in respect to the uncontrolled case.

Influence of uncertainties of ground motion: The design methodology is further used for the design of weakening and damping considering a set of records, and the performance of the case study building is examined. A group of ground motions defined as ‘MCEER series’ [21] has
FIGURE 11. Hysteretic loops at the first- and fifth-story level for uncontrolled case (a,d); for active-controlled case (b,e) and for weakened and damped case (c,f) of the five-story building.

been selected for this purpose. This series consists of 100 synthetic near fault ground motions corresponding to different hazards with return periods of 250, 500, 1000, and 2500 years or 20, 10, 5, and 2% probability of exceeding (PE) in 50 years, respectively. They have been generated combining a white noise and a spectrum that is based on a physical model, the ‘specific barrier model’ [22] that has been calibrated using actual near fault records. Since the effect of weakening the structure should mostly affect the response of structures in strong motions, where yielding occurs, only the set of 25 records corresponding to 2% of the PE in 50 years has been used as input for the optimization procedure described above.

Control forces are applied at every story and the maximum horizontal control force has a saturation limit of 42% of the total yielding force at the first story, equal to $U_{\text{max}} = 10000 \text{kN}$. The upper and lower bounds of the optimization parameters remain the same as in the previous case. The results of the optimization procedure are shown in Table III, where it is observed that dampers and weakening are required at all levels.

Although the optimization was executed using only the 2% in 50 years ground motions, it is interesting to study the effect of weakening and damping the structure in all levels of seismic intensities. Hence, nonlinear dynamic analyses were performed using the ensemble of all 100 earthquake ground motions.

The maximum inter-story drift $x_i$ and the maximum absolute acceleration $\ddot{x}_i$ at 1st, 2nd, 3rd, and 5th floor are presented in Figures 12 and 13 for all records considered.

The short horizontal dotted line in Figure 12 represents the yielding inter-story drifts at the selected stories. In order to investigate the effect of damping and the effect of weakening separately, as well as their combined effect, the response of ‘weakening plus damping’ is compared with the ‘damping distribution’, which uses the same added damping with no structural modifications (or weakening), as well as with the ‘uncontrolled’ response. The results show a reduction of both
Table III. Final locations and capacities of weakened stories and viscous dampers.

<table>
<thead>
<tr>
<th>Story level</th>
<th>Distribution</th>
<th>Drift $x_i$ (%)</th>
<th>$\ddot{x}_a$ (m/s²)</th>
<th>Drift $x_i$ (%)</th>
<th>$\ddot{x}_a$ (m/s²)</th>
<th>Drift $x_i$ (%)</th>
<th>$\ddot{x}_a$ (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>$c_{\text{opt}}$ (kN·s/m)</td>
<td>$\beta (F_y/F_y0)$</td>
<td></td>
<td>$x_i$ (%)</td>
<td>$\ddot{x}_a$ (m/s²)</td>
<td></td>
<td>$x_i$ (%)</td>
</tr>
<tr>
<td>5</td>
<td>5.000</td>
<td>0.50</td>
<td>1.40</td>
<td>10.48</td>
<td>0.83</td>
<td>9.70</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>20.500</td>
<td>0.50</td>
<td>0.92</td>
<td>11.62</td>
<td>0.63</td>
<td>8.97</td>
<td>0.66</td>
</tr>
<tr>
<td>3</td>
<td>39.100</td>
<td>0.50</td>
<td>0.70</td>
<td>10.77</td>
<td>0.54</td>
<td>8.33</td>
<td>0.58</td>
</tr>
<tr>
<td>2</td>
<td>84.100</td>
<td>0.50</td>
<td>1.02</td>
<td>9.99</td>
<td>0.51</td>
<td>7.92</td>
<td>0.52</td>
</tr>
<tr>
<td>1</td>
<td>119.700</td>
<td>0.50</td>
<td>1.36</td>
<td>9.66</td>
<td>0.52</td>
<td>8.15</td>
<td>0.54</td>
</tr>
</tbody>
</table>

When comparing the response of the proposed weakening and damping scheme with ‘adding damping only’, a similar response in terms of drifts is observed while a larger reduction of accelerations is obtained with the proposed method. In fact, both distributions are able to lead to a reduction in drifts of about 40%, but when considering acceleration the proposed method is able to obtain a 20% more reduction with respect to the distribution with only damping.

CONCLUDING REMARKS

This paper presents a design methodology for location and capacities of dampers and weakening connections for controlled performance of inelastic structures. Such methodology can be used for design of seismic retrofit or in new design of controlled structures. The methodology is based on implementing a nonlinear actively controlled system by using passive means obtained through an iterative optimization algorithm based on the minimization of the difference between the response of the active and the equivalent passive implementation. Stiffness and strength are decoupled during the optimization procedure. This allows determining the location of weakened structural elements, such as ‘rocking columns’ [15], which are able to control strength keeping the stiffness constant. To illustrate the methodology, the procedure is applied to a five-story shear-type nonlinear building tested using El Centro ground motion as well as using an ensemble of simulated ground motions.
motions, showing the effectiveness of the final design in reducing both inter-story drifts and peak total accelerations. A sensitivity analysis for design parameters is presented to facilitate rational choices.

Figure 12. Maximum of inter-story drift response for different hazard levels.
Figure 13. Maximum of acceleration response for different hazard levels.
APPENDIX A

This appendix describes a modified procedure of Yang et al. [20] to design the controller for nonlinear structures that is valid for nonlinear shear-type buildings under the assumption that each nonlinear story unit is installed with one controller, i.e. \( r \geq l \).

The first step consists of converting the state equation of motion, into the so-called regular form by the following transformation [17]. Let

\[
Y = Dz \tag{A1}
\]

in which \( D \) is the following transformation matrix:

\[
D = \begin{bmatrix} I_{2n-r} & -B_1B_2^{-1} \\ 0 & I_r \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{A2}
\]

where \( I_{2n-r} \) and \( I_r \), are respectively, \((2n-r) \times (2n-r)\) and \((r \times r)\) identity matrices, and \( B_1 \) and \( B_2 \) are \((2n-r) \times r\) and \( r \times r\) submatrices obtained from the partition of the \( B \) matrix. Note that the \((r \times r)B_2\) matrix should be nonsingular. If the \( B_2 \) matrix is singular in the original state equation, then the state equation should be rearranged such that \( B_2 \) is nonsingular. The transformation (A2) leads to the following matrices in the new coordinate system:

\[
\tilde{A} = DA D^{-1}, \quad \tilde{P} = PD^{-1}, \quad \tilde{B} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \quad \tilde{B}^* = \begin{bmatrix} 0 \\ \tilde{B}_2 \end{bmatrix} \tag{A3}
\]

where \( \tilde{B}_2 \) is a \((r \times l)\) matrix obtained from the \( B_2 \) matrix by eliminating \((r-l)\) columns. Let \( Y, \tilde{A}, \) and \( \tilde{P} \) be partitioned as follows:

\[
Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix}, \quad \tilde{P} = \begin{bmatrix} \tilde{P}_1 & \tilde{P}_2 \end{bmatrix} \tag{A4}
\]

in which \( Y_1 \) and \( Y_2 \) are \(2n-r\) and \( r\) vectors, respectively, and \( \tilde{A}_{11}, \tilde{A}_{22}, \tilde{P}_1, \) and \( \tilde{P}_2 \) are \((2n-r) \times (2n-r), r \times r, r \times (2n-r),\) and \( r \times r\) matrices, respectively. Substituting (A4) into the equation of motion (3), one obtains the linear equations of motion on the sliding surface in the new coordinate system

\[
\dot{Y}_1 = \tilde{A}_{11}Y_1 + \tilde{A}_{12}Y_2 \tag{A5}
\]

\[
S = \tilde{P}_1Y_1 + \tilde{P}_2Y_2 \tag{A6}
\]

For simplicity \( \tilde{P}_2 \) is chosen to be an identity matrix, \( \tilde{P}_2 = I_r \), so \( Y_2 = -\tilde{P}_1Y_1 \). The problem reduces to determine the \( P_1 \) matrix. After determining \( P_1 \), the unknown matrix \( P \) is obtained.

The design of the sliding surface \( S = Pz = 0 \) is obtained by minimizing the integral of the quadratic function of the state vector [23] in which \( Q \) is a \((2n \times 2n)\) positive-definite weighting matrix that after coordinate transformation becomes

\[
T = (D^{-1})^TQD^{-1}, \quad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{A7}
\]
where \( \mathbf{T}_{11} \) and \( \mathbf{T}_{22} \) are \((2n-r) \times (2n-r)\) and \((r \times r)\) matrices, respectively. The matrix \( \mathbf{P}_1 \) is, thus, determined by the following equation:

\[
\mathbf{P}_1 = -0.5 \mathbf{T}_{22}^{-1} (\mathbf{\bar{A}}_{12}^T \mathbf{\hat{P}} + 2 \mathbf{T}_{21})
\]  

(A8)

in which \( \mathbf{\hat{P}} \) is a \((2n-r) \times (2n-r)\) Ricatti matrix satisfying the following Ricatti equation:

\[
\mathbf{\hat{A}}^T \mathbf{\hat{P}} + \mathbf{\hat{P}} \mathbf{\hat{A}} - 0.5 \mathbf{\hat{P}} \mathbf{\bar{A}}_{12} \mathbf{T}_{22}^{-1} \mathbf{\bar{A}}_{12}^T \mathbf{\hat{P}} = -2(\mathbf{T}_{11} - \mathbf{T}_{12} \mathbf{T}_{22}^{-1} \mathbf{T}_{12}^T)
\]  

(A9)

where

\[
\mathbf{\hat{A}} = \mathbf{\bar{A}}_{11} - \mathbf{\bar{A}}_{12} \mathbf{T}_{22}^{-1} \mathbf{T}_{21}
\]  

(A10)

Back substituting, the \( \mathbf{P} \) matrix is determined

\[
\mathbf{P} = \mathbf{\bar{P}} \mathbf{D} = [\mathbf{\bar{P}}_1 \mathbf{I}_r] \mathbf{D}
\]  

(A11)

The design of the controller that drives the state trajectory into the sliding surface \( \mathbf{S} = 0 \) is determined using the Lyapunov function. The control force vector is given by

\[
\mathbf{u}(t) = \mathbf{G}(t) - \mathbf{\bar{d}} \mathbf{\lambda}^T
\]  

(A12)

where

\[
\mathbf{\lambda} = \mathbf{S}^T \mathbf{P} \mathbf{B}, \quad \mathbf{G}(t) = - (\mathbf{P} \mathbf{B})^{-1} \mathbf{P} (\mathbf{A} \mathbf{z}(t) - \mathbf{\bar{B}} \mathbf{f}_s(t) + \mathbf{e}(t))
\]  

(A13)

and \( \mathbf{\bar{d}} \) is a \((r \times r)\) diagonal matrix with diagonal elements \( \delta_1, \delta_2, \ldots, \delta_r \) that define the sliding margin in which \( \delta_i \geq 0 \). Hence, the control force is a linear function of the state space variables plus a nonlinear term that describes the nonlinear characteristic of the structure and the ground motion input. In practical applications controllers are saturated, since the control force \( u_i(t) \) is saturated (or bounded) at \( \bar{u}_{i\text{max}} \), so the continuous controller is given in Equation (5) in the text.

APPENDIX B: MATRICES USED IN NUMERICAL EXAMPLE

The following matrices have been used in the numerical example. A part of the system matrix after coordinate transformation:

\[
\mathbf{\bar{A}}_{21} = \begin{bmatrix}
-2368.7 & 2210.8 & 0 & 0 & 0 \\
2368.7 & -4421.6 & 1985 & 0 & 0 \\
0 & 2210.8 & -3970 & 1421.2 & 0 \\
0 & 0 & 1985 & -2842.4 & 789.6 \\
0 & 0 & 0 & 1421.2 & -1579.2
\end{bmatrix}
\]  

(B1)

The diagonal weight matrix after coordinate transformation

\[
\mathbf{T}_{[10 \times 10]} = \text{diag}(10^6 \ldots \ 10^6 \ 1 \ldots 1)
\]  

(B2)
The controller location matrix

\[
B_2 = \begin{bmatrix}
1 & -1 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 \\
0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & -1 & 2 \\
\end{bmatrix} \times 10^{-3} \tag{B3}
\]

The matrix that determine the sliding surface

\[
\bar{P}_1 = 1000 \cdot I_{5 \times 5} \tag{B4}
\]

where \( I \) is the identity matrix.

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REFERENCES


