First Italian Meeting on Probability and Mathematical Statistics
Torino, 19-22/6/2017

Book of abstracts

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1 Program and order in which abstracts are presented in the book

The order in which abstracts are presented is as follows: invited talks first, then invited sessions, contributed sessions, contributed talks and posters. Within each category we use the order in which they appear in the program which is included below. Enjoy the meeting, the organizers.

MONDAY 19th - Cavallerizza Reale

- 8.30-9.30: Registration
- 9.30-10.10: Welcome
- 10.10-10.50: Invited speaker: E. Regazzini
- 10.50-11.20: *Coffee break*
- 11.20-12.50: Contributed Sessions

  Room 1 G. Pistone - *Information and Stochastic Geometry* (G. Pistone - E. Ferrucci - L. Malagò)

- 12.50-14.20: *Lunch*
- 14.20-15.00: Invited speaker: P. Baldi
- 15.00-16.30: Invited Sessions

  Room 1 F. Fagnola - *Quantum Probability* (P. Gibilisco - M. Gregoratti - V. Crismale)

  Room 2 I. Prünster - *Bayesian Nonparametrics* (A. Guglielmi - X. Nguyen - M. Ruggiero)

- 16.30-17.00: *Coffee break*
- 17.00-18.30: Invited Sessions

  Room 1 D. Marinucci - *Geometry of Random Fields* (V. Cammarota - M. Rossi - G. Peccati)

  Room 2 L. Sacerdote - *Stochastic models in Biology and Medicine* (M.Tamborrino - U. Picchini - G. Scalia Tomba)

TUESDAY 20th - Politecnico di Torino

- 9.00-10.30: Contributed Sessions

  Room 1I C. Fontana - *Information and Arbitrage in Financial Markets* (B. Acciaio - W.J. Runggaldier - C. Fontana)

  Room 3I F.R. Nardi, A. Zocca - *Metastability of Interacting Particle Systems* (E. Scoppola - E.N.M. Cirillo - F.R. Nardi)

  Room 2I E. Riccomagno - *Algebraic Statistics* (R. Fontana - M. Leonelli - F. Rapallo))

  Room 5I C. Costantini - *Asymptotics for Measure Valued Processes* (C. Giardinà - D. Spanò - C. Costantini)

- 10.30-11.00: *Coffee break*
- 11.00-11.40: Invited speaker: F. Pratelli
- 11.40-13.10: Poster Session
- 13.10-14.30: *Lunch*
- 14.30-16.00: Invited Sessions

  Room 1I C. Ceci, A. Pascucci - *Stochastic processes and applications to Finance and Insurance* (G. Ferrari - K. Colaneri - S. Pagliarani)

  Room 3I F. Martinelli, P. Dai Pra - *Stochastic Dynamics in Statistical Physics* (C. Toninelli - F. Toninelli - M. Mariani)

- 16.00-16.30: *Coffee break*
- 16.30-18.00: Contributed Sessions

  Room 1I P. Siorpaes - *Martingale Optimal Transport* (L. Campi - P. Siorpaes - S. De Marco)

  Room 3I F. Morandin - *Stochastic Fluid Dynamics* (D. Barbato - B. Ferrario - F. Morandin)

  Room 2I G. Como, F. Fagnani - *Stochastic Network Systems: Opinion Dynamics, Robustness, and Epidemics* (F. Fagnani - G. Como - L. Zino)

  Room 5I P. Rigo - *Asymptotic Behavior of Conditional Probabilities* (P. Rigo - E. Dolera - E. Mainini)

EVENING: *Social Dinner*

WEDNESDAY 21st - Politecnico di Torino

- 9.30-10.30: Contributed Talks

  Room 1I Contributed Talks:

  1. P. Semeraro - *Multivariate marked Poisson processes and market related multidimensional information flows*
  2. M. Piccirilli - *Additive energy forward curves under the Heath-Jarrow-Morton framework*
  3. R.M. Mininni - *A new approach to CIR Short-Term Interest Rates Modeling*

  Room 3I Contributed Talks:

  1. E. Bandini - *Existence and uniqueness for BSDEs driven by a general random measure, possibly non quasi-left-continuous*
  2. N. Foresta - *Reflected BSDE driven by marked point process and optimal stopping*
  3. E. Issoglio - *FBSDEs with distributional coefficients*

  Room 5I Contributed Talks:

  1. A. Zocca - *Stochastic modeling and control of energy networks under uncertainty*
  2. A. Ghiglietti - *Systems of reinforced stochastic processes with a network-based interaction*
  3. B. Martinucci - *A continuous-time random walk on a star graph and its diffusion approximation*

  Room 2I Contributed Talks:

  1. A. Lanconelli - *Prohorov-type local limit theorems on Gaussian spaces*
  2. Y.G. Lu - *Central limit theorem on* $\otimes \mathbf{M}_2$ *with the Jordan-Wigner embedding*
  3. D. Trevisan - *A PDE approach to a 2-dimensional matching problem*

- 10.30-11.00: *Coffee break*

- 11.00-12.30: Invited Sessions

  Room 1I F. Pellerey - *Dependence Modeling* (F. Durante - G. Puccetti - E. Di Bernardino)

  Room 3I F. Flandoli - *Stochastic Differential Equations* (L. Beghin - L. Caramellino - F. Masiero)

- 12.30-14.00: *Lunch*

- 14.00-15.30: Contributed Sessions

  Room 1I M. Frittelli - *Probability Methods in Robust Finance* (G. Callegaro - M. Maggis - C.A. Munari)

  Room 3I B. Ferrario - *Stochastic PDE's* (L.A. Bianchi - S. Bonaccorsi - E. Priola)

  Room 5I M. Ruggiero - *Bayesian Nonparametrics* (F. Bassetti - F. Camerlenghi - P. De Blasi)

  Room 2I D. Spanò - *Stochastic Processes in Discrete Structures and their Limit Behaviour* (E. Candellero - G. Cannizzaro - A. Chiarini)

- 15.30-16.00: *Coffee break*

- 16.00-17.00: Contributed Talks

  Room 1I Contributed Talks:

  1. S. Scotti - *Optimal Investment in Markets with Over and Under-Reaction to Information*
  2. A. Calzolari - *Martingale representations in progressive enlargement by the reference filtration of a semi-martingale*
  3. B. Torti - *Martingale representations in markets driven by processes sharing accessible jump times*

  Room 3I Contributed Talks:

  1. T. De Angelis - *Optimal stopping and Skorokhod embedding*
  2. M. Longobardi - *A study of inactivity times of coherent systems with dependent components*
  3. Giovanni Conforti - *A second order ODE in Wasserstein space for the Schrödinger bridge*

  Room 5I Contributed Talks:

  1. E. De Vito - *Reconstruction trees*
  2. A. De Gregorio - *Test statistics for stochastic differential equations sampled at discrete times*
  3. G. Sanfilippo - *Compounds of conditionals and iterated conditioning under coherence*

- 17.00-19.00: Open Session - Towards the future

THURSDAY 22nd - Politecnico di Torino

- 9.00-10.30: Contributed Sessions

  Room 1I P. Rigo, B. Vantaggi - *Some Recent Developments about Finitely Additive Probability Measures* (P. Berti - B. Vantaggi - G. Cassese)

  Room 3I F. Confortola, G. Guatteri, F. Masiero - *Some Topics on Path-dependent Stochastic Equations* (A. Cosso - C. Di Girolami - G. Zanco)

  Room 5I A. Stauffer - *Stongly Correlated Random Interacting Systems* (A. Caraceni - A. Cipriani - L. Taggi)

  Room 2I B. D'Auria - *Diffusion Processes: Inference and Applications* (A. Di Crescenzo - G. Albano - B. D'Auria)

- 10.30-11.00: *Coffee break*

- 11.00-12.40: Contributed Talks and Contributed Session

  Room 1I Contributed Session: M. Rossi - *Limit Theorems in Probability and Applications* (C. Durastanti - M. Dal Borgo - R. Maffucci - A. Vidotto)

  Room 3I Contributed Talks:

  1. A. Cecchin - *Probabilistic approach to finite state space mean field games*
  2. T. Vargiolu - *Verification theorem for stochastic impulse non-zero sum games and applications*
  3. S. Federico - *Verification theorems for stochastic optimal control problems in Hilbert spaces by means of a generalized Dynkin formula*
  4. A. Calvia - *Filtering and control of time-homogeneous pure jump Markov processes with noise-free observation*
  5. A. Balata - *Regress Later Monte Carlo for Controlled Markov Processes*

  Room 5I Contributed Talks:

  1. S. Mazzucchi - *High order heat-type equations and random walks on the complex plane*
  2. L. Andreis - *Ergodicity of a system of interacting random walks with asymmetric interaction*
  3. D. Tovazzi - *Collective periodic behavior in spin-flip models with dissipation*
  4. S. Ugolini - *Entropy chaos and Bose-Einstein Condensation*
  5. M. D'Ovidio - *Skew diffusions across Koch interfaces*

  Room 2I Contributed Talks:

  1. B. Toaldo - *Semi-Markov processes and their Kolmogorov's equations*
  2. C. Ricciuti - *Non-homogeneous subordinators and their connection with semi-Markov processes*
  3. E. Mariucci - *A compound Poisson approximation to estimate the Lévy density*
  4. E. Pirozzi - *On a Class of Gauss-Markov processes for neuronal models with jumps*
  5. G. D'Onofrio - *On the reference frame invariance of stimulus-specific information measures*

- 12.40-14.00: *Lunch*

2 Invited talks

A journey through Probability and Statistics in Italy in the 20th century

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A recollection of some of the most significant contributions to Probability and Statistics by Italian mathematicians over the 20th century seems definitely suited to open the *First Italian Meeting on Probability and Mathematical Statistics*. The journey will start from the efforts of Guido Castelnuovo (1865–1952) aiming at promoting both the study and the applications of Probability and Statistics and will move on to the analysis of the crucial results obtained by Francesco P. Cantelli (1875–1966) concerning the convergence of sequences of random numbers, in particular the first formulation of a strong law of large numbers dating back to 1917. Since 1926 a vital role has, then, been played by Bruno de Finetti (1906–1985) who, between 1926 and 1931, launched – among others – completely new ideas concerning: (a) stochastic processes with independent and stationary increments; (b) the subjective interpretation of probability and the ensuing mathematical theory based on the "axiom of coherence"; (c) the probabilistic foundations of the inductive reasoning by means of the concept of exchangeability. Since the 50's, de Finetti's ideas mentioned in (b) and (c) are considered decisive toward the modern regeneration of the Bayesian theory of Statistics. From a statistical viewpoint, the figure of Corrado Gini (1884–1965) stood out because of both his distinguished research activity and his capability in reorganizing the entire Italian official statistics system. I shall confine myself to illustrating his project toward an organic treatment of the so–called descriptive statistics, including the pioneering idea of dissimilarity (*dissomiglianza*) between probability distributions, nowadays well–known as Wasserstein distance. The journey will, then, be completed by a quick glance at the fundamental function the journals *Metron* and *Giornale dell'Istituto Italiano degli Attuari* – founded by Gini and Cantelli, respectively – had in spreading, at an international high–level, new methods and results, at least till the outbeak of the Second World War.

Large Deviations and conditioned diffusions

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The talk is about old and new results concerning Large Deviation estimates (sharp and non-sharp) for conditioned diffusions as the conditioning time tends to 0.

The space L^0 between Probability and Analysis.

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The space L^0, i.e. the space of all random variables on a probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with the topology of convergence in probability, is usually considered not very interesting: it is metric complete but not locally convex and therefore usual results of Functional Analysis cannot be applied.

Nevertheless it was considered an important object in some problems (in particular in the seventies for the construction of general Stochastic Integration and some years later for the foundation of Mathematical Finance), and the investigation of the properties of this space increased and new properties were found.

The object of my talk is to recall these problems and to expose main results that were obtained.

Acknowledgments: I want to thank the organizers for their very kind invitation.

3 Invited sessions

3.1 Quantum Probability (F. Fagnola)

Operators means in quantum probability: the decomposition formula for the quantum Fisher-Rao metrics and the generalized Rao inequality for mean expectation

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The fascinating fields of numerical and operator means have some deep applications in Quantum Probability. The deepest result of this kind is certainly the Petz classification theorem showing that the family of Quantum Fisher Information is in a bijective correspondence with the family of Kubo-Ando operator means (actually the same is true for the Quantum Covariance, still due to Petz). It is actually correct to talk about Quantum Fisher-Rao Information (or metric) because Petz theorem makes use of the fundamental observation by Rao showing that Fisher Information is indeed a Riemannian metric (under suitable regularity hypotheses) on statistical models.

In my talk I am going to discuss the following decomposition formula for Quantum Fisher Information

$$f(0)\langle i[\rho, A], i[\rho, B]\rangle_{\rho,f} = \mathrm{Cov}_\rho(A, B) - \mathrm{Cov}_\rho^{\tilde{f}}(A, B). \qquad (1)$$

showing that each QFI is (up to a scalar) the difference of the "standard" quantum covariance and the covariance associated to the $\tilde{f}$ function whose definition and properties will be briefly discussed [1, 2]. Present and potential applications of the above formula will be presented.

Secondly I will discuss a result by Rao regarding the harmonic mean (denoted by $m_h(\cdot,\cdot)$) of two positive random variables X and Y. Rao proved that

$$\mathrm{E}(m_h(X, Y) \geq m_h(\mathrm{E}(X), \mathrm{E}(Y)). \qquad (2)$$

and he was also able to prove the formula (**??**) in the matrix case. I will discuss the paper [3] where the above result has been generalized to any concave mean in the commutative case and to any Kubo-Ando mean in the operator case.

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Measurement uncertainty relations for quantum observables: Relative entropy formulation

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The first part of the talk will give an introduction to measurement uncertainty relations in Quantum Probability for Classical Probabilists. Then, the second part will present our new informational-theoretic formulation of the measurement uncertainty relations, based on the notion of relative entropy between probability measures.

In the case of a finite-dimensional system, we quantify the total error affecting an approximate joint measurement of two discrete quantum observables, we prove the general properties of its minimum value (the uncertainty lower bound) and we study the corresponding optimal approximate joint measurements. The new error bound, which we name *entropic incompatibility degree*, turns out to enjoy many key features: among the main ones, it is state independent and tight, it shares the desirable invariance properties, and it vanishes if and only if the two observables are compatible. By exploiting the symmetry properties of the target observables, exact values and lower bounds are computed in two different concrete examples: (1) a couple of spin-1/2 components (not necessarily orthogonal); (2) two Fourier conjugate mutually unbiased bases in prime power dimension.

This is joint work with Alberto Barchielli (Politecnico di Milano) and Alessandro Toigo (Politecnico di Milano).

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Monotone Central Limit Theorems

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In Quantum Probability one generally deals with non-commutative random variables. Dropping the commutativity entails many consequences, one of them concerning independence. Contrarily to the classical case, where one notion of independence is possible, in the non-commutative setting one finds many inequivalent notions of independence. Among them, the monotone independence, which emerged about twenty years ago in the study of quantum electrodynamics [3], is one of the most important. In [4] it was proven a Central Limit Theorem for normalised sums of monotonically independent and identically distributed self-adjoint operators (random variables). There it was shown the weakly convergence for these random variables to the arcsine distribution, thus suggesting this law as a monotone analogue of the normal one.

On the other hand, the study of the asymptotic behaviour of sums of random variables realising the monotone commutation rules often needs to consider operators which are not self-adjoint. In this case the above mentioned Central Limit Theorem cannot be applied. In the talk we present a solution to this problem. More in detail, after a quick introduction on definitions and properties of monotone random variables [1], we present a non-commutative Central Limit Theorem for general, not necessarily self-adjoint monotone operators [2]. This result, which includes the previous one as a particular case, seems naturally addressed for applications in Quantum Physics and Applied Mathematics, above all in Quantum Information and Computing.

In the final part we outline how to achieve a functional counterpart of our theorem. The result entails the weak convergence of the involved processes to the so-called monotone Brownian motion.

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3.2 Bayesian Nonparametrics (I. Prünster)

Bayesian nonparametric covariate-driven clustering

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Bayesian nonparametric priors are becoming a versatile tool for clustering problems; in such a context, defining the model boils down to assign the prior for the random partition itself and to flexibly assign the cluster-specific distribution, since, conditionally on the partition, data are assumed iid within each cluster and independent between different clusters. However, assigning the prior for the random partition parameter it is not an easy task in general. Here, we aim at taking into account possible patterns within available covariates, which can be either continuous or categorical: the additional covariate information should drive the prior knowledge on the random partition by increasing the probability that two items with similar covariates belong to the same cluster. This is done through a covariate-dependent nonparametric prior, thus departing from the standard exchangeable assumption. We start from [1], who developed the covariate-dependent product partition

*Speaker

model, where the covariates enter into the prior on the partition of our data via a similarity function among covariates that multiplies a cohesion function. However, their model is equivalent to jointly model the vector (y, x) of the response variable and covariates. Our aim is to propose a new similarity function lying outside this framework, but also including dependence on covariates in the class of normalized completely random measures. The model is fitted to data from a biomedical real application.

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Singularity structure of parameter space and posterior contraction in finite mixture models

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Singularities of a statistical model are the elements of the model's parameter space which make the corresponding Fisher information matrix degenerate. These are the points for which estimation techniques such as the maximum likelihood estimator and standard Bayesian procedures do not admit the root-n parametric rate of convergence. We propose a general framework for the identification of singularity structures of the parameter space of finite mixtures, and study the impacts of the singularity levels on parameter estimation in a compact parameter space. Our study makes explicit the links between model singularities, parameter estimation convergence rates, and the algebraic geometry of the parameter space for mixtures of continuous distributions. The theory is applied to establish concrete convergence rates of both Bayesian and maximum likelihood parameter estimation for finite mixtures of skewnormal distributions. This rich and increasingly popular mixture model is shown to exhibit a remarkably complex range of asymptotic behaviors which have not been hitherto reported in the literature.

*Speaker

Sequential Bayesian inference for measure-valued Markov processes via duality

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We exploit the duality properties of two families of Markov processes, each including widely used finite- and infinite-dimensional subcases, to derive their closed form marginal posterior distributions given discretely collected data, where the law of the process is interpreted as the prior. From a different viewpoint, we extend classic characterisations of posterior distributions under Dirichlet process and gamma random measures priors to a dynamic framework.

More specifically, we consider the problem of learning, from indirect observations, two families of time-dependent processes of interest in Bayesian nonparametrics: the first is a dependent Dirichlet process driven by a Fleming–Viot model, and the data are random samples from the process state at discrete times; the second is a collection of dependent gamma random measures driven by a Dawson–Watanabe model, and the data are collected according to a Poisson point process with intensity given by the process state at discrete times. The driving processes are diffusions taking values in the space of discrete measures whose support varies with time, they are stationary and reversible with respect to Dirichlet and gamma priors respectively, and include Wright–Fisher and Cox–Ingersoll–Ross models as special cases.

A common methodology is developed to obtain in closed form the time-marginal posteriors given past and present data. This is based on the projective properties of the signals and on certain duality properties of their finite-dimensional projections. The time-marginal posteriors are shown to belong to classes of finite mixtures of Dirichlet processes and gamma random measures for the two models respectively, yielding conjugacy of these classes to the type of data we consider. We provide explicit results on the parameters of the mixture components and on the mixing weights, which are time-varying and drive the mixtures towards the respective priors in absence of further data. Explicit algorithms are provided to recursively compute the parameters of the mixtures.

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3.3 Geometry of Random Fields (D. Marinucci)

On critical points and excursion sets of random Laplace eigenfunctions on compact manifolds

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We study the critical points and the excursion sets of smooth Gaussian random fields on compact real-analytic manifolds (e.g. sphere and flat torus).

We discuss the limiting distribution and asymptotic fluctuations of the number of critical points of random Laplace eigenfunctions in the high energy limit; this requires a careful investigation of the validity of the Kac-Rice formula in nonstandard circumstances. Building upon such results and Wiener-Itô chaos decomposition, we derive a Central Limit Theorem for the Euler characteristic of the excursion sets.

Some applications to data analysis in Cosmology are also discussed.

Acknowledgments

The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreements no 277742 and no 335141.

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*Speaker

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Nodal sets in Wiener chaoses*

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In this talk we investigate the *nodal* geometry of *random* Laplacian eigenfunctions on compact Riemannian manifolds. In particular we will show that, in a Gaussian setting, Wiener-Îto chaos expansion is a powerful tool to study *local* properties of the nodal sets.

Indeed, we will present some recent results concerning the asymptotic behavior of the zeros of Gaussian Laplacian eigenfunctions on two-dimensional manifolds (like the sphere, the torus and the Euclidean plane), which rely on a pervasive use of chaotic expansions.

For all the quantities we will consider (the nodal length [3, 5, 6], the number of nodal intersections [7], the number of phase singularities [2]) it turns out that, in the limit, only one term in the chaotic series expansion collects all information about the distribution, whereas all the remaining chaotic components are negligible. This phenomenon has been observed, for other functionals, also in [4, 1]. In particular, it makes easier the understanding of the asymptotic variance and law of the functional of interest. The geometry of the underlying manifold however influences the results, for instance non-Gaussian second order fluctuations are observed on the torus, while we have an asymptotic Gaussian behavior on the sphere.

This talk is based on joint works with F. Dalmao, D. Marinucci, I. Nourdin, G. Peccati, and I. Wigman.

Acknowledgments

The research leading to the works this talk is based on has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreements no 277742 *Pascal* (PI: D. Marinucci) and by the grant F1R-MTH-PUL-15STAR (STARS) at University of Luxembourg (PI: G. Peccati).

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Non-universal nodal fluctuations of arithmetic random waves

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First studied by Rudnick and Wigman, arithmetic random waves are (real or complex) Gaussian Laplace eigenfunctions on the two-dimensional torus. In this talk, I will describe the asymptotic behaviour of two geometric quantities associated with the zero set of such objects: (1) the nodal length (real case), and (2) the number of phase singularities (complex case). We will show that both quantities behave non-centrally and non-universally in the high-energy limit. The non-central behaviour can be understood in terms of an underlying four-dimensional quantitative central limit theorem, and has to be studied by means of highly non-trivial arithmetic considerations. The crucial technical difference between the real and complex case resides in the behaviour of the so-called 'Leray measure', roughly corresponding to the 'occupation density at zero' associated with a given wave. A subsequent talk by M. Rossi will explain in full generality the connection between these findings and the so-called 'Berry Cancellation phenomenon', according to which several geometric quantities associated with the zeros of random Laplace eigenfunctions display smaller fluctuations than the ones associated with non-zero level sets. Joint works with D. Marinucci, M. Rossi and I. Wigman (2016), F. Dalmao, M. Rossi and I. Nourdin (2016) and M. Rossi (2017).

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*Speaker

3.4 Stochastic models in Biology and Medicine (L. Sacerdote)

Statistical Inference for Perturbed Stochastic Processes with Application to Neuroscience

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A latent internal process describes the state of some system, e.g. the membrane potential evaluation of a neuron, the social tension in a political conflict, the price of a stock, the strength of an industrial component or the health status of a person. When this process reaches a predefined threshold, the process terminates and an observable event occurs, e.g. a neuron releases an electrical impulse (also known as action potential or spike), the stock is sold/bought, the political conflict finishes/explodes, the industrial component breaks down or the person dies. Imagine an intervention, e.g., an input current, a speculation strategy, a political decision, a maintenance of a component or a medical treatment, is initiated to the process before the event occurs. How can we evaluate whether the intervention had an effect? How can we detect the type of stimulus applied only observing the events following the intervention? What can be said if both the time of the intervention and the type of stimulus are unknown? Imagine now that the intervention has an unknown intensity level. What is the highest decoding accuracy of the intensity level? How does this discrimination change if the system is observed for a longer time? Answering these questions is particularly difficult because the latent internal process describing the state of the system is perturbed, i.e. observed only on top of an indistinguishable background noise. From a mathematical point of view, the described problem is modeled by stochastic point processes obtained as hitting times of perturbed stochastic processes. A study of the decoding accuracy of the stimulus level based on either the first event after the intervention (assuming the change point to be either known or unknown) or the rate of events on a certain observation time window were performed, yielding counter-intuitive results, representing a novel manifestation of the noise-aided signal enhancement, which differs fundamentally from the usual kinds reported on, such as standard stochastic resonance. Our results are discussed in the framework of neuroscience, and in particular of information transfer in neural systems, but the same scenario can be found in many fields, such as reliability theory, social sciences, finance, biology or medicine.

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Inference via Bayesian synthetic likelihoods for a mixed-effects SDE model of tumor growth

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In [1] we consider parameter estimation for a state-space model fitted on repeated measurements of tumor data from several mice (hearefter "subjects"). We construct a nonlinear mixed-effects model where individual dynamics are expressed by stochastic differential equations (SDE). Nonlinear SDE state-space models are notoriously difficult to fit: however, advancements in parameter estimation using sequential Monte Carlo methods have made (exact) Bayesian inference for SDE models approachable, if not always straightforward. Instead, here we use summary statistics to encode information pertaining between-subjects variation, as well as individual variation, and resort to a synthetic likelihood approximation [2] for parameter estimation. In particular, we take advantage of the Bayesian (pseudo-marginal) synthetic likelihood approach in [3].

We consider longitudinal data from measurements of tumor volumes in mice. Mice are divided in three groups and administered three different treatments. The dataset is challenging because of sparse observations. Synthetic likelihoods based on summary statistics are able to reproduce the results from exact Bayesian inference and, when data are available from a non-negligible number of subjects, are able to identify a specific treatment to be more effective in reducing tumor growth.

Acknowledgments

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*Speaker

Inference in the early phase of an epidemic

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In several recent epidemic outbreaks (SARS, A(H1N1) flu, Ebola,...), efficient data collection has allowed inference on important epidemic parameters such as R_0, exponential increase rate (r), generation time distribution characteristics, case fatality rate (CFR), already in the early phase of spread (see e.g. [1]). However, statistical analysis of such data poses interesting non-standard problems. These problems will be discussed and some solution proposals presented.

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3.5 Stochastic processes and applications to Finance and Insurance (C. Ceci, A. Pascucci)

On the Optimal Management of Public Debt: a Singular Stochastic Control Problem*

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Consider the problem of a government that wants to reduce the debt-to-GDP (gross domestic product) ratio of a country. The government aims at choosing a debt reduction policy which minimises the total expected cost of having debt, plus the total expected cost of interventions on the debt ratio. We model this problem as a singular stochastic control problem over an infinite time-horizon. In a general not necessarily Markovian framework, we first show by probabilistic arguments that the optimal debt reduction policy can be expressed in terms of the optimal stopping rule of an auxiliary optimal stopping problem. We then exploit such link to characterise the optimal control in a two-dimensional Markovian setting in which the state variables are the level of the debt-to-GDP ratio

*Speaker

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†Speaker

and the current inflation rate of the country. The latter follows uncontrolled Ornstein-Uhlenbeck dynamics and affects the growth rate of the debt ratio. We show that it is optimal for the government to adopt a policy that keeps the debt-to-GDP ratio under an inflation-dependent ceiling. This curve is given in terms of the solution of a nonlinear integral equation arising in the study of a fully two-dimensional optimal stopping problem. s

Value adjustments and dynamic hedging of reinsurance counterparty risk under partial information

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Reinsurance counterparty risk represents the risk that a reinsurance company fails to honor her obligations from a reinsurance treaty, for instance because the company defaults prior to maturity of the contract. While this risk is of high concern to practitioners and regulators for instance under the Solvency II regulatory regime, there is only very little quantitative research on measuring and hedging reinsurance counterparty risk.

In this paper we attempt to fill this gap. We compute valuation adjustments for reinsurance counterparty risk and we study the hedging of this risk by trading in credit default swaps on the reinsurance company. Perfect hedging is typically not possible and we resort to the (local) risk-minimization approach. We consider a partial information framework where the intensity of the loss process of the primary insurance contract is unobservable and correlated to the default intensity of the reinsurer. Moreover there might be direct contagion effects. To determine the hedging strategy we make use of an orthogonal decomposition of the market value of the reinsurance contract into a hedgeable and a non-hedgeable part called the Galtchouk-Kunita-Watanabe decomposition under partial information. Moreover we characterize the optimal hedging strategy in the full and the partial information framework by means of predictable projections. Stochastic filtering will be used to compute value adjustments and hedging strategy under partial information.

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Analytical approximations of non-linear SDEs of McKean-Vlasov type

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We provide analytical approximations for the law of the solutions to a certain class of scalar McKean-Vlasov stochastic differential equations (MKV-SDEs) with random initial datum. "Propagation of chaos" results ([1]) connect this class of SDEs with the macroscopic limiting behavior of a particle, evolving within a mean-field interaction particle system, as the total number of particles tends to infinity. Here we assume the mean-field interaction only acting on the drift of each particle, this giving rise to a MKV-SDE where the drift coefficient depends on the law of the unknown solution. By perturbing the non-linear forward Kolmogorov equation associated to the MKV-SDE, we perform a two-steps approximating procedure that decouples the McKean-Vlasov interaction from the standard dependence on the state-variables. The first step yields an expansion for the marginal distribution at a given time, whereas the second yields an expansion for the transition density. Both the approximating series turn out to be asymptotically convergent in the limit of short times and small noise, the convergence order for the latter expansion being higher than for the former. The resulting approximation formulas are expressed in semi-closed form and can be then regarded as a viable alternative to the numerical simulation of the large-particle system, which can be computationally very expensive. Moreover, these results pave the way for further extensions of this approach to more general dynamics and to high-dimensional settings.

Acknowledgments

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*Speaker

3.6 Stochastic Dynamics in Statistical Physics (F. Martinelli, P. Dai Pra)

Metastability in non-reversible diffusion processes

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I will discuss of possible generalization of the methods of potential theory in the context of finite-dimensional diffusion processes. These tools can be applied to partly proof some well-known (but heuristic) asymptotic formulas in the low-temperature limit.

Bootstrap percolation and kinetically constrained spin models: critical time and length scales

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Recent years have seen a great deal of progress in understanding the behavior of bootstrap percolation models, a particular class of monotone cellular automata. In the two dimensional lattice there is now a quite satisfactory understanding of their evolution starting from a random initial condition, with a strikingly beautiful universality picture for their critical behavior. Much less is known for their non-monotone stochastic counterpart, namely kinetically constrained models (KCM). In KCM each vertex is resampled (independently) at rate one by tossing a p-coin iff it can be infected in the next step by the bootstrap model. In particular infection can also heal, hence the non-monotonicity. Besides the connection with bootstrap percolation, KCM have an interest in their own : when p->0 they display some of the most striking features of the liquid/glass transition, a major and still largely open problem in condensed matter physics. In this talk I will discuss some recent results on the characteristic time scales of KCM as p ? 0 and the connection with the critical behavior of the corresponding bootstrap models.

[Joint work with Fabio Martinelli]

Discrete interface dynamics and hydrodynamic limit

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Dimer models provide natural models of (2+1)-dimensional random discrete interfaces and of stochastic interface dynamics.I will discuss two examples of such dynamics, a reversible and an irreversible one. In both cases we can prove the convergence of the stochastic interface evolution to a deterministic PDE after suitable space-time rescaling.

Joint work with B. Laslier and M. Legras.

3.7 Dependence Modeling (F. Pellerey)

COPULAS FOR NON-CONTINUOUS RANDOM VECTORS

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According to Sklar's Theorem it is known that the probability law of any random vector can be expressed as the composition of the distribution functions of all one-dimensional margins and a suitable copula. However, while the copula associated with a random vector is unique when all margins are continuous, in the non-continuous case, various copulas can be associated with the same random vector. In the literature, the concept of *subcopula* has been introduced to denote the function, defined on a suitable subset of the copula domain, that summarizes the dependence information in a (possibly non-continuous) random vector.

Here, the structure of the class of subcopulas is investigated and some of its analytical properties are formulated. Moreover, extension procedures are presented in a high-dimensional framework to transform a specific subcopula to a copula. Finally, convergence results are given in order to check how some of these extensions approximate (in different metrics) a target copula.

Practical implications in copula-based inferential procedures are also discussed.

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Multivariate notions of extremal dependence and the swapping algorithm

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First, we consider several multivariate extensions of comonotonicity and countermonotonicity. We show that naive extensions do not enjoy some of the main properties of the univariate concepts. In order to have these properties, more structures are needed than in the univariate case and we define extremal multivariate dependence concepts based on optimization properties. Optimal measures maximizing (minimizing) the expected inner product of two marginals are called c-co(unter)monotonic, as they generalize the case of maximal (minimal) correlation to multivariate marginal distributions.

Then, we introduce a new algorithm, called the swapping algorithm, to approximate numerically the minimal and maximal expected inner product of two random vectors with given marginal distributions. As a direct application to mass transportation problems, the algorithm computes an approximation of the L2-Wasserstein distance between two multivariate measures.

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Estimation of directional extreme risk regions at high levels

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In multivariate extreme value analysis, the focus is on the quantification of the dependent multivariate risks outside of the observable sampling zone, which implies that a region of interest is located at high levels. This work provides an *out-sample* estimation method for the recently introduced *Directional Multivariate Quantiles* and a characterization of the risk region at high levels.

The estimation of extreme level curves is important for identifying extreme events and for characterizing the joint tails of multidimensional distributions. They are usually considered as quantiles at high levels; that is, they are linked with a probability α of occurrence of certain event, where α is a very small number. This proposal considers values of α lower or equal than $1/n$, where n denotes the sample size, which implies that the number of data points that fall beyond the quantile curve is small and can even be zero; thus we are outside of the observable region, or in other words, in the framework of *out-sample* estimation. This lack of relevant data points does the estimation difficult, making it necessary to introduce tools from the multivariate extreme value theory.

The main purpose of this paper is to provide an *out-sample* estimation method for the directional multivariate quantiles recently introduced in [11] and [12]. In these papers, the directional setting refers to the inclusion of a parameter of direction $\mathbf{u}$ that allows the analysis of data by looking at the cloud of observations from different perspectives. Accurate assessments of these quantiles are sought in a diversity of applications from financial risk management to environmental impact assessment.

Both scenarios, *in-sample* and *out-sample*, have been widely studied in the univariate setting and recently the literature has focused on the extension to the multivariate context. Some relevant references for the level curve estimations using either joint distribution or survival functions are for

*Speaker

instance [4, 8, 1, 2, 5]. Works based on copulas are also [3, 6, 10, 9]. These works have introduced proposals in both contexts *in-sample* (for $\alpha > 1/n$) and *outsample*, but most of them present the theory or have applications only in the bivariate case.

As we have mentioned before, the methodology developed in this work includes a directional notion and one can find in the literature a few references dealing with this notion. For instance, [7] studied bounds for multivariate financial risks, highlighting the utility of the analysis considering two particular directions. [1] presented a bivariate quantile application to air quality where the directions are related to the four classical orthants.

Therefore, inspired in the work of [4] where an *out-sample* estimator for bivariate level curves of a distribution function F was established, the contributions of this paper are three fold:

1. to include the directional framework given in [11, 12],
2. to provide the expression of the estimator for those directional high level sets in a general dimension d;
3. to present a non-parametric estimator for these high level directional quantiles.

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3.8 Stochastic Differential Equations (F. Flandoli)

Fractional generators and time-changed stochastic processes

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We present a number of recent results on stochastic processes whose transition densities satisfy partial differential equations of fractional order. We discuss their representations as time-changed processes with independent subordinators (or their inverses). In particular, we deal with some extensions of space-fractional diffusion equations, which are satisfied by stable laws. On the other hand, we also cover differential equations solved by densities of geometric stable processes; we exploit their representation as compositions of stable processes with an independent Gamma subordinator. Some time-inhomogeneous extensions of the previous results are also considered.

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Total variation distance between stochastic polynomials and invariance principles

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The goal of this talk is to estimate the total variation distance between two general stochastic polynomials and, as a consequence, to study an invariance principle for such polynomials. This generalizes known results concerning the total variation distance between two multiple stochastic integrals on one hand, and invariance principles in Kolmogorov distance for multi-linear stochastic polynomials on the other hand. The results are applied to the study of the asymptotic behavior of U-statistics associated to polynomial kernels and to an example of Central Limit Theorem associated to quadratic forms. From a joint work with Vlad Bally (Université Paris-Est, Marne-la-Vellée).

BSDEs related to stochastic optimal control problems with delay

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In this talk we present Backward Stochastic Differential Equations (BSDEs) and we show how BSDEs arise as adjoint equations in the formulation of the stochastic Pontryagin maximum principle. When we deal with stochastic optimal control problem with delay in the state the adjoint BSDE turns out to be anticipating, see [1].
We introduce a new class of anticipating BSDEs suitable to treat general control problems with delay in the state via the stochastic maximum principle.
Part of the talk is based on [2].

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4 Contributed sessions

4.1 Information and Stochastic Geometry (G. Pistone)

Projecting the Fokker-Plank equation on a finite-dimensional statistical manifold

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We apply the L^2-based Hellinger-Fisher-Rao vector-field projection by Brigo, Hanzon and LeGland [2] to finite dimensional approximations of the Fokker-Planck equation [4] on exponential families [1]. We show that this vector field projection is equilvalent to an assumed density "moment matching" approximation based on expectation parameters. We derive an algebraic relation that allows to recover canonical parameters from expectation parmeters for polynomial exponent families. For general exponential families we show that if the sufficient statistics are chosen among the diffusion eigen-functions, the finite dimensional projection or the equivalent assumed density approximation provide the exact maximum likelihood density. The same result had been derived earlier by Brigo and Pistone [3] in the infinite-dimensional Orlicz based geometry [5] as opposed to the L^2-structure used here.

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Information Geometry of Stochastic Optimization

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Information Geometry [3] is an interdisciplinary and expanding research field at the intersection of statistics and differential geometry, which studies the geometry of statistical models, represented as manifolds of probability distributions. Notably, Information Geometry provides a principled framework for the analysis and design of natural Riemannian gradient descent algorithms for the optimization of functions defined over statistical models, with applications in machine learning, statistical inference, information theory, stochastic optimization, and several fields in computer science, such as robotics and computer vision.

The task of optimizing a function whose variables are the parameters of a statistical model is widespread in data science, think for example to the optimization of the expected value of a function with respect to a distribution in a statistical model, the maximization of the likelihood, or more in general the minimization of a loss function. Whenever the closed formula for the solution of the problem is unknown, gradient descent methods constitute a classical approach to optimization. However, it is a well-known result in statistics that the geometry of a statistical model is not Euclidean, instead the unique metric which is invariant to reparameterization is the Fisher information metric. It follows that the direction of maximum decrement of a function over a statistical model is given by the Riemannian natural gradient, first proposed by Amari [2].

In the first part of the presentation we will focus on Riemannian methods based on gradient descent, for the optimization of the stochastic relaxation of black-box functions, i.e., the optimization of the expected value of a function by natural gradient descent. In the optimization of continuos functions, a standard choice for the stochastic relaxation is the use of Gaussian distributions, parametrized by the mean vector and the covariance matrix. For large dimensions, a quadratic number of parameters can be unfeasible, thus sub-models parametrized by a smaller number of variables are often adopted. In the talk we will discuss sub-models in the Gaussian distribution parametrized by sparse inverse covariance matrices, which determine computational complexities for the evaluation of the Riemannian natural gradient which are linear in the number of non-zero entries.

Despite the directness of first-order methods, there are situations where taking into account the information on the Hessian of the function to be optimized gives an advantage, for instance for ill-conditions problems for which gradient methods may converge too slowly. Similarly to the natural gradient, also the definition of the Hessian of a function depends on the metric, so that second-order methods over statistical manifolds need to be generalized to the Riemannian geometry of the search space [1].

When we move to the second-order geometry of a differentiable manifold, the notion of covariant derivative is required for the parallel transport between tangent spaces, in particular to compute directional derivatives of vector fields over a manifold.

In the second part of the talk we will focus on second-order methods for the optimization of the stochastic relaxation of continuos functions, based on Gaussian distributions. The application of such methods to the optimization over statistical manifolds using second-order Riemannian optimization algorithms is a novel and promising area of research, indeed even if Information Geometry and second-order manifold optimization are well consolidated fields, surprisingly little work has been done at the intersection of the two.

Acknowledgments

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SDEs on Manifolds and Their Optimal Projections on Submanifolds

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We begin by reviewing the differences between the Itô and Stratonovich definitions of the stochastic integral, how the former is easier to interpret probabilistically, while properties of the latter make it the mainstream choice for geometric applications. Although Itô and Stratonovich stochastic differential equations (SDEs) are equivalent (through a transformation of the drift term), numerical approaches for the two theories differ. We discuss the definition of SDEs on manifolds, and the 2-jet representation introduced in [1] as a convergent numerical scheme.

Next we consider the problem of optimally approximating SDEs on submanifolds, see [2]. This allows one to systematically develop low dimensional approximations to high dimensional SDEs using differential geometric techniques. The corresponding problem in the case of ODEs has a clear solution, which doesn't extend to the stochastic case in a straightforward manner. We discuss three different types of projections and compare their optimality. Applications include optimal finite dimensional approximations of infinite dimensional stochastic filters in signal processing.

Acknowledgments

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4.2 Information and Arbitrage in Financial Markets (C. Fontana)

Causal optimal transport and its links to enlargement of filtrations and stochastic optimization problems

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The martingale part in the semimartingale decomposition of a Brownian motion, with respect to an enlarged filtration, is an anticipative mapping of said Brownian motion. In analogy to optimal transport theory, I will define causal transport plans in the context of enlargement of filtrations, as the Kantorovich counterparts of the aforementioned non-adapted mappings. I will present a necessary and sufficient condition for a Brownian motion to remain a semimartingale in an enlarged filtration, in terms of certain minimization problems over sets of causal transport plans. The latter will be also used in order to give an estimate of the value of having additional information, for some classical stochastic optimization problems.

*Speaker

Optimal arbitrage and portfolio optimization for market models satisfying weaker no-arbitrage conditions than NFLVR

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The classical no-arbitrage condition of NFLVR is often too strong and can be weakened thereby still allowing to solve meaningfully standard problems in mathematical finance. A weaker condition to this effect is NUPBR (NA1). For market models satisfying NUPBR, but where NFLVR does not hold, classical arbitrage is thus possible. In view of constructing such market models we consider models with insider information and for such models we discuss, besides optimal arbitrage, also the possibility of solving portfolio optimization problems by analogy to classical duality even under absence of an ELMM. [1]

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*Presenting

The Price of Informational Arbitrage

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In the context of a general complete semimartingale financial model, we consider the utility maximization problem for two types of agents:

(i) an ordinary agent having access to the publicly available information;

(ii) an informed agent having access to some private information.

In particular, we are interested in the case where the private information can potentially generate arbitrage opportunities but not arbitrages of the first kind, according to [1]. We aim at characterizing the price at which an agent is indifferent between buying the private information and observing only the publicly available information, knowing that the private information could lead to arbitrage opportunities. By relying on the martingale representation result recently established in [3], we generalize the results of [2]. In general terms, the present work aims at answering the following question: "*what price would you be prepared to pay for some private information that will allow you to make arbitrage profits*"?

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*Speaker

4.3 Metastability of Interacting Particle Systems (F. R. Nardi, A. Zocca)

Irreversible stochastic dynamics: effects of boundary conditions

Elisabetta Scoppola

Some results on metastability for irreversible dynamics are recalled and in particular the effects of boundary conditions are analysed in a simple Ising-type spin system. In particular some recent results obtained in collaboration with Aldo Procacci and Benedetto Scoppola in [6] are presented. Irreversible dynamics turn out to be a challenging problem since they are the main ingredient in the study of non-equilibrium statistical mechanics. Indeed many interesting physical systems can not be described in terms of equilibrium: for instance non-Hamiltonian evolutions, systems with external non-conservative forces, or systems with thermostats or reservoirs. Such systems exhibit non zero currents of matter or energy flowing in an *irreversible* way. For this kind of problems it is necessary to consider non-equilibrium statistical mechanics. Actually we can say that the description of non-equilibrium systems represents one of the “grand challenges" in statistical mechanics.

In this frame the main point is to describe *Non-Equilibrium Stationary States* (NESS), “in understanding the properties of states which are in stationary nonequilibrium: thus establishing a clear separation between properties of evolution towards stationarity (or equilibrium) and properties of the stationary states themselves: a distinction which until the 1970's was rather blurred." as mentioned in the beautiful book by Gallavotti [3].

Irreversible dynamics play in this context a crucial role. The invariant measures of irreversible dynamics are stationary states but they describe non zero currents of probability, and hence they are NESS. A famous example is given by the TASEP model, in which particles hop only to the right, entering from a left reservoir with a given rate and leaving the system from the site L with another rate.

In the context of Markovian dynamics, given any two states i and j in some configuration space $\mathcal{X}$, the irreversibility is defined by transition probabilities $P(i,j)$ violating the detailed balance condition

$$\pi(j)P(j,i) = \pi(i)P(i,j) \qquad \forall i,j \in \mathcal{X}$$

This means that there are non zero probability currents. Indeed given a pair of states $i,j \in \mathcal{X}$ define the probability current (or flow of probability) from j to i at time t the asymmetric function on $\mathcal{X} \times \mathcal{X}$:

$$K_t(j,i) = P^t(j)P(j,i) - P^t(i)P(i,j)$$

where $P^t(\cdot)$ represents the probability of the state $\cdot$ at time t.

The continuity equation for $P^t(i)$, gives

$$P^{t+1}(i) - P^t(i) = \sum_j P^t(j)P(j,i) - P^t(i)\sum_j P(i,j) =$$

$$= \sum_{j \neq i} \Big(P^t(j)P(j,i) - P^t(i)P(i,j)\Big) = \sum_{j\neq i} K_t(j,i) = -(\mathrm{div}\, K_t)(i)$$

Stationarity implies

$$0 = \sum_{j\neq i} \Big(\pi(j)P(j,i) - \pi(i)P(i,j)\Big) = \sum_{j \neq i} K(j,i) \qquad \forall i \tag{1}$$

being $K(j,i) = \pi(j)P(j,i) - \pi(i)P(i,j)$, the stationary probability current (or stationary flow of probability) from j to i, a divergence free flow. This flow K is proportional to the antisymmetric part of the conductance associated to the chain and it is also considered for instance in [4].
Actually the presence of currents can be used to detect irreversible dynamics without using the invariant measure. This is done by the Kolmogorov criterion for reversibility: the Markov dynamics with transition probabilities $P(i,j)$ is reversible if and only if for any loop of states: $i_o, i_1, i_2, ..., i_n, i_o$ we have

$$P(i_0,i_1)P(i_1,i_2)....P(i_n,i_0) = P(i_0,i_n)....P(i_2,i_1)P(i_1,i_0).$$

This means that the dynamics is irreversible if there is a loop with a stationary current.

Beside their crucial role in the understanding of non-equilibrium statistical mechanics, irreversible dynamics have been frequently considered in the literature in order to speed up simulations. Indeed in some case rigorous control of mixing time of irreversible dynamics has been obtained. See for instance [1].

Several problems arise when considering irreversible dynamics. Indeed some tools frequently used in the study of convergence to equilibrium are strongly related to reversibility as spectral representation or the potential theoretical approach. Recently some progress has been done to extend some of these tools to non reversible dynamics. See for instance the extension of the Dirichlet principle to non reversible Markov chains obtained in [4].
In this paper we want to stress the main difficulty related to irreversibility: while detailed balance is a crucial tool to control the invariant measure of reversible dynamics, in the irreversible case the control of the invariant measure can be quite complicated, and in particular it is difficult to study its sensitivity to boundary conditions. Very recent results have been obtained in this direction in [2] where irreversible dynamics are constructed with a given Gibbsian stationary measure by exploiting cyclic decomposition of divergence free flows.
In some case it is possible to verify that the equation for the invariant measure (1) is satisfied by a suitable Gibbs measure, as proved below in the (easy) case of empty boundary conditions. This is also the case of 2-dimensional Ising model with asymmetric interaction discussed in [5] with periodic boundary condition. In general, due to the presence of probability currents, the verification of equation (1) typically involves non local argument and so the invariant measure strongly depends on boundary conditions.

We consider a one dimensional spin system on the discrete interval $[1, L] \equiv \{1, 2, ..., L\}$ with a single-spin-flip Markovian dynamics $\{X_t\}_{t\in\mathbb{N}}$, defined on $\mathcal{X} := \{-1, 1\}^L$ by the following transition probabilities

$$P(\sigma, \sigma^{(i)}) = \frac{1}{L} e^{-2J(\sigma_i \sigma_{i-1}+1)} \tag{2}$$

where $\sigma^{(i)}$ is the configuration obtained from σ flipping the spin in the site $i \in \{1, 2, ..., L\}$. This means that at each time a site i is chosen uniformly at random in $\{1, 2, ..., L\}$ and the spin is flipped in this site with probability one if it is opposite to its left neighbour, σ_{i-1}, or with probability e^{-4J} if it is parallel to σ_{i-1}. We will consider two different boundary conditions:

- the empty boundary condition corresponding to $\sigma_0 = 0$;
- the $+$ boundary condition corresponding to $\sigma_0 = +1$.

The chain is irreducible and aperiodic so that in both cases there exists a unique invariant measure. Our goal is to compare the invariant measures of the Markov chains corresponding to these two different boundary conditions in a very low temperature regime, i.e., when the parameter J is sufficiently large w.r.t. L.
We shall prove that while in the case of empty boundary conditions the stationary distribution is the Gibbs measure, in the case of + boundary condition the stationary measure changes

drastically. Due to the particular low-temperature regime we are able to write the stationary distribution as an absolutely convergent expansion in e^{-4J}. This expansion is easily controlled in this case, but it could be a general tool in order to control the invariant measure at a very low temperature in more general contexts. We control completely the first order of such expansion, and we show that it has several interesting features. In particular, the presence of probability currents implies that the boundary conditions do not have the effect of a conditioning, as in the case of the Gibbs measure. The boundary conditions actually modify the stationary distribution and the effect of their presence decay very slowly in the distance i from the boundary, namely as $\frac{1}{\sqrt{i}}$. Moreover, the presence of boundary conditions makes the probabilities of interval of minus spins dependent on their length, producing macroscopical effects on the magnetization. Interesting combinatorial identities are involved in the proofs.

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Metastability for general dynamics with rare transitions: escape time and critical configurations

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Contributed talk

"This is joint work with E. Cirillo and F. Nardi.

Metastability is a physical phenomenon ubiquitous in first order phase transitions. A fruitful mathematical way to approach this phenomenon is the study of rare transitions Markov chains. For Metropolis chains associated with Statistical Mechanics systems, this phenomenon has been described in an elegant way in terms of the energy landscape associated to the Hamiltonian of the system. In this paper, we provide a similar description in the general rare transitions setup. Beside their theoretical content, we believe that our results are a useful tool to approach metastability for non–Metropolis systems such as Probabilistic Cellular Automata.

*Speaker

Competing Metastable States for general rare transition dynamics and applications to Blume-Capel model and Probabilistic Cellular Automata*

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Contributed talk

The study of systems with multiple (not necessarily degenerate) metastable states presents subtle difficulties from the mathematical point of view related to the variational problem that has to be solved in these cases. First, we prove sufficient conditions to identify multiple metastable states. Since this analysis typically involves non–trivial technical issues, we give different conditions that can be chosen appropriately depending on the specific model under study. We show how these results can be used to attack the problem of multiple metastable states. Second, we consider the problem of non degenerate in energy metastable states forming a series in the framework of reversible finite state space Markov chains. We assume that, starting from the state at higher energy, the system visits with high probability the second one before reaching the stable state. In this framework we give a sharp estimate of the transition time from the metastable state at higher energy to the stable state and, on the proper exponential time scale, we prove an addition rule. As application of the theory, we study the Blume–Capel model in the zero chemical potential case and to a probabilistic cellular automata that happen to have two multiple not degenerate in energy metastable states. We estimate in probability, in law and in expectation the time for the transition from the metastable states to the stable state. Moreover, we identify the set of critical configurations that represent the minimal gate for the transition.

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4.4 Algebraic Statistics (E. Riccomagno)

Simulation of multivariate binary distributions

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We present a new computational procedure to simulate multivariate distributions of binary variables with fixed marginal distributions and given higher-order moments. Among the others this procedure allows us to establish if a given correlation matrix is compatible with the assigned margins and, whenever it is, to easily construct one of the corresponding joint density. It also provides a general expression for the bounds that each correlation must satisfy to be compatible with the assigned margins.

We express each Fréchet class of multivariate Bernoulli distributions with given margins as the convex hull of the *ray densities*, which are densities that belong to the same Fréchet class. Such representation is based on a polynomial expression of the distributions of a Fréchet class. Then we reduce the problem of finding a density of a Fréchet class with given correlation matrix to the solution of a linear system of equations.

An algorithm and its use on some examples is shown.

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Sensitivity analysis in graphical models: a polynomial approach

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Sensitivity analysis comprises a suite of methods to investigate the effects of the inputs in a statistical model to outputs of interest [1]. These methods have received great attention for a class of statistical graphical models usually called *Bayesian networks* (BNs), in particular for cases where all variables take values in finite discrete spaces. A discrete BN consists of two components: a directed acyclic graph with vertices the variables of interests and conditional probability tables (CPTs) for each vertex given its parents.

Sensitivity analysis in discrete BNs usually entails the variation of some of the numerical entries of the CPTs and the quantification of the consequent change on output probabilities of interest. The conditional probabilities from the same CPTs need to be *covaried* in order to respect the sum to one condition of probabilities. The most commonly used type of covariation is the so-called *proportional scheme*, which imposes that each covarying parameter has the same proportion of the remaining probability mass as it originally had [7].

For discrete BNs and when only one input parameter is varied (the so called one-way sensitivity analysis), proportional covariation leads to the smaller CD distance [3] between the original and the varied distribution amongst all possible covariation schemes [2]. For multi-way sensitivity analyses, namely when more than one parameter is varied contemporaneously, very little is known. For a

*Speaker

large class of discrete graphical models including BNs and for a specific class of multi-way parameter variations called single full CPT analyses, that optimality result has been extended [6] relying on a polynomial representation of statistical models that, as a special case, includes the standard polynomial representation of discrete BNs [5].

In this work following [6] we introduce a new characterization of sensitivity analysis using *information geometry* and specifically *I-projections* [4]. Preliminary results show that covarying the relevant probabilities in an optimal way corresponds to computing the I-projection of the original probability distribution into a well-defined space. Optimality results of [2] and [6] can be replicated under this formalism. Furthermore, this combination of the polynomial approach with concepts from information geometry enables us to extend the optimality of proportional covariation to new classes of multi-way sensitivity analysis.

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On the regularity of multi-level Orthogonal Arrays

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In this work we analyze the notion of regular fraction for symmetric multi-level designs with qualitative factors. A regular fraction is a special Orthogonal Array. We give some results in order to check if a given Orthogonal Array is isomorphic to a regular fraction, under permutation of factors or factor levels. Our results have an immediate algorithmic counterpart, since they actually yield the relevant permutations.

Regular fractions and Orthogonal Arrays

Let $X_1, \ldots, X_d$ be d qualitative factors with p levels (p prime). We use here the complex coding of the factor levels and we denote with $\mathcal{D}$ the full-factorial design $\mathcal{D} = \Omega_p^d$, where $\Omega_p = \{\omega_0, \ldots, \omega_{p-1}\} \subset \mathbb{C}$ is the set of the p-th roots of the unity.

A fraction $\mathcal{F} \subset \mathcal{D}$ with n points (without replicates) is written as a $n \times d$ table, or design matrix, with values in Ω_p, where each row represents a design point and each column represents a factor. The indicator function of $\mathcal{F}$ is a function $F : \mathcal{D} \to \mathbb{C}$ such that $F(\zeta) = 1$ for $\zeta \in \mathcal{F}$ and $F(\zeta) = 0$ for $\zeta \in \mathcal{D} \setminus \mathcal{F}$. Following [1], the indicator function can be expressed in polynomial form as

$$F(\zeta) = \sum_{\alpha \in L} b_\alpha X^\alpha(\zeta) \,, \qquad b_\alpha \in \mathbb{C},\ \zeta \in \mathcal{D} \,, \tag{1}$$

where $L = \{0, \ldots, p-1\}^d$, and X^α is a vector notation for $X_1^{\alpha_1} \cdots X_d^{\alpha_d}$.

A fraction $\mathcal{F}$ is regular if there exist a sup-group $\mathcal{L}$ of L and a group homomorphism e from $\mathcal{L}$ to Ω_p such that the equations

$$X^\alpha = e(\alpha) \qquad \alpha \in \mathcal{L} \tag{2}$$

define the fraction $\mathcal{F}$, i.e., they are a set of generating equations.

If the factor X_i is transformed into $\omega_h X_i^k$ for some $h = 0, \ldots, p-1$ and $k = 1, \ldots, p-1$, its levels are permuted. A regular fraction with such a transformation on one or more factors is transformed into a regular fraction. Notice that if $p > 3$ not all the level permutations are of this type.

Let I be a set of factors, $I \subseteq \{X_1, \ldots, X_d\}$. A fraction $\mathcal{F}$ factorially projects on the I-factors if the projection is a full factorial design where each point appears equally often. A fraction $\mathcal{F}$ is an Orthogonal Array of strength t if it factorially projects on any I-factors with $\#I = t$. For an Orthogonal Array of strength t, all the coefficients of the indicator function up to the order t are equal to zero.

The set of all Orthogonal Arrays with n design points on $\mathcal{D}$ and strength t is denoted with $OA(n, \mathcal{D}, t)$. Two (symmetric) Orthogonal Arrays $\mathcal{F}_1, \mathcal{F}_2 \in OA(n, \mathcal{D}, t)$ are isomorphic if $\mathcal{F}_1$ can be obtained from $\mathcal{F}_2$ by reordering the design points (i.e., the rows of the design matrix), by relabeling the factors (i.e., by permuting the columns of the design matrix), by permuting the levels of one or more factors, see [2]. While the permutation of the rows does not affect the indicator function, and the permutation of the columns simply yields a reordering of the coefficients of the indicator function in Eq. (1), the last condition is less simple to check.

*Speaker

Regularity and permutations

We focus here on Orthogonal Arrays with strength 2, $\mathcal{F} \in OA(n, \mathcal{D}, 2)$, and therefore we look at generating equations involving three factors. Without loss of generality, let us consider the factors X_1, X_2, X_3. Let

$$X_1^{\alpha_1} X_2^{\alpha_2} X_3^{\alpha_3} = \omega_k \tag{3}$$

be a generating equation of $\mathcal{F}$, with $\alpha_1, \alpha_2, \alpha_3 \in \{1, \ldots, p-1\}$ and $\omega_k \in \Omega_p$. For brevity, we say that X_1, X_2, X_3 form a generating equation of the Orthogonal Array $\mathcal{F}$.

Consider now the $p \times p$ table $C = X_3(X_1, X_2)$ containing the values of X_3 as a function of X_1 and X_2, i.e., $C_{h,k} = x_3$ given $x_1 = h$ and $x_2 = k$. This table may be regarded as a $p \times p$ latin square with values in Ω_p. The main result can be stated as follows.

Let X_1, X_2, X_3 be three factors and let $X_3(X_1, X_2)$ be the corresponding latin square.

(a) If X_1, X_2, X_3 form a generating equation, then $X_3(X_1, X_2)$ has rank 1 in Ω_p, i.e., all 2×2 minors of $X_3(X_1, X_2)$ vanish in Ω_p;

(b) If there is a permutation π_3 of Ω_p such that $(\pi_3(X_3))(X_1, X_2)$ has rank 1 in Ω_p, then there exist permutations π_1 and π_2 such that $\pi_1(X_1), \pi_2(X_2), \pi_3(X_3)$ form a generating equation.

The above result can be used for the definition of an algorithm in the general setting of an Orthogonal Array with an arbitrary number of factors and with generic strength t along the following lines:

1. For generating equations involving $r > 3$ factors. The above result must be applied slice by slice to 3 factors with the remaining $(r-3)$ factors fixed at given levels.
2. For Orthogonal Arrays with more than three factors. All possible choices of r factors must be considered.

In both cases some compatibility conditions among the permutations must be taken into account.

Further investigations will include the study of the constraints induced by the permutations of the factor levels on the coefficients of the indicator function. In fact, the b_α's in Eq. (1) encode the generating equations and each permutation of the factor levels transform such equations through a polynomial map. Finally, we will provide some insights on how to define permutation-invariant measures to discriminate between Orthogonal Arrays, as partially outlined in [3], where the notion of mean aberration has been introduced.

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4.5 Asymptotics for Measure Valued Processes (C. Costantini)

Duality in population models: an algebraic approach*

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Duality theory plays a key role in mathematical population genetics models, and in particular in the study of asymptotic properties. Typically, forward in time evolution is related by duality to backward random genealogies. The fundamental construct here is Kingman's coalescent (1982), which describes the genealogy of a random sample from a population.

In this talk, a novel approach to duality theory of Markov processes will be described. In particular, the scheme will be explained using classical Lie algebras for some well-known process in population dynamics, such as multi-type Moran models and multi-type Wright-Fisher diffusions (neutral or including mutations) [1].

The algebraic approach, besides recovering the classical dualities with a coalescent, yields novel dualities between two forward processes. If time allows we will also consider some new processes that arise by considering deformed algebras, that amounts to introducing genetic selection in the population model [2, 3].

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Fixation and duality in a Xi-Flemig-Viot model with selection

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Ξ-Fleming-Viot models are the most general class of measure-valued processes arising as scaling limit of a population genetics model with exchangeable offspring distribution (Cannings models). They are jump-diffusion "allele-frequency"processes characterised by a dual coalescent genealogy

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which may allow for simultaneous multiple mergers. In this talk we will introduce a generalisation of Cannings models allowing for several types of selection, thus implying a partially exchangeable offspring distribution. We shall focus on the behaviour of a two-type model, where the two types have different fitness strength. The construction provides an almost sure dual relation between frequency process and ancestral process. We will derive its scaling limit frequency process and the corresponding dual ancestral process, the latter resulting in a branching-coalescing chain with arbitrarily-sized jumps. We will focus on the fate of the selectively weak allele and, using duality, we will investigate in which cases the selective type goes to fixation with probability one. (Joint work with Adrián González Casanova (WIAS Berlin))

Wright–Fisher construction of the two-parameter Poisson–Dirichlet diffusion

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The two-parameter Poisson–Dirichlet diffusion, introduced in [3], extends the infinitely-many-neutral-alleles diffusion model, related to Kingman's one-parameter Poisson–Dirichlet distribution and to certain Fleming–Viot processes. The additional parameter has been shown to regulate the clustering structure of the population (see e.g. [4]), but is yet to be fully understood in the way it governs the reproductive process.

In this talk we shed some light on these dynamics by providing a finite-population construction, with finitely many species or alleles, in analogy to what is done in [2] for the infinitely-many-neutral-alleles diffusion model. Our model is a K-allele Wright–Fisher model for a population of size N, involving a uniform mutation pattern and a specific state-dependent migration mechanism. Suitably scaled, this process converges in distribution to a K-dimensional Wright–Fisher diffusion process as $N \to \infty$. The descending order statistics of the K-dimensional diffusion converge in distribution to the two-parameter Poisson–Dirichlet diffusion as $K \to \infty$.

Convergence of the finite-dimensional diffusion to the two-parameter Poisson–Dirichlet diffusion depends on a delicate balance between reinforcement and redistributive effects in the migration mechanism. Moreover the proof of convergence is nontrivial because the generators do not converge on a core. Our strategy for overcoming this difficulty is to prove *a priori* that in the limit there is no "loss of mass", i.e., that, for each limit point of the sequence of finite-dimensional diffusions (after a reordering of components by size), allele frequencies sum to one.

We also show that the two-parameter Poisson–Dirichlet distribution is the weak limit of the stationary distributions of the Wright–Fisher diffusions (modified to account for the rearranging of components in descending order), by analogy to what happens in the one-parameter case, where these stationary distributions are symmetric Dirichlet distributions.

This talk is based on a joint work with P. De Blasi, S. N. Ethier, M. Ruggiero and D. Spanò ([1]).

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4.6 Martingale Optimal Transport (P. Siorpaes)

On the support of extremal martingale measures with given marginals: the countable case*

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We investigate the supports of extremal martingale measures with pre-specified marginals in a two-period setting. First, we establish in full generality the equivalence between the extremality of a given measure Q and the denseness in $L^1(Q)$ of a suitable linear subspace, which can be seen as the set of all semi-static trading strategies. Moreover, when the supports of both marginals are countable, we focus on the slightly stronger notion of weak exact predictable representation property (henceforth, WEP) and provide two combinatorial sufficient conditions, called "2-link property" and "full erasability", on how the points in the supports are linked to each other for granting extremality. Finally, when the support of the first marginal is a finite set, we give a necessary and sufficient condition for the WEP to hold in terms of the new concept of 2-net.

Short description of the main results

In this paper we are interested in describing the supports of extremal martingale measures, defined on the product space $\mathbb{R}_+^2 = (0, \infty)^2$ equipped with its Borel σ-field, under the constraints of having given marginals μ and ν. The set of all such measures, which is nonempty if and only if μ is smaller than ν in the convex order, is at the core of *martingale optimal transport*, a new field of research that has been introduced by [3] in the discrete-time case and by [14] in the continuous-time case. The martingale optimal transport problem is a variant of the classical Monge-Kantorovich optimal transport problem (see [31]), and it consists in optimizing a given functional over the set $\mathcal{M}(\mu, \nu)$ of all probability measures with given marginals μ and ν and satisfying the martingale property. The latter property is what makes the difference with the classical optimal transport problem and it is motivated by financial applications. An important growing literature originated from the seminal paper [17], which started the model-free approach to derivative pricing using techniques based on the Skorokhod embedding problem. Within this approach, only very weak assumptions are made, namely the price process of the underlying is a martingale (to rule-out arbitrage opportunities) and its marginals are given by the observation of European Call prices (via the so-called Breeden-Litzenberger formula). Hence, computing for instance the super-replication price of some derivative

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†Speaker

boils down to maximizing the expected value of its pay-off, say f, over the set $\mathcal{M}(\mu, \nu)$, yielding the following martingale transport problem:

$$\sup_{Q \in \mathcal{M}(\mu,\nu)} Q(f), \tag{1}$$

where $Q(f)$ denotes the expectation of f under Q. This problem has been studied in great depth in [4] for a large class of payoffs. The results therein have been further generalized in [15]. In the papers [19, 18] the (martingale) optimal transport has been found for $f(x, y) = \pm|x - y|$.

Our interest in the extremal elements of the set $\mathcal{M}(\mu, \nu)$ is fundamentally motivated by the model-free approach. Indeed, the notion of extremal measures is intimately related to that of optimizers (see, for instance, Bauer Maximum Principle, [2, Sec. 7.69]). Therefore, understanding the support of extremal measures can give insights on the solutions of martingale optimal transport problems such as (1). Another motivation for this study comes as a consequence of our first result, which roughly states the following equivalence:

A martingale measure $Q \in \mathcal{M}(\mu, \nu)$ is extremal if and only if every derivative can be approximately replicated on the support of Q by semi-static strategies.

This can be seen as an extension, to the model-free setting, of the well-known equivalence in the classical setting between "market completeness" and extremality of Q in the set of all martingale measures without constraints on the marginals (see, e.g. [23] for the discrete-time case), which is in turn the financial translation of one of the most important results in martingale theory, namely that extremality is equivalent to the predictable representation property (see [10] in discrete-time and, e.g., [29, Theorem 4.7, Ch. V] in continuous-time).

Therefore, in the model-free setting, knowing the support of extremal measures in $\mathcal{M}(\mu, \nu)$ gives a way to generate models where any derivative can be (approximately) replicated by semi-static strategies.

Besides the financial motivation, the problem of characterizing the supports of extremal measures is mathematically interesting on its own and it has quite a long history. Indeed, there is a rich literature on the support of extremal probability measures with given marginals (without the martingale property), which goes back to a paper by Birkhoff [7], where a complete description of extremal measures is given in the finite case, i.e. both marginals have finite supports. The main result therein establishes that a probability measure with given marginals is extremal if and only if its support does not contain any cycle. Many papers followed, e.g. [5, 6, 11, 13, 16, 21, 22, 24, 26, 27] among others, giving different kinds of characterizations in the finite or countable case and going from functional analysis to combinatorics. In particular [11] extends to the countable case Birkhoff's result about absence of cycles in the support of extremal measures. In the general case, the problem of giving a complete description of extremal measures with given marginals is still open.

Inspired by this literature, the present paper provides, in full generality, a characterization of extremality in the martingale case in terms of a weaker form of the predictable representation property. In the more specific case of marginals with countable supports, we define a slightly stronger property (called WEP) which allows us to focus on the combinatorial properties of the support of a given measure in $\mathcal{M}(\mu, \nu)$. Therefore, we propose two sufficient conditions, called "2-link property" and "full erasability", having a strong combinatorial flavour. Three important examples satisfy those criteria and hence they are extremal measures: the binomial tree, the left curtain introduced in [4], and Hobson and Klimmek's trinomial tree (cf. [18]). Moreover, those criteria are very easy to implement for generating many other examples of extremal supports. Finally, we introduce the new notion of 2-net, which allows to formulate a necessary and sufficient condition for the WEP when the support of the first marginal is finite.

– A full version of the paper can be found at: arxiv.org/abs/1607.07197 –

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The Martingale Polar Sets

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Martingale optimal transport (MOT) is a variant of the classical optimal transport problem where a martingale constraint is imposed on the coupling. As shown by Beiglböck, Nutz and Touzi in the forthcoming paper [1], in dimension one there is no duality gap and that the dual problem admits an optimizer. A key step towards this achievement is the characterization of the polar sets of the family of all martingale couplings. Here we extend this characterization to arbitrary finite dimension through a deeper study of the convex order.

*Speaker

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Robust hedging of VIX options: a constrained Martingale Optimal Transport problem

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VIX options traded on the CBOE have become popular volatility derivatives. As S&P500 vanilla options and VIX both depend on S&P500 volatility dynamics, it is important to understand the link between these products. In this talk, we obtain model-free bounds and super-replication strategies for VIX options using vanilla options on the S&P500 and VIX futures. This leads us to introduce a new *martingale optimal transport* problem, that (in its general form) we solve numerically. Analytical lower and upper bounds are also provided: we characterize the class of S&P500 marginal distributions for which these explicit bounds are optimal, and illustrate numerically that they appear to be optimal for the marginal distributions implied by the market. Interestingly, theses bounds seem to highlight some potential arbitrage opportunities contained in VIX options (at least on some specific trading day).

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4.7 Stochastic Fluid Dynamics (F. Morandin)

Stationary distribution for stochastic inviscid shell models

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We consider a stochastic differential system and aim to study the uniqueness of its stationary invariant solutions. We show that it is possible deduce the uniqueness of stationary solutions via an optimal transport argument. The idea is to introduce a distance between probability measures as cost function to be minimized, in such a way that the existence of two different solutions would contradict the Kantorovich's formulation for the optimal transport problem.

Stochastic Navier-Stokes equations in $\mathbb{R}^d$

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We consider the stochastic damped Navier-Stokes equations

$$\begin{cases} \partial_t v + [-\nu\Delta v + \gamma v + (v\cdot\nabla)v + \nabla p]\ dt = G(v)\ \partial_t w + f\ dt \\ \nabla\cdot v = 0 \end{cases} \tag{1}$$

where the unknowns are the vector velocity $v = v(t,\xi)$ and the scalar pressure $p = p(t,\xi)$ for $t \geq 0$ and $\xi \in \mathbb{R}^d$. By $\nu > 0$ we denote the kinematic viscosity and by $\gamma \geq 0$ the sticky viscosity. On the right hand side $\partial_t w$ is a space-time white noise and f is a deterministic forcing term; we consider a multiplicative term $G(v)$ keeping track of the fact that the noise may depend on the velocity. The low regularity of this term $G(v)$ is the peculiarity of our problem; the covariance of the noise is not too regular, so Itô calculus cannot be applied in the space of finite energy vector fields.

We prove the existence of martingale solutions for $d = 2$ or $d = 3$ and, for $d = 2$, the pathwise uniqueness of solutions. Moreover, assuming $\gamma > 0$ we prove existence of invariant measures for $d = 2$ and existence of stationary solutions for $d = 3$.

The techniques involved are different from those for the bounded spatial domain case, since the embedding $H^a \subset H^b$ (with $a > b$) is continuous but not compact. In particular, the proof of existence of a weak solution is obtained by Galerkin approximation and proving its tightness (this requires new compactness results), whereas the proof of the existence of an invariant measure is based on the technique working with weak topologies introduced by Maslowski and Seidler.

Structure function for an intermittent dyadic model*

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We study a generalization of the original tree-indexed dyadic model by Katz and Pavlović, for the turbulent energy cascade of three-dimensional Euler equation. We allow the coefficients to vary with some restrictions, thus giving the model a realistic spatial intermittency. By introducing a forcing term on the first component, the fixed point of the dynamics is well defined and some geometric properties of the physical solution may be proved. In particular the exponent of the structure function is concave in accordance with other theoretical and experimental models.

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*Speaker
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†Speaker

4.8 Stochastic Network Systems: Opinion Dynamics, Robustness, and Epidemics (G. Como, F. Fagnani)

Stochastic matrices and invariant probabilities in networks: new questions for classical concepts

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Since its first appearance in the pioneering work by Markov in 1906, stochastic matrices and corresponding invariant probabilities have played an increasingly crucial role in many different fields of mathematics and science, often in contexts not directly related to the original probabilistic interpretation. Notably, stochastic matrices are the key constitutive element in the De-Groot averaging model proposed in social science as a simple model for the evolution of opinions. Similar models have more recently appeared in sensor and computer network literature as basic algorithms implemented to achieve a variety of goals as decentralized computation of global functions, clock synchronization, load balancing. Further on, in the increasing attention to network, stochastic matrices have showed up in the definition of centralities of nodes: the well-known Bonacich and page-rank centralities are indeed nothing else but invariant probabilities of particular random walks on a given graph. The fundamental results obtained by Perron and Frobenius in 1907-08 on the spectral properties of non-negative matrices are the key tools used in the analysis of these models.

In this talk, we first briefly review some of these recent applications of the concepts of stochastic matrix and invariant probability to the above mentioned fields. After, we consider fundamental questions that these new applicative settings naturally generate and that typically were not been previously addressed. In all the mentioned applications, the invariant probability plays a fundamental role and a key question is to investigate its stability under a perturbation of the underlying stochastic matrix. The typical scenario we have in mind is that of a stochastic matrix constructed as a random walk on a given graph and the perturbation affecting a 'small' number of rows. This perturbation may model the presence of a minority of heterogeneous units with different features. Alternatively, it could be a perturbation of local nature, in a graph sense, where a node or a set of nodes add/delete or, more generally 'rewire' their outgoing or incoming edges in order to suitably reshape the invariant probability. In this last context, natural optimization problems emerge like for instance how to determine the rewiring maximizing the centrality of a node. The main theoretical result presented is in the context of a large scale limit. It consists in a fundamental limitation on the total variation of the invariant probability when the stochastic matrix is modified in a limited number of rows. A final application showing the resilience of the page-rank centrality used by many web engines completes the presentation.

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Influence, polarization, and opinion fluctuations in social networks

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In this talk, tractable models of opinion dynamics are considered accounting for long-run disagreements and persistent opinion fluctuations in social networks. The considered models involve inhomogeneous stochastic averaging processes of continuous opinion dynamics in a society consisting of two types of agents: *regular agents*, who update their beliefs according to information that they receive from their social neighbors; and *stubborn agents*, who never update their opinions and might represent leaders, political parties or media sources attempting to influence the beliefs in the rest of the society. When the society contains stubborn agents with different opinions, the belief dynamics never lead to a consensus (among the regular agents). Instead, beliefs in the society fail to converge almost surely, the belief profile keeps on fluctuating in an ergodic fashion, and it converges in law to a non-degenerate random vector. The structure of the graph describing the social network and the location of the stubborn agents within it shape the opinion dynamics. The expected belief vector is proved to evolve according to an ordinary differential equation coinciding with the Kolmogorov backward equation of a continuous-time Markov chain on the graph with absorbing states corresponding to the stubborn agents, and hence to converge to a harmonic vector, with every regular agent's value being the weighted average of its neighbors' values, and boundary conditions corresponding to the stubborn agents' beliefs. Expected cross-products of the agents' beliefs allow for a similar characterization in terms of coupled Markov chains on the graph describing the social network.

We prove that, in large-scale societies which are *highly fluid*, meaning that the product of the mixing time of the Markov chain on the graph describing the social network and the relative size of the linkages to stubborn agents vanishes as the population size grows large, a condition of *homogeneous influence* emerges, whereby the stationary beliefs' marginal distributions of most of the regular agents have approximately equal first and second moment. We also present a sufficient condition for polarization across a cut splitting the network in two parts each containing only stubborn nodes of one opinion: if the ratio between the stationary flow across the cut and the aggregate centrality of the stubborn nodes with one opinion is vanishing in large scale limit, and the network is sufficiently expansive in the neighborhood of these stubborn nodes, then the opinions of the agents on the corresponding side of the cut concentrate on the the value of the stubborn node. Finally, steady state opinion fluctuations are studied and their level of synchronicity is related to the network structure. The talk is based partly on [1] and partly on a recent work by the same authors currently in preparation.

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Spreading Processes in Large Scale Graphs

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Analysis of Spreading Processes

Many diffusion dynamics, such as a disease outbreak in a geographic area or the propagation of an opinion within a population, can be modeled into the framework of spreading processes over networks. In this framework, a population and the connections between the individuals belonging to this population are represented through a graph. Each node of the graph (which we refer to as an agent) has a binary state, which determine, e.g., whether the corresponding agent is infected or not. Agents update their state spontaneously or after pair-wise interactions with another agents. Both spontaneous changes and interactions are regulated by stochastic Poisson clocks. Therefore, such models induce Markov jump processes on the space of all possible nodes configurations. Despite the markovianity of the process, its direct analysis is unfeasible for large-scale graphs, since the dimension of the state space grows exponentially in the size of the population.

Typically, this issue is tackled by projecting the Markov process on a lower-dimensional space. The classical choice consists in considering a stochastic process counting the fraction of individuals for each possible state. We notice that the dimension of the state space of this process grows linearly with the size of the population.However, since the lower-dimensional process is in general non-markovian, but in the trivial case of a complete graph, the analytical study of many spreading processes is limited to the case of a complete graph, yielding the mean field approach of the dynamics.

On the other hand, in order not to neglect the topological structure of the network in the analysis of the process, the non-markovian lower-dimensional process can be studied by means of a stochastic domination technique. Specifically, after having highlighted the dependence of the transition rates of the stochastic process on the magnitude of the boundary between agents with different states, we can construct a pair of topology-based Markov processes, where one dominates the original process an the other one is dominated by it. Then, through the analysis of these two Markov processes, we obtain very interesting results for the spreading process on a general graph, which relate the behavior of the system with both the model parameters and the topological ones.

In this contribution, first we present the development of these stochastic domination techniques for the analysis of the susceptible-infected-susceptible (SIS) model. Then, we apply these techniques to the analysis of two real-world inspired models, obtaining interesting topology-related results.

The SIS Model

Here, we show how stochastic domination techniques can be developed and used in the analysis of the simple SIS model, proving the existence of two regimes (namely, fast extinction and "endemization") depending on model and graph parameters, through a first-order analysis of the two bounding Markov processes obtained with stochastic domination.

A Model for the Diffusion of a New Product

Here, we present a model for the diffusion of a new product in a large-scale population. We apply stochastic domination techniques in the analysis of this model, showing how the parameters of the model, the topology of the graph and, possibly, the initial diffusion of the product, determine whether the spread of the item is successful or not. The main novelty of this model, with respect to the SIS model, consists in the dependence of the behavior of the model on the initial condition. This dependence will pose some issues in the analysis of the process, which can be addressed by a second-order analysis.

An Evolutionary Model

Here, we present a model for a controlled introduction of a mutant species in a geographic area. Stochastic domination techniques allows for the analytical study of the time needed for the mutant species to spread in the whole geographic area, depending on the model parameters, the control, and on the graph topology.

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4.9 Asymptotic Behavior of Conditional Probabilities (P. Rigo)

Asymptotics of predictive measures for exchangeable sequences

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Let (X_n) be a sequence of random variables, with values in a measurable space $(S, \mathcal{B})$ and adapted to a filtration $(\mathcal{G}_n)$, and let

$$\mu_n = (1/n) \sum_{i=1}^{n} \delta_{X_i} \quad \text{and} \quad a_n(\cdot) = P(X_{n+1} \in \cdot \mid \mathcal{G}_n)$$

be the empirical and the predictive measures. Fix $\mathcal{D} \subset \mathcal{B}$ and define

$$\| \mu_n - a_n \| = \sup_{B \in \mathcal{D}} | \mu_n(B) - a_n(B) | .$$

Under some conditions, $\mu_n(B) - a_n(B) \xrightarrow{a.s.} 0$ for fixed $B \in \mathcal{B}$. In that case, a (natural) question is whether the convergence is uniform over $\mathcal{D}$. Such a question arises in several frameworks, including predictive inference, Bayesian consistency and frequentistic approximation of Bayesian procedures.

Conditions for $\| \mu_n - a_n \| \to 0$, almost surely or in probability, are given. Also, to determine the rate of convergence, the asymptotic behavior of $r_n \| \mu_n - a_n \|$ is investigated for suitable constants r_n. Special attention is paid to

$$r_n = \sqrt{n} \quad \text{and} \quad r_n = \sqrt{\frac{n}{\log \log n}}.$$

The sequence (X_n) is exchangeable or, more generally, conditionally identically distributed.

Why and how frequentistic procedures are approximations of Bayesian ones

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The talk explains a part of a work in progress with D.M. Cifarelli and E. Regazzini, dealing with an asymptotic property of posterior distributions arising in a Bayesian nonparametric setting, as more and more data are collected. Confining to the case in which the observable quantities are modeled by an exchangeable sequence $\{\tilde{\xi}_n\}_{n \geq 1}$ of random variables, taking values in some topologically nice space $\mathbb{X}$, the attention is focused on the posterior distribution of either the random probability

*Speaker

measure $\tilde{\mathfrak{p}}$ such that $\mathsf{P}[\tilde{\xi}_1 \in A_1, \ldots, \tilde{\xi}_n \in A_n \mid \tilde{\mathfrak{p}}] = \prod_{i=1}^n \tilde{\mathfrak{p}}(A_i)$ holds for all $n \in \mathbb{N}$ and measurable sets $A_1, \ldots, A_n$, or some specific functional $\tilde{\tau}$ of $\tilde{\mathfrak{p}}$ itself belonging to the present class of statistical functionals: U-statistics; M-estimators; L-estimators; von Mises functionals. In many practical situations, the posterior distributions of $\tilde{\mathfrak{p}}$ and $\tilde{\tau}$ are very diffucult to compute but, on the other hand, more tractable estimators, say $\hat{\mathfrak{p}}_n$ and $\hat{\tau}_n$, respectively, are available from frequentist (or, generally, non-orthodox Bayesian) statistical literature. Thus, assuming classical consistency of these estimators, our main results provide explicit rates of convergence to zero of the p-Wasserstein distance (reducing to Gini's dissimilarity index when $p = 1$) between the posterior distribution of $\tilde{\mathfrak{p}}$ ($\tilde{\tau}$, respectively), given $\tilde{\xi}_1, \ldots, \tilde{\xi}_n$, and the unit mass centered at $\hat{\mathfrak{p}}_n$ ($\hat{\tau}_n$, respectively), as $n \to \infty$. This phenomenon shows why and how a Bayesian statistician can utilize suitable non-Bayesian procedures as an approximation, the error depending on the prior distribution only by simple, synthetical measures of its, and going to zero at an explicit rate. Characteristic features of our study, with respect to more common literature on Bayesian consistency, are: first, comparisons are made between entities which depend on the n past observations only; second, the approximations are studied under the actual (exchangeable) law of the $\tilde{\xi}_n$'s, and not under hypothetical product laws $\mathfrak{p}_0^\infty$.

Stability properties of posterior distributions: an optimal transport approach

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We investigate the dependence of posterior probability distributions on observed data, in the framework of dominated statistical models. We discuss quantitative Lipschitz estimates with respect to the Wasserstein distance coming from optimal transportation.

4.10 Probability Methods in Robust Finance (M. Frittelli)

Robust pricing of structured contracts: a utility indifference approach*

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In this paper, we study the pricing and hedging of structured products in energy markets, such as swing and virtual gas storage, using the exponential utility indifference pricing approach in a general incomplete multivariate market model driven by finitely many stochastic factors. The buyer of such contracts is allowed to trade in the forward market in order to hedge the risk of his position. We fully characterize the buyer?s utility indifference price of a given product in terms of continuous viscosity solutions of suitable nonlinear PDEs. This gives a way to identify reasonable candidates for the optimal exercise strategy for the structured product as well as for the corresponding hedging strategy. Moreover, in a model with two correlated assets, one traded and one nontraded, we obtain a representation of the price as the value function of an auxiliary simpler optimization problem under a risk neutral probability, that can be viewed as a perturbation of the minimal entropy martingale measure. Finally, numerical results are provided.

Pointwise Arbitrage Pricing Theory in Discrete Time

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joint with Matteo Burzoni, M. Frittelli, Z. Hou, J. Obloj

The *State Preference Model* or *Asset Pricing Model* is the base of most mathematical description of Financial Markets. It postulates that the price of d financial assets is known at a certain initial time $t_0 = 0$ (today), while the price at future times $t > 0$ is unknown and therefore it is given by a certain random outcome. To formalize such a model we essentially need to fix a quadruple $(X, \mathcal{F}, \mathbb{F}, S)$, where X is the set of events, $\mathcal{F}$ a σ-algebra, $\mathbb{F} := \{\mathcal{F}_t\}_{t\in I} \subseteq \mathcal{F}$ the filtration such that the d-dimensional process $S := (S_t)_{t\in I}$ is adapted. We can therefore assert that, at this stage, no probability measure needs to be specified or required for the Financial Market model $(X, \mathcal{F}, \mathbb{F}, S)$.

Among many others, one fundamental reason for producing such models is to assign a reasonable price to contracts which are not liquid enough for having a market-determined price. For this reason

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the theory of pricing by no arbitrage has been successfully developed over the last 50 years, supported by the economically meaningful argument that it should not be possible to trade in the market and to obtain a positive gain without taking any risk. Absence of arbitrage and existence of reasonable pricing mechanisms are typically shown to be equivalent concepts, a result known as Fundamental Theorem of Asset Pricing (FTAP). A first intuition for this equivalence can be accredited to De Finetti for his work on *coherence* and *previsions* (see [deF70]), while the first systematic approach for understanding the deep relation among no arbitrage pricing and risk-neutral pricing can be found in the work of Ross on Arbitrage Pricing Theory (see e.g. [Ross76, Ross77]). Later on in the case of X being a finite set of events a version of the FTAP was proven by Harrison and Pliska [HP81] (see also [HK79, K81]). Namely, let $X = \{\omega_1, \dots, \omega_n\}$ and $s = (s^1, \dots, s^d)$ the initial prices of d assets with random outcome $S(\omega) = (S^1(\omega), \dots, S^d(\omega))$ for any $\omega \in X$. We have the following equivalence

$$\begin{array}{c} \nexists H \in \mathbb{R}^d \text{ such that } H \cdot s \leq 0 \\ \text{and } H \cdot S(\omega) \geq 0 \text{ with } > \text{ for some } \omega \in X \end{array} \iff \begin{array}{c} \exists Q \in \mathfrak{P} \text{ such that } E_Q[S^i] = s^i \\ \text{and } Q(\omega_i) > 0 \ \forall i = 1, \dots, n \end{array} \tag{1}$$

where $\mathfrak{P}$ is the class of probability measures on X.

It is immediately clear that in the finite setting no probability measure is needed for the specification of the model since impossible events are automatically excluded from the construction of the state space X. On the other hand, linear pricing rules consistent with the observed prices $s^1, \dots s^d$, and not violating the No Arbitrage condition, turn out to be (risk-neutral) probabilities with full support, that is, they assign positive measure to any state of the world. By introducing a *reference probability measure* P with full support and defining an arbitrage as a portfolio with $H \cdot s \leq 0$, $P(H \cdot S(\omega) \geq 0) = 1$ and $P(H \cdot S(\omega) > 0) > 0$, the thesis in Theorem 1 can be restated as

$$\text{There is No Arbitrage} \iff \exists Q \sim P \text{ such that } E_Q[S^i] = s^i \quad \forall i = 1, \dots d \tag{2}$$

The identification suggested by (2), allows non-trivial extensions of the FTAP to the case of a general space X, and was finally proven by Dalang-Morton-Willinger in the celebrated work [DMW90], by use of measurable selection arguments.

This approach has become prominent in the whole field of Mathematical Finance, nevertheless, the apparently innocuous passage of introducing a reference probability measure P is, at the matter of facts, a model assumption. Especially after the recent financial crises, this aspect was criticized and new and challenging questions related to Knightian Uncertainty were posed in several branches of Mathematical.

An important stream of research in this direction aims at extending the probabilistic framework of [DMW90], to a framework which allows for a set of possible priors $\mathcal{P}$. The class $\mathcal{P}$ represents a collection of plausible (probabilistic) models for the market which might be also orthogonal to each other. This leads naturally to the theory of Quasi-sure Stochastic Analysis as in [DM06, STZ11a]. Important results in terms of FTAP are provided by Bouchard and Nutz [BN15]. Under some technical conditions on the state space and the set $\mathcal{P}$ they provide a version of the FTAP, as well as a superhedging duality, which specializes to the classical case when $\mathcal{P} = \{P\}$ is a singleton. On the other extreme, in the case of full ambiguity, $\mathcal{P}$ coincides with the whole set of probability measures and hence the description of the model become pathwise. We might hence observe that the evolution of the theory is somehow circular: starting from a pathwise model a first extension is of probabilistic nature ([DMW90]), and a second extension has, as a particular case, a pathwise extension ([BN15]).

Instead of pursuing this road, in this paper we take a different approach. In particular we take a step back and we propose an alternative, pathwise, method for model construction which is, on one hand, a generalization of the classical finite state space model already used in [HP81] but different from the one suggested by (2), and on the other hand it might include the probabilistic framework of [DMW90] as a special case. More specifically, in this paper the *agent beliefs* are specified through a selection of admissible paths which we denote by $\Omega \subseteq X$. Such beliefs may have a probabilistic nature or not. In this way the agent is allowed to incorporate all the additional information she has at her disposal. The set of beliefs need only to fulfill a minimal measurability requirement (and differently from [CK16] need not to be closed). The use of an arbitrary selection of path appeared already in the literature (see [HO15] for further details) and can be intended as a prediction set where the price process will take values. We show that the choice of an appropriate subset of paths leads to probabilistic notions of arbitrage as well as the probabilistic version of the Fundamental Theorem of Asset pricing.

Another fundamental question of Mathematical Finance is the duality between prices of contracts and prices of superhedging strategies. This problem has been investigated with a pathwise approach over the last decades. A first stream of literature originates from the work of Hobson [Ho98], where the main goal is to find optimal bounds for the price of some exotic option g, which only depends on observable quantities. A rich offspring of papers deal with the same question as, for example, [BHR01, CO11, DOR14]. One of the key ingredient is the set of risk-neutral probability measures which are consistent with the price of some vanilla options that are liquidly traded in the market. The problem of the existence of such martingale measures and its relation to arbitrage considerations is only partially described (see for example [DH07]). Importanlty, in this paper it has been shown, through an example, that the dichotomy absence of arbitrage might break down when a possible disagreement on the effective arbitrage strategy among agents occur. The equivalence between the two concepts has been recovered under some assumptions in Riedel [Ri15] in a topological one-period setup and in Acciaio at al. [ABPS13] where the class of admissible strategies is given by dynamic positions in the single (canonical) asset S and static positions are allowed in a finite number of vanilla options (whose initial prices are known and without loss of generality equal to 0) among a possible uncountable collection Φ. A rigorous analysis of the phenomenon highlithed by [DH07] is given by [BFM16] where it is also shown that several notions of Arbitrage can be studied within the same framework. The setting considered does not allow for semi-static trading but no restrictions are imposed on S which may describe generic financial securities (for examples, stocks and/or options) and in particular it is not necessary the canonical process.

Our main contributions can be resumed as follows.

1. We provide a statement of the Fundamental Theorem of Asset Pricing, for a market where the agent beliefs are modeled by a set Ω and dynamic trading on stocks is combined with static trading on options. We therefore generalize the results of [BFM16], where no options are statically traded and $\Omega = X$. In particular we exploit a (simple) *conditional* construction of a so-called Arbitrage Aggregator for the price process S, as well as an iterative scheme for identifying the class of polar sets with respect to martingale measure for S calibrated to the options Φ.

2. We prove in full generality the superheging duality. As already observed in [BFM16b] but also in [BNT16] in the context of martingale optimal transport, it is crucial to consider $\mathcal{M}$-q.s. inequalities over pathwise inequalities in order to avoid duality gaps. In this paper this feature is achieved through the set of *efficient* trajectories Ω^*_Φ which only depends on Ω and the market. The set Ω^*_Φ recollects all trajectories which are supported by a finite support martingale measure and are calibrated on options prices. This duality generalizes the results of [BFM16b] in two directions. First, it includes the possibility of choosing a subset of beliefs Ω on which the hedging is required. It is typically argued that pathwise models for hedging provides outputs which are too wide to be have a relevance in practice, especially when considering all the possible trajectories (e.g. canonical process). This feature allows to shrink the prices interval, compatible with no arbitrage, to a more useful one. Second it avoids the use of a restrictive assumption introduced in [BFM16b] Theorem 1.2 to treat the case of semistatic trading strategies.

3. We show that our approach is in accordance with the probabilistic one. If a modeler has some probabilistic beliefs regarding the possible evolution of the market, we can construct a suitable subset of trajectories on which probabilistic and pathwise arbitrage considerations coincide.

4. We generalize the results of [ABPS13] for a multi-dimensional non-canonical stock process. We work under the assumption that no arbitrage opportunities can be created on dynamic

trading only and assume the existence an option ϕ_0 convex and with superlinear growth. We show that pathwise superhedging is obtained in two cases: if ϕ_0 does not leads to new arbitrage opportunities then the result follows from an application of a minimax Theorem (which also ensure the existence of a minimizer). In the second case, the option ϕ_0 actually serves as an arbitrage which lifts the superreplication from $\Omega^*_{\phi_0}$ to a superreplication on the entire set of paths Ω. We conclude that a duality between no-arbitrage prices and exact pathwise superhedging is more delicate and it can be obtained at the cost of exploiting arbitrage opportunities.

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Fatou Property, Representations, and Extensions of Law-Invariant Risk Measures on General Orlicz Spaces

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We provide a variety of results for (quasi)convex, law-invariant functionals defined on a general Orlicz space, which extend well-known results in the setting of bounded random variables. First, we show that Delbaen's dual characterization of the Fatou property, which no longer holds in a general Orlicz space, continues to hold under the assumption of law-invariance. Second, we identify the range of Orlicz spaces where the characterization of the Fatou property in terms of norm lower semicontinuity by Jouini, Schachermayer and Touzi still holds. Third, we extend Kusuoka's dual representation to a general Orlicz space. Finally, we prove a version of the extension result by Filipovic and Svindland by replacing norm lower semicontinuity with the (generally non-equivalent) Fatou property. Our results have natural applications to the theory of risk measures. The talk is based on joint work with Niushan Gao, Denny Leung, and Foivos Xanthos.

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4.11 Stochastic PDE's (B. Ferrario)

Modulation equations for stochastic Swift-Hohenberg equation

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We consider a mathematical model for the Rayleigh-Benard convection, the stochastic Swift-Hohenberg equation:

$$\partial_t u = -(1 + \partial_x^2)^2 u + \varepsilon^2 \nu u - u^3 + \varepsilon^{\frac{3}{2}} \xi(t, x).$$

Near its change of stability, the fluid's motion can be described in a multiscale setting as the product of a slowly varying amplitude equation and a faster periodic wave. After an introduction to the problem in its deterministic setting, we'll review some known stochastic results and see some recent developments in the unbounded space domain setting.

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Stochastic differential equations with memory effects*

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We study an evolution equation of the form

$$\mathrm{d}x(t) = b(x_t)\,\mathrm{d}t + \varepsilon\sigma(x_t)\,\mathrm{d}W(t), \qquad x_0 = \phi,$$

where $\{W(t)\}$ is a Brownian motion and $x_t = x(t + \cdot)$ denotes the history process of the solution $x(t)$. We discuss a power series expansion of the solution in terms of the small parameter ε, and relate it to the large deviation principle and the Laplace principle for the solution.

Well-posedness of semilinear stochastic wave equations with Hölder continuous coefficients

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We prove that semilinear stochastic abstract wave equations, including wave and plate equations, are well-posed in the strong sense with an α-Hölder continuous drift coefficient, if $\alpha \in (2/3, 1)$. The uniqueness may fail for the corresponding deterministic PDE and well-posedness is restored by adding an external random forcing of white noise type. This shows a kind of regularization by noise for the semilinear wave equation. To prove the result we introduce an approach based on backward stochastic differential equations. We also establish regularizing properties of the transition semigroup associated to the stochastic wave equation by using control theoretic results.

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4.12 Bayesian Nonparametrics (M. Ruggiero)

Dirichlet mixture models for Calibration and Combination

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We introduce a Bayesian approach to predictive density calibration and combination that accounts for parameter uncertainty and model set incompleteness through the use of random calibration functionals and random combination weights. Building on the work of [2], we use infinite beta mixtures for the calibration. The proposed Bayesian nonparametric approach takes advantage of the flexibility of Dirichlet process mixtures to achieve any continuous deformation of linearly combined predictive distributions. We provide some conditions under which the proposed probabilistic calibration converges in terms of weak posterior consistency to the true underlying density for both cases of i.i.d. and Markovian observations.

Combination and calibration

Let $F_1, \ldots, F_M$ be a set of predictive cumulative distribution functions (cdfs) for a real-valued variable of interest, Y, which might be based on distinct statistical models or experts. Following the forecast combination and calibration literature, we assume the cdfs are externally provided. We consider combination formulas $\{\mathfrak{F}_{\boldsymbol{\theta}} : \boldsymbol{\theta} \in \Theta\}$ that map the M-tuple $(F_1, \ldots, F_M)$ into a single, aggregated predictive cdf, $F(\cdot|\boldsymbol{\theta}) = \mathfrak{F}_{\theta}(\cdot|F_1, \ldots, F_M)$. Given a sequence of observations, $y_1, \ldots, y_T$, the cdf evaluated on one observation, e.g. $F(y_t|\boldsymbol{\theta})$, is referred as probability integral transform (PIT). We say that the PITs, $F(y_1|\boldsymbol{\theta}), \ldots, F(y_T|\boldsymbol{\theta})$, are well calibrated (or probabilistically calibrated) if their distribution is uniform. As noted in [2], well calibration is a critical requirement for probabilistic forecasts and checks for the uniformity of the PITs have formed a cornerstone of density forecast evaluation.

The aggregation method introduced in [2] considers the beta transformed linear pool

$$F(y|\boldsymbol{\theta}) = B_{\alpha,\beta}\left(\sum_{m=1}^{M} \omega_m F_m(y)\right) \tag{1}$$

where $\boldsymbol{\theta} = (\alpha, \beta, \boldsymbol{\omega})$, $B_{\alpha,\beta}$ denotes the cdf of the standard beta distribution with parameters $\alpha > 0$ and $\beta > 0$ and $\boldsymbol{\omega}$ belongs to the unit simplex Δ_M in $\mathbb{R}^M$.

We extend (1) by introducing an aggregation method based infinite beta mixture combination formulas and propose a Bayesian non-parametric approach to estimate this mixture.

The training data comprise the predictive cdfs $F_{1t}, \ldots, F_{Mt}$ and pdfs $f_{1t}, \ldots, f_{Mt}$, which are conditional on the information available at time $t-1$, along with the respective realization, y_t, at time $t = 1, \ldots, T$, respectively.

For $\boldsymbol{\omega} \in \Delta_M$ define the combined cdf as $H_t(y_t|\boldsymbol{\omega}) = \sum_{m=1}^{M} \omega_m F_{mt}(y_t)$ for $t = 1, \ldots, T$. The calibrated cdf is

$$F_t(y_t|\boldsymbol{\theta}) = B^*_{\mu,\nu}\left(H_t(y_t|\boldsymbol{\omega})\right)$$

where $\boldsymbol{\theta} = (\mu, \nu, \boldsymbol{\omega})$ with $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_M)$ and $B^*_{\mu,\nu}(y) = B(\mu\nu, (1-\mu)\nu)^{-1} \int_0^y x^{\mu\nu-1}(1-x)^{(1-\mu)\nu-1}dx$. The corresponding calibrated density is $f_t(y_t|\boldsymbol{\theta}) = b^*_{\mu,\nu}\left(H_t(y_t|\boldsymbol{\omega})\right) h_t(y_t|\boldsymbol{\omega})$, where $b^*_{\mu,\nu}$ is the density of $B^*_{\mu,\nu}$ and $h_t(y_t|\boldsymbol{\omega}) = \sum_{m=1}^{M} \omega_m f_{mt}(y_t)$.

*Speaker

At this stage, we assume a nonparametric hierarchical prior for $\boldsymbol{\theta}$. More precisely, we assume that $\boldsymbol{\theta} \sim G(\boldsymbol{\theta})$ where G is Dirichlet process (DP) with concentration parameter ψ and base measure G_0. The stick-breaking representation of the DP allows us to write the combination and calibration model in terms of infinite mixtures of random beta distributions, that is

$$f_t(y_t|G) = \int f_t(y_t|\boldsymbol{\theta})G(d\boldsymbol{\theta}) = \sum_{k=1}^{\infty} w_k b^*_{\mu_k,\nu_k}\left(H_t(y_t|\boldsymbol{\omega}_k)\right) h_t(y_t|\boldsymbol{\omega}_k),$$

where the random weights w_k are generated by the stick-breaking construction $w_k = v_k \prod_{l=1}^{k-1}(1-v_l)$, v_l being i.i.d. random variables $Beta(1,\varphi)$. The atoms $\boldsymbol{\theta}_k = (\mu_k, \nu_k, \boldsymbol{\omega}_k)$ are i.i.d. random variables from the base measure G_0.

There are many possible simulation methods for performing inference on infinite mixture models resulting from a Dirichlet prior assumption. In [1] we rely on the slice sampling algorithm proposed in [3] and we study the performance of the proposed calibration formula in simulation examples with fat tails and multimodal densities. We also apply it to density forecasts of daily S&P returns and daily maximum wind speed at the Frankfurt airport.

Consistency results for i.i.d. observations

In [1] we provide some conditions under which the proposed probabilistic calibration formula converges to the true underlying density, implying uniformity of the PITs in the limit. The convergence is studied in terms of (Bayesian) weak posterior consistency. We assume that the observations are i.i.d., and hence we drop from H and h the observation index t. For some preliminary result on the Markovian case see [1].

Theorem 1 *Assume that there is a point $\boldsymbol{\omega}$ in the interior of Δ_M such that $h(\cdot|\boldsymbol{\omega})$ is continuous and that, for every compact set $C \subset \mathcal{Y}$, $\inf_{y \in C} h(y|\boldsymbol{\omega}) > 0$. Assume also that the true density f_0 is continuous on $\mathcal{Y}$ and that*

$$\begin{aligned} &\int [|\log(H(y|\boldsymbol{\omega}))| + |\log(1 - H(y|\boldsymbol{\omega}))|] f_0(y)dy < +\infty \\ &\textit{and} \quad \int f_0(y)\log(f_0(y)/h(y|\boldsymbol{\omega}))dy < +\infty. \end{aligned} \tag{2}$$

If G_0 has full support, then the posterior is weakly consistent at f_0.

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Dependent hierarchical processes for multi-armed bandits

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In Bayesian nonparametrics, a large amount of literature has been developed for exchangeable observations. However in a large variety of applications exchangeability is a quite restrictive assumption, for example this happens when data are generated by different experiments: even if these experiments may be related, they have some specific features that induce heterogeneity across observations. In such a situation data are usually divided into different groups, and a more appropriate assumption is partial exchangeability.
In particular the construction of dependent random probability measures to deal with partially exchangeable observations has recently attracted great attention in Bayesian nonparametric literature. Here we define and investigate a general class of nonparametric priors, called *hierarchical processes*, and based on transformations of completely random measures. We derive all the analytical properties, namely the random partition structure, the predictive distributions and the posterior characterization of the processes (see [2]).
Our theoretical findings form the backbone to develop suitable sampling schemes in a large number of inferential problems, here we focus on multi–armed bandits. In such a situation one is typically provided with J populations of plants or animals composed by species with unknown proportions. In the same spirit of [1], we consider the problem of sequentially sampling these populations in order to observe the greatest number of different species or to re-observe a species having a specified abundance in the sample available up to now. The algorithm we develop is based on the Hierarchical Pitman-Yor process and the parameters are updated through a particle filter step. Finally we show the benefits of the Bayesian nonparametric approach with respect to the traditional Upper Confidence Bound (UCB) algorithm [3].
The talk is based on a series of joint works with M. Battiston, S. Favaro, A. Lijoi, P. Orbanz and I. Prünster.

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On the two-parameter Poisson-Dirichlet process truncation error*

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We undertake a thorough asymptotic investigation of the truncation error in the stick-breaking representation of the two-parameter Poisson-Dirichlet process. First, we consider the limiting distribution of a (properly centered) log transform of the truncation error and we show that this limit in distribution can be described in terms of the alpha-diversity of the process, thus obtaining back a result due to Jim Pitman. By an application of of Esseen's lemma, the rate of convergence in law is proved to be as fast as n^{-1}. Second, we complement with the derivation of the asymptotic distribution of the stopping time which defines the truncation level necessary to account for a given approximation accuracy. Finally, we derive a large deviation principle for the truncation error. The usefulness of these theoretical results is demonstrated on statistical applications to Bayesian nonparametric mixture models.

4.13 Stochastic Processes in Discrete Structures and Their Limit Behaviour (D. Spanò)

Percolation and isoperimetric inequalities

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In this talk we will discuss some relations between percolation on a given graph G and its geometry. There are several interesting questions relating various properties of G such as growth (or dimension) and the process of percolation on it. In particular we will look for conditions under which its critical percolation threshold is non-trivial, that is: $p_c(G)$ is strictly between zero and one. In a very influential paper on this subject, Benjamini and Schramm asked whether it was true that for every graph satisfying $\dim(G) > 1$, one has $p_c(G) < 1$. We will explain this question in detail and present some recent results that have been obtained in this direction.

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The KPZ Equation as a scaling limit of discrete and continuous systems

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The KPZ equation is presumed to be a universal object even though it is dramatically ill-posed. The Cole-Hopf solution proposed by Bertini and Giacomin, even if it was shown to be the physically correct one, was proved not to be able to fully capture such a universality. Recently two new notions have been established thanks to which the claim of universality has been partially confirmed. In this talk we will see how it is possible, thanks to these techniques, to show that certain continuous and discrete systems, when suitably rescaled, converge to the solution of the KPZ equation.

Invariance principle for the degenerate dynamic random conductance model

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We study a continuous-time random walk on $\mathbb{Z}^d$ in an environment of dynamic random conductances. We assume that the law of the conductances is ergodic with respect to space-time shifts. We prove a quenched invariance principle for the random walk under some moment conditions on the environment. The key result on the sublinearity of the corrector is obtained by Moser's iteration scheme. Time permitted, we will discuss a quenched local central limit theorem for the above mentioned random walk, the crucial estimates will rely on De Giorgi's iteration technique. This is joint work with S. Andres, J-D. Deuschel and M. Slowik.

[*] Speaker

4.14 Some Recent Developments about Finitely Additive Probability Measures (P. Rigo, B. Vantaggi)

A unifying view on some problems in probability theory

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A number of (existence) issues, ranging from probability to statistics and finance, reduce to the problem stated below. Among other things, we mention de Finetti's coherence principle, equivalent martingale measures, equivalent measures with given marginals, stationary and reversible Markov chains, and compatibility of conditional distributions.

Problem: Let $(\Omega, \mathcal{A})$ be a measurable space and L a linear space of real random variables on $(\Omega, \mathcal{A})$. Is there a probability P on $\mathcal{A}$ such that

$$E_P|X| < \infty \quad \text{and} \quad E_P X = 0 \quad \text{for all } X \in L \text{ ?}$$

Such a P may be finitely additive or countably additive. Further, in addition to "nullify" L, P may be asked to satisfy

$$P \sim P_0 \quad \text{or} \quad P \ll P_0$$

where P_0 is a reference probability measure on $\mathcal{A}$.

Some solutions to the previous problem are provided. The probability P is finitely additive in a few of them and countably additive in the others.

Envelopes of probabilities

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In several fields such as econometrics, finance, statistics and game theory, the finitely additive setting allows to obtain more complete mathematical results and to overcome some measurability, topological and conditioning issues. A classical example, just to cite one, is the *mass transportation problem* where finite additivity reveals to be crucial to have duality under very general conditions.

In general, the price we pay for working with finite additivity is the loss of uniqueness. So, in this environment it is important, besides proving the existence of a probability space encoding the available information, to determine the whole class of compatible probability measures, by studying the related *lower* and *upper envelopes*.

*Speaker

A distinguished problem where probability envelopes come to the fore is *identifiability*, which is an essential property of a statistical model though often a critical issue when latent variables are considered (see for example [1, 4]). A statistical model is said to be *strictly identifiable* if a joint distribution for that model uniquely determines the parameters characterizing the joint distribution. Strict identifiability fails when working with latent variables since relabeling the levels of a latent variable leads to the same probability distribution for different parameters (namely, a *label swapping* issue). For this reason, different weaker identifiability notions have been proposed such as *local identifiability* and *generic identifiability* [1]. Local identifiability corresponds to the local invertibility of the parametrization map from the parametrization space to the natural parameter space. In [7] conditions based on the topology of graphs are provided for locally and generically identifiable concentration graph models. This characterization provides also a way to determine the subspace where the local identifiability breaks down and so it allows to consider the relevant set of "equivalent" probability distributions. Such class of probability measures can be characterized through the corresponding envelopes.

In the realm of Bayesian statistics, the problem of considering a class of probabilities is not recent, in fact in the seminal paper [5] by Dubins, the notion of *strategy* σ together with the ensuing concepts of *conglomerability* and *disintegrability* with respect to a *(finitely additive) prior probability* π are presented and it is proved that the assessment $\{\pi, \sigma\}$ can always be extended, generally not in a unique way, to a full conditional probability. The extension of an assessment $\{\pi, \sigma\}$ is particularly meaningful in statistics limit theorems, stochastic processes and their applications. An open problem in this context is to characterize the whole class of full conditional probabilities extending an assessment $\{\pi, \sigma\}$, so, a first aim is to provide a closed form expression for the envelopes of such class.

Generally, this class of extensions can contain full conditional probabilities failing conglomerability, that is a regularity condition often required in applications, since non-conglomerable extensions of $\{\pi, \sigma\}$ can show a pathological behaviour as they could not be approximated in the total variation norm by conglomerable ones [5]. Thus, there is an advantage in restricting to extensions meeting this property. Moreover, as is well-known, conglomerability reduces to disintegrability when σ is integrable with respect to π (see [2]).

Then, the class of conglomerable full conditional probabilities extending $\{\pi, \sigma\}$ is considered and a closed form expression for the envelopes of such class is provided [6]. Furthermore, a conditional version of conglomerability is introduced in order to reinforce the conglomerability constraint on those conditional events $F|K$'s whose conditioning event K has null conglomerable joint probability. This is reached by requiring conglomerability to hold with respect to a full conditional prior probability extending π. Hence, the class of conditionally conglomerable full conditional probabilities extending $\{\pi, \sigma\}$ is considered and its envelopes are characterized.

The lower envelope of such class is a totally monotone capacity on a specific subfamily of conditional events. As a consequence, this allows to compute (as a Choquet integral [3]) the corresponding lower conditional prevision on a suitable class of conditional bounded random quantities. However, the lower envelope of conditionally conglomerable extensions is generally not 2-monotone [6].

Starting from the aforementioned characterizations (in particular referring to those under conglomerability) we address the so-called multiple prior problem in which the aim is to study the class of full conditional probabilities extending a strategy σ and a set of priors $\mathbf{P}$, limiting to the case where the pointwise infimum $\inf \mathbf{P}$ is a 2-monotone capacity.

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A Finitely Additive Version of the Theorem of Halmos and Savage

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In this work it is proved an extension to the finitely additive setting of the classical theorem of Halmos and Savage on the existence, in a dominated family of probabilities, of a dominating countable sub family. From this initial result we deduce a number of corollaries, such as the classical result of Drewnowski concerning countable additivity on sequences of disjoint sets. In addition we prove a new characterization of weak compactness in the space of finitely additive set functions.

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4.15 Some Topics on Path-dependent Stochastic Equations (F. Confortola, G. Guatteri, F. Masiero)

Randomization method in stochastic optimal control

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The talk is about a recently introduced methodology in stochastic optimal control theory (see paper [2]), known as randomization method, firstly developed for classical Markovian control problem in [1]. The randomization method consists, in a first step, in replacing the control by an exogenous process independent of the driving noise and in formulating an auxiliary ("randomized") control problem where optimization is performed over changes of equivalent probability measures affecting the characteristics of the exogenous process. We will discuss the main features of this approach, showing that the randomization method allows for greater generality beyond the Markovian case. In particular, we may consider stochastic control problems with path-dependence in the coefficients (with respect to both the state and the control), without requiring any non-degeneracy condition on the controlled equation.

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Stochastic calculus for non-semimartingales in Banach spaces, an infinite dimensional PDE and some stability results

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Abstract This research develops some aspects of stochastic calculus via regularization for processes with values in a general Banach space B. A new concept of quadratic variation which depends on a particular subspace is introduced. An Itô formula and stability results for processes admitting this kind of quadratic variation are presented. Particular interest is devoted to the case when B is the space of real continuous functions defined on $[-T, 0]$, $T > 0$ and the process is the window process $X(\cdot)$ associated with a continuous real process X which, at time t, it takes into account the past of the process. If X is a finite quadratic variation process (for instance Dirichlet, weak Dirichlet), it is possible to represent a large class of path-dependent random variable h as a real number plus a real forward integral in a semiexplicite form. This representation result of h makes use of a functional solving an infinite dimensional partial differential equation. This decomposition

generalizes, in some cases, the Clark-Ocone formula which is true when X is the standard Brownian motion W. Some stability results will be given explicitly.
This is a joint work with Francesco Russo (ENSTA ParisTech Paris).

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Path-dependent PDEs as infinite dimensional equations on continuous functions

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Given a path-dependent SDE in $\mathbb{R}^d$

$$\begin{cases} dX(t) = b_t(X_t)dt + c_t(X_t)dW(t), \quad t \in [t_0, T], \\ X_{t_0} = \gamma_{t_0} \end{cases} \tag{1}$$

where W is a d-dimensional Brownian motion and $X_t := \{X(s)\}_{s\in[0,t]}$, we can reformulate it as the state-dependent SDE for the process $Y(t) = \left(X(t), X_{[t-T,t]}\right)^\top$ in the space $\mathcal{C} := \mathbb{R}^d \times C\left([-T,0];\mathbb{R}^d\right)$ (the so-called product framework)

$$\begin{cases} dY(t) = \left[AY(t) + B(t, Y(t))\right]dt + C(t, Y(t))dW(t), \quad t \in [t_0, T], \\ Y(t_0) = y \end{cases} \tag{2}$$

which separates the *present* $X(t)$ from the *past* $X_{[t-T,t]}$. Here A is the derivative operator and B and C are defined in terms of b and c, respectively.

This framework allows to formally link to the SDE (1) an infinite dimensional PDE of the form

$$\begin{cases} \frac{\partial u}{\partial t}(t,y) + \langle Du(t,y), Ay + B(t,y)\rangle + \frac{1}{2}Tr\left[C(t,y)C(t,y)^* D^2 u(t,y)\right] = 0 \\ u(T,\cdot) = \Phi. \end{cases} \tag{3}$$

We will show how to give meaning to such PDE in the space $\mathcal{C}$ and that, under suitable assumptions on the coefficients and on the datum Φ, it has a unique classical solution. These results are first obtained on spaces of L^p functions and then extended via approximations to the space of continuous functions.

We will also deal with semilinear extensions of the PDE (**??**), that are studied using forward-backward SDEs in the space $\mathcal{C}$.

The analysis above provides an insight on the analytical structure of the so-called *horizontal derivative* of a path-dependent functional, as defined in the functional Itô calculus ([1, 2]), here denoted by $\mathcal{D}_t$. Proving the equivalence

$$\frac{\partial}{\partial t} + A = \mathcal{D}_t \tag{4}$$

we are able to rephrase our infinite-dimensional PDEs as path-dependent PDEs and to deduce existence results for the latter.

We will eventually show how this equivalence reflects also in Itô-type formulae for functions on $\mathcal{C}$, that constitute the infinite-dimensional counterpart of the change of variable formula given in functional Itô calculus.

This talk is based on the articles [3] and [4] and on a working paper with Carlo Orrieri.

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4.16 Strongly Correlated Random Interacting Systems (A. Stauffer)

A stroll around random quadrangulations of the plane

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The investigation of large random combinatorial objects has spawned a very active research field in the study of local and scaling limits of random planar maps.

Within this broader topic, a position of particular interest is occupied by the so-called Uniform Infinite Quadrangulation of the Plane (UIPQ), and by its several variants, which include the Uniform Infinite Quadrangulation of the Half-Plane (UIHPQ), both with a general and a simple boundary. The two latter objects, since their introduction by Miermont and Curien, have undergone a thorough investigation (by the Author and Curien, among others) resulting in a rather in depth description of their metric properties. We shall see how the simple boundary UIHPQ relates to an annealed model of self-avoiding walk on random quadrangulations, and how metric information obtained for the UIHPQ can be used to study quantities such as the displacement of the self-avoiding walk from the origin, as well as to ultimately determine how the biasing of random quadrangulations by the number of their self-avoiding walks affects their local limit.

Approximating conditional distributions*

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Given a random variable or process X, what can one say about Y, the law of X conditioned upon an observable ϕ? If X and X' are close, are Y and Y' so? We propose a new framework to treat conditional distributions which fits in well with the Stein–Chen method. Our technique consists in three main steps:

1. from an integration-by-parts formula for the unconditional law X we derive an integration-by-parts formula for the conditional distribution Y;

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†Speaker

2. we derive a Stein equation whose solution is bounded via ad hoc couplings;
3. we bound the distance between conditional measures with the method of exchangeable pairs.

This method provides quantitative bounds in several examples:

- the transportation distance between the solutions of the discrete and continuous filtering problem;
- the distance between bridges of Markov processes (for example, random walks on the hypercube and Langevin dynamics);
- the distance between bridges and discrete schemes approximating them.

Ensembles of self-avoiding polygons*

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Self-avoiding random walks are by now a classical topic of modern probability theory, although many questions still remain to be answered; we refer to the classic book [3]. A variant of self-avoiding walks are self-avoiding polygons, where the last step of the self-avoiding walk has to come back to the point of origin. In most of the literature, a single random walk or polygon is the object of study. In the present work, we instead investigate properties of random polygons interacting with an environment consisting of other random polygons. As we will detail below, a natural way to view these systems is as random permutations on a graph.

Specifically, we are interested in the behavior of step-weighted self-avoiding polygon ensembles. In the context of the single step-weighted single self-avoiding walk, the following interesting and rather complete picture is known: fix a sequence of growing subsets Λ_n of $\mathbb{Z}^d$, for example the cubes of side length n. Fix in addition, for each n, two points a and z at opposite ends of Λ_n, and consider the set of all self-avoiding walks starting in a and ending in z. Let $\alpha \in \mathbb{R}$, and assign to each such self-avoiding walk X the weight $\exp(-\alpha|X|)$, where $|X|$ is the number of steps that X takes. Write μ for the connective constant of the d-dimensional cubic lattice. When $\alpha > \log \mu$, it is known that the shape of X converges to a straight line as $n \to \infty$, when scaled by $1/n$.

For $\alpha < \log \mu$, on the other hand, the results are entirely different. As Dominil-Copin, Kozma and Yadin have recently shown [2], in this case the rescaled self-avoiding walk becomes *weakly space filling*, meaning that it will only leave holes of logarithmic size in the graph. [2].

Ensembles of self-avoiding interacting polygons Let us now present our model of interacting, self-avoiding loops, and outline our main results. Let V be a finite set and $E \subset \{\{x, y\} : x, y \in V, x \neq y\}$, so that $G = (V, E)$ is a finite, simple, undirected graph; We consider the set S_G of

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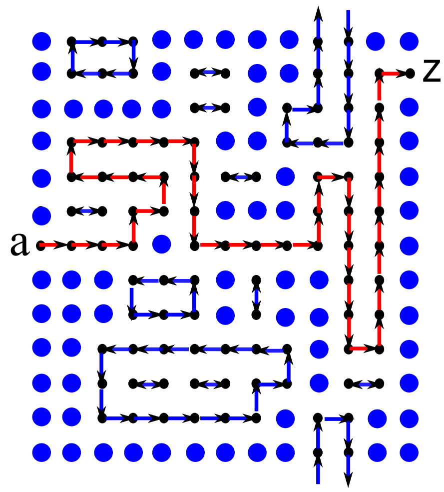

Figure 1: Representation of a bijection with a forced open cycle between a and z, when $\Lambda \subset \mathbb{Z}^2$ is a box with cylinder boundary conditions. If $\pi(x) = x$, then a circle is drawn at x, while if $\pi(x)$ is a neighbour of x, then an arrow is directed from x to $\pi(x)$.

permutations $\pi : V \to V$ such that for all $z \in V$, either $\pi(z) = z$ or $\{z, \pi(z)\} \in E$. The energy $\mathcal{H}_G$ of a permutation π is just the total number of edges used by that permutation, i.e.

$$\mathcal{H}_G(\pi) = \sum_{z \in V} \mathbb{1}\{\ \pi(z) \neq z\ \}. \tag{1}$$

For $\alpha > 0$, the relevant probability measure then is

$$\mathbb{P}_G(\pi) = \frac{e^{-\alpha \mathcal{H}_G(\pi)}}{Z(G)}, \tag{2}$$

where the *partition function* $Z(G)$ normalizes the Boltzmann weights $\exp -\alpha\mathcal{H}_G(\pi)$ to a probability measure.

Questions and results We can now ask the same questions as discussed above: if we make α small, will the cycle starting from the point a be weakly space filling with positive probability? Will a cycle connecting a to z collapse to a straight line in the scaling limit when α is large? Where is the boundary α_c between these two behaviours, assuming there is one? We can only give partial answers to theses questions. We have no result about the existence of a space filling cycle, which is unfortunate since this is by far the most interesting question. Instead, we show that *if* there is a regime of space filling cycles, it must start at lower α than for the case of the self-avoiding polygon. More precisely, in the case where G is a subgraph of a vertex transitive graph, and when μ is the *cyclic* connective constant of that graph, then we identify an $\alpha_c < \log \mu$ so that for all $\alpha > \alpha_c$, and uniformly in the size of G, the length of a cycle through a given point has exponential tails. Thus in the interval $(\alpha_c, \log \mu)$ the single self-avoiding loop is weakly space filling while the self-avoiding loop embedded into an ensemble of other such loops is very short.

Our second main result requires most of the work and it involves the model where a cycle originating in a point a and ending in a point z (on the opposite side of the graph) is forced through the system. Figure 1 shows a typical configuration. We give a positive answer to the question about collapse to a straight line for large α. Unlike in the case of the single self-avoiding loop, we do not have a good quantitative estimate on the threshold above which this behavior holds, and we cannot quite control the scaling well enough to prove the convergence to a Brownian bridge. The reason for these shortcomings is that we have to fight much more serious correlations than are present in the self-avoiding walk case.

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4.17 Diffusion Processes: Inference and Applications (B. D'Auria)

An Exponential Time-Based Transformation of Reflected Diffusions*

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Determining probability density functions (pdf's) of diffusion processes is relevant in the theory of stochastic processes and their applications. Fruitful approaches leading to closed-form results for transition pdf's are based on suitable transformations of diffusion processes. We recall the space-time transformations proposed by Cherkasov [4], Ricciardi [11], Bluman [1], Sacerdote and Ricciardi [12], and Kwok [10], for the mapping of diffusion processes into the Wiener process. Other general criteria are given in Bluman and Shtelen [2], involving nonlocal transformations, and in Borodin [3], for transformations of jump-diffusion processes.

We aim to propose a method able to determine the pdf and the mean of certain one-dimensional time-homogeneous diffusion processes. The method is based on a transformation involving two diffusion processes with state-space $[0, +\infty)$, having the same infinitesimal variance, and where the drift of the new process, say $Y(t)$, is a suitable modification of the drift of the former process, say $X(t)$. The nature of the endpoints of such processes is specified as follows:

- 0 is a reflecting regular or an entrance endpoint for both processes $X(t)$ and $Y(t)$;
- $+\infty$ is an attracting natural endpoint for $Y(t)$, whereas it is either attracting or non attracting for $X(t)$.

*This work is partially supported by the GNCS of INdAM.

†Speaker

The considered transformation involves the hazard rate function of $X(t)$ evaluated at a random exponential time and conditioned to zero initial state. Specifically, the proposed method allows us to obtain the pdf and mean of $Y(t)$ conditional on $Y(0) = 0$, as well as for the density of $Y(t)$ evaluated at 0 conditional on $Y(0) = y$ (when 0 is a reflecting endpoint).

The proposed method leads to diffusion processes that are suitable for describing populations subject to rapid growth, such as bacterial growth (cf. Golding *et al.* [8], for instance) and buffer occupation of stochastic networks in heavy traffic (see, e.g. Kushner [9]). For the new process $Y(t)$, we get the expressions of the Laplace transform of the first-passage-time density through a lower constant boundary. In addition, we are able to express the pdf of $Y(t)$ in terms of the density of process $X(t)$ in the presence of catastrophes, this extending previous results given in [5]. We also provide an interpretation of $Y(t)$ as a diffusion in a decreasing potential.

We study in detail the diffusion processes obtained when $X(t)$ is the Wiener, Ornstein-Uhlenbeck, Bessel and Rayleigh process.

Finally, we point out that the proposed method has been successfully adopted recently also in the context of birth-death processes (see [6]).

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Non parametric inference in diffusion processes: bootstrap performance in short time series

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Diffusion processes are commonly used in a lot of fields ranking from economics to biology, from genetics to engineering. In particular they are able to model stochastic phenomena, such us dynamics of financial securities and short-term loan rates (see, for example, [1] and [2]). Several methods for the inference for discretely observed diffusions have been proposed in the literature. Most of them are essentially based on MLE or its generalizations. Alternative methods have also been proposed: they include techniques based on estimating functions (see [3]), indirect inference (see [5]) and efficient method of moments (see [4]). Also numerical approximations to the unknown likelihood function (see [6]) lead to efficient estimators.

In our talk we focus on two well known diffusion processes, Vasicek and CIR models, defined respectively:

$$dX_t = k(\alpha - X_t)dt + \sigma dB_t,$$

and

$$dX_t = k(\alpha - X_t)dt + \sigma\sqrt{X_t}dB_t.$$

Let $\boldsymbol{\theta} = (k, \alpha, \sigma)$ be the vector of the unknown parameters. Sample properties of MLE estimators for $\boldsymbol{\theta}$ have been analysed by Tang and Chen ([7]) when the sample size n tends to infinity. Moreover, they have proposed a parametric bootstrap procedure to reduce the bias of the drift estimates; a simulation study shows an improvement in the involved estimates also by looking at the mean square error.

However, in many applications of these models, data are yearly or quarterly observed, so in the estimation of the parameter $\boldsymbol{\theta}$ the condition $n \to \infty$ means to observe the phenomenon for a very long period and most likely such kinds of time series present structural breaks. This is the case in which Vasicek and CIR models are used in insurance for the valuation of life insurance contracts (see, for example, [8]) or also to model short-term interest rates (see, for example, [9] and [10]).

In this talk we focus on small sample properties of some alternative estimation procedures in Vasicek and CIR models. In particular, we consider short time series, with a length T between 10 and 100, tipically values observed in these contexts. We perform a simulation study in order to investigate which properties of $\boldsymbol{\theta}$ estimator remain still valid. Moreover, we also investigate what extend the estimator accuracy remains accettable for very short time series, for example 20-30 yearly observations.

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Conditioned Stochastic Differential Equations with finance applications*

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This work aims to extend known techniques of initial enlargement of filtration to cases where the information does not directly involve the final price of the asset or the terminal value of the stochastic driving Wiener process but the final value of functionals of the parametrizing processes, such as the trend, the volatility and the interest rate processes. In particular this work analyzes a specific example that models the interest rate process by an Ornstein–Uhlenbeck process. The proposed initial enlargement of the natural filtration includes information about functionals of this process. Starting by this example it will be discussed the extension of the concept of Conditioned Stochastic Differential Equations introduced in [1] and then extended in [2] to these more general framework.

Acknowledgments

The author thanks the ICMAT (Madrid, Spain) Research Institute that kindly hosted him while developing this project.

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4.18 Limit Theorems in Probability and Applications (M. Rossi)

Stein-Malliavin method meets wavelets: an overview on some recent results*

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In this talk, we describe a recent stream of research on normal approximations for linear and nonlinear functionals built over a homogeneous spherical Poisson field by means of spherical wavelets, namely, the so called needlets. More specifically, on the one hand, using recent results from [6], based on Malliavin calculus of variations and Stein's method, allows one to assess the rate of convergence to Gaussianity for a triangular array of statistics with growing dimensions (in the linear case) and for several kinds of U-statistics. On the other hand, as aforementioned, these objects are built by using decompositions in terms of needlets, introduced in the literature by [5] and characterized by several pivotal properties. In particular, we are referring to the concentration properties they feature in both the real and the frequency domains.

The combination of these two techniques provides therefore a powerful tool to investigate in the high-frequency limit some standard problems in statistical inference, such as: testing for the functional form of an unknown density function (cf. [4]), estimation of the variance (see [3]), comparison between two unknown density functions - the so-called two sample problem (see [2]).

Even if these topics can be in general considered among the most common problems in statistical inference, our investigations are developed under circumstances which are not standard, for several reasons. First, we shall consider the case of directional data, assumed to be sampled over compact manifold, even if for the sake of the simplicity we will consider the d-dimensional sphere. Second, in view of the concentration properties of the needlets, these procedures can be considered as local, in the sense that we allow for the possibility that not all the manifold is observable. Hence, we can consider as the sample domain only subsets of the compact manifold, as opposed to most of the existing statistical procedures. Third, and most innovative, we consider classes of "high-frequency" tests, where the number of procedures to be implemented is itself a function of the number of observations available. Especially for the latter consideration, the Malliavin-Stein techniques derived in [6] turn out to be of the greatest practical importance. Indeed, they allow, for instance, to determine how many joint procedures can be run while maintaining an acceptable Gaussian approximation for the resulting statistics, e.g., those where the dimension of a given statistic increases with the number of observations. Our principal motivation originates from the implementation of wavelet systems, when more and more data become available so that a higher number of wavelet coefficients is evaluated. Hence, we consider sequences of Poisson fields, whose intensity grows monotonically. In this case, the rate of convergence to Gaussianity of the statistics taken into account is locally related to the scale parameter of the corresponding wavelet transform. The final part of this talk will introduce a further extension of these results, concerning U-statistics of order

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n developed in Besov spaces over compact manifolds (see [1]).
This talk resumes joint works with S. Bourguin, D. Marinucci and G. Peccati.

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Mod-Gaussian convergence for random matrix ensembles

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We introduce the framework of mod-Gaussian convergence developed by V. Féray, P.-L. Méliot and A. Nikeghbali in [1]. Roughly speaking, given a sequence of random variables $(X_n)_{n\in\mathbb{N}}$ which do not converge in law, we look for a normalization of their moment generating function, which forces convergence to happen. In particular, when this normalization writes like the moment generating function of a Gaussian random variable with mean zero and variance blowing up to infinity with n, we shall speak of mod-Gaussian convergence for the sequence $(X_n)_{n\in\mathbb{N}}$.
Using the framework of mod-Gaussian convergence, we prove precise large deviations and central limit theorems for log-determinants of some random matrix ensembles, like the complex Gaussian, the β-Laguerre and β-Jacobi ensemble. In order to prove such results, we look for the asymptotic expansion of the moment generating function of such log-determinants in some region of the complex plane.
Additionally, our approach allows us to identify the scale at which the central limit theorem still holds i.e. the scale up to which the Gaussian approximation for the tail of the distribution is valid.

*Speaker

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Nodal intersections for random waves against a segment on the 2-dimensional torus

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We will consider random Gaussian eigenfunctions of the Laplacian on the standard torus, and investigate the number of nodal intersections against a straight line segment. The expected intersection number, against a generic smooth curve, is universally proportional to the length of the reference curve, times the wavenumber, independent of the geometry. The first to consider this problem were Rudnick and Wigman [1], who found precise asymptotics for the nodal intersections variance in the high-energy limit, in the case of a curve with nowhere vanishing curvature.

We shall focus on the nodal intersections variance in the case of a straight line segment (the other extreme), and discuss how this problem is related to counting the zeros of a stationary Gaussian process.

Acknowledgements

This work was carried out as part of the speaker's PhD thesis at King's College London, under the supervision of Igor Wigman. The speaker's PhD is funded by a Graduate Teaching Scholarship, Department of Mathematics. Special thanks to Igor Wigman for his guidance and helpful corrections. Special thanks to Zeév Rudnick for suggesting this very interesting problem, and for helpful communications and corrections.

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An improved second-order Gaussian Poincaré inequality and its applications*

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In a recent paper [2] by Chatterjee, a second-order Gaussian Poincaré inequality is established as a useful tool for the normal approximation of a functional over a Gaussian vector. This idea was soon generalised by Nourdin, Peccati et Reinert [3] in a streamlined fashion to the infinite-dimension setting, using Malliavin calculus.

In this talk we present an improved version of this inequality as well as its application to the celebrated Sinai-Soshnikov CLT [4] and Breuer-Major theorem [1].

This is a joint work with G. Peccati.

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5 Contributed talks

5.1 Wednesday 21, 9.30-10.30 Room 1I

Multivariate marked Poisson processes and market related multidimensional information flows

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We propose a new interpretation of multidimensional information flows and their relation to market movements. The new conceptualization hinges on results of two distinct mathematical theories, Lévy processes and marked Poisson processes, bridged in [1] and applied here in the context of finance. Specifically, in this paper we construct a class of multivariate Gaussian marked Poisson processes to model asset returns. The model proposed accommodates the cross section properties of trades, allows for returns to be correlated conditional on trading activity, and preserves normality of returns conditional on trading activity. Intuitively, a marked Poisson process is constructed by attaching to the atoms of a Poisson random measure a collection of random variables, marks, conditionally independent of the random measure. In this framework, the Poisson random measure is a measure of the trading activity on the collection of assets up to a given time, and marks represent returns conditional on the trading activity. We specify marks to have a multivariate Gaussian distribution in order to have normality of asset returns conditional on the trading activity, and, at the same time, we specify the Poisson measure to recover a one factor structure of trade information, according to [2] findings. We prove that the Poisson random measure defines a factor based multivariate subordinator and the class introduced generalizes the class of time changed Brownian motions - $\rho\alpha$-models - in [3] and [4] and spans a wider range of linear correlations. By further specifying the random measure we could obtain processes of variance gamma, normal inverse Gaussian and generalized hyperbolic types. However, for our first application we focus on the normal inverse Gaussian specification, obtained by choosing a Poisson measure of inverse Gaussian type. The new class of processes is fully characterized through its Lévy triplet, and the characteristic function is given in closed form. As a first application, we perform a sensitivity analysis of the linear and non linear dependence structure of a two dimensional price process calibrated to the daily log-returns of Goldman Sachs and Morgan Stanley US asset prices.

[*]Speaker

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Additive energy forward curves under the Heath-Jarrow-Morton framework

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In energy markets the most traded products are usually forward contracts. Generally, these can be of two types: in our terminology, forward and futures. As, for instance, in stock markets, the former consist of an agreement to buy a certain underlying at a fixed price to be given at a fixed date, whereas who sells a futures contract commits to deliver, say, the power *over a fixed period*. This implies that abitrage-free futures prices consist of (in our continuous time setting, integral) averages of forward prices (cf. [2]). Our purpose is to design a Heath-Jarrow-Morton framework, both for forwards and futures, which exhibits the following features. Firstly, the dynamics of the processes are additive and mean-reverting: the additivity property generally allows to find closed-form formulae and suitable calibration procedures for average based contracts (see [3]), while mean reversion can be empirically observed. Furthermore, we want to preserve the Markovianity of the modeling stochastic processes. Finally and most importantly, in a market model consisting of contracts of any maturity date or delivery period, we require that no arbitrage opportunities are possible. This forces us to find a change of measure that turns into martingales the forward and futures prices of *any* maturity or delivery. We consider the forward prices for each maturity that can be represented as affine functions of a universal source of randomness. In the Brownian setting, we are able to completely characterize the models which allow for the above mentioned martingale property. We discuss the possibility of introducing a Lévy component either as stochastic driving factors, or in the role of stochastic volatility (in analogy to [1]).

*Speaker

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A New Approach to CIR Short-Term Interest Rates Modelling

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It is well known that the Cox-Ingersoll-Ross (CIR) stochastic model

$$dr(t) = k[\theta - r(t)]dt + \sigma\sqrt{r(t)}dW(t), \quad r(0) = r_0 > 0, \tag{1}$$

to study the term structure of interest rates, as introduced in 1985 [2], is inadequate for modelling the current market environment with negative short interest rates. Moreover, the diffusion term in the rate dynamics (1) goes to zero when short rates are small; both volatility σ and long-run mean θ do not change with time; they do not fit with the skewed (fat tails) distribution of the interest rates, etc.

Several different extensions of the original model have been proposed to date, with the aim of overcoming the limitations of the CIR model: from one-factor models including time-varying coefficients or jump diffusions to multi-factor models. All these extensions preserve the positivity of interest rates; in some cases the analytical tractability of the basic model is violated.

The aim of the present work is then to provide a new numerical methodology in the CIR framework, which we call the *CIR# model*, that fits well the term structure of short interest rates as observed in real markets. Our approach is based on a proper translation of interest rates such that the market volatility structure is preserved as well as the analytical tractability of the original CIR model.

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5.2 Wednesday 21, 9.30-10.30 Room 3I

Existence and uniqueness for BSDEs driven by a general random measure, possibly non quasi-left-continuous

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We study the following backward stochastic differential equation on finite time horizon driven by an integer-valued random measure μ on $\mathbb{R}_+ \times E$, where E is a Lusin space, with compensator $\nu(dt, dx) = dA_t\, \phi_t(dx)$:

$$Y_t = \xi + \int_{(t,T]} f(s, Y_{s-}, Z_s(\cdot))\, dA_s - \int_{(t,T]} \int_E Z_s(x)\, (\mu - \nu)(ds, dx), \qquad 0 \leq t \leq T.$$

The generator f satisfies, as usual, a uniform Lipschitz condition with respect to its last two arguments. In the literature, the existence and uniqueness for the above equation in the present general setting has only been established when A is continuous or deterministic. The general case, i.e. A is a right-continuous nondecreasing predictable process, is addressed here. These results are then applied in the study of control problems related to piecewise deterministic Markov processes by means of BSDEs methods.

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*Speaker

Reflected BSDE driven by marked point process and optimal stopping.

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We define a class of reflected backward stochastic differential equation (RBSDE) driven by a marked point process (MPP) and a Brownian motion, where the solution is constrained to stay above a given process. Under usual assumptions we obtain existence and uniqueness in the case where only a marked point process is involved, as well as in the case with both MPP and Brownian motion. We use the equation to represent the value function of an optimal stopping problem, and we characterize the optimal strategy.

Keywords: reflected backward stochastic differential equations, optimal stopping, marked point process.

FBSDEs with distributional coefficients

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In this talk I will present some recent results about systems of forward-backward stochastic differential equations (FBSDEs) where some of the coefficients are Schwartz distributions, in particular they are elements of a fractional Sobolev space of negative order. A notion of virtual solution is introduced in order to make sense of the singular integrals that appear in the FBSDE, and we use the theory of PDEs with distributional coefficients to do so. In this new setting we show the existence and uniqueness of a solution and the validity of the so-called non-linear Feynman-Kac formula.

This talk is based on a joint work with Shuai Jing (ArXiv:1605.01558).

5.3 Wednesday 21, 9.30-10.30 Room 5I

Stochastic modeling and control of energy networks under uncertainty*

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Power grids are increasingly affected by uncertainty due to the intermittent nature of renewable generation. In this talk I will present a stylized model for energy network under uncertainty, aiming to get insight in the interplay between renewable energy and grid reliability.

The physical network is modeled by a weighted graph G, where nodes represent buses and edges represent transmission lines. The power injected or consumed in the network nodes is described by a power injection vector p, modeled as a random vector or multidimensional stochastic process, that uniquely determines the current flows f in the network edges under the DC power flow approximation. This stochastic model is leveraged to analyze two different aspects, namely (i) stochastic optimization of energy storage and (ii) line failure probabilities.

The first part of the talk is based on [1] and investigate a scenario where energy storage devices ("batteries") that can coordinate their operations are added to the energy network. Such batteries can both charge using the network current excess or discharge to meet the network current demand. Either way, the presence of batteries can be leveraged to mitigate the intrinsic uncertainty in the power generation and demand and, hence, transport the energy more efficiently through the network.

The performance metric that we consider is the expected total heat loss $\mathbb{E}H(\alpha)$ when control α is used, where the *total heat loss* H is a random variable (or stochastic process) can be expressed as a quadratic for of the power injection vector p. I will show how the expected total heat loss $\mathbb{E}H(\alpha)$ depends on the network structure and on the batteries operations via the control α. Furthermore, in the case where the power injections are modeled by Ornstein-Uhlenbeck processes, I will show how we derive the dynamical optimal control $\alpha(t)$ for the batteries over a finite time interval.

I will then discuss some recent work [2], where the same stochastic model is used in a static scenario to derive upper bounds for *line failure probabilities* leveraging concentration inequalities for the maximum of multivariate Gaussian random variables. Given an average power injection vector μ, such upper bounds are of the form

$$\mathbb{P}_\mu(\text{line failure}) \leq \exp\Big(-\frac{(1-r(\mu))^2}{2\max_i \sigma_i^2}\Big),$$

where $r(\mu) = \mathbb{E}\max_i |f_i|$ and $\sigma_i^2 = \mathrm{Var}(f_i)$, for $i = 1, \ldots, m$. Understanding how the likelihood of such events depends on the underlying stochastic process and network structure becomes crucial to get insight in how cascade failure, i.e. blackouts, occur.

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Systems of reinforced stochastic processes with a network-based interaction

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Several areas of research are interested in the formal characterization of the dynamic of systems composed by elements that evolve interacting among each other. Significant examples can be found for instance in neuroscience, biology, social and computer sciences, where often the interaction forces the components of the system to *synchronize* along time, that is when all the elements asymptotically adopt a common behavior. In particular, we consider the synchronization phenomenon in systems whose elements evolve with a reinforcement mechanism, i.e. the occurrence of a given event increases the probability that the same event occurs in the future. This type of processes have always known a great interest in applied probability since there are several random phenomena considered in different scientific fields that can be modeled with the dynamic of a reinforced stochastic process. The behavior of systems of reinforced processes has been initially studied in [4, 5] for Pólya urn processes, and then extended in [3] for a more general class of reinforced stochastic processes. In these works, the elements of the system are connected among each other with a mean-field interaction that leads to the synchronization phenomenon. Systems with a general network-based interaction, that includes the mean-field as a special case, have been studied in [2], where the components of the system are represented by generalized Friedman's urns, which are not reinforced processes and hence synchronization does not hold. In [1] we have studied the dynamic of a system composed by the type of reinforced stochastic processes introduced in [3] with the network-based interaction studied in [2]. This allows us to study the relationship between the topology of the network of the interactions and asymptotic behavior of the system.

Formally, the model presented in [1] is the following. Consider a weighted directed graph $G = (V, E)$, where $V = \{1, ..., N\}$, $N \geq 1$, denotes the set of vertices, $E \subseteq V \times V$ the set of edges, and $W = [w_{j,k}]_{j,k \in V \times V}$ the corresponding weighted adjacency matrix, with $\sum_{j=1}^{N} w_{j,k} = 1$. For each edge $(j,k) \in E$, the weight $w_{j,k} \geq 0$ quantifies how much the element positioned in the vertex j has a direct influence on the elements positioned in the vertex k (when $(j,k) \notin E$ we set $w_{j,k} = 0$). We suppose to have a reinforced stochastic process located at each vertex $j \in V$, which is described by $X^j = (X_{n,j})_{n \geq 1}$ and $Z^j = (Z_{n,j})_{n \geq 0}$ such that, for each $n \geq 0$ the random variables $\{X_{n+1,j} : j = 1, \ldots, N\}$ take values in $\{0, 1\}$ and are conditional independent given $\mathcal{F}_n$ with

$$P(X_{n+1,j} = 1 \,|\, \mathcal{F}_n) = \sum_{k=1}^{N} w_{k,j} Z_{n,k},$$

where, for each $k \in V$,

$$Z_{n,k} = (1 - r_{n-1}) Z_{n-1,k} + r_{n-1} X_{n,k},$$

with $0 \leq r_n < 1$, $Z_{0,k}$ random variables with values in $[0, 1]$ and $\mathcal{F}_n = \sigma(Z_{0,k} : k \in V) \vee \sigma(X_{m,j} : j \in V, m \leq n)$.

Under suitable condition on the sequence $(r_n)_n$, in [1] we prove that the stochastic processes $\{(Z_{n,j})_n : 1 \leq j \leq N\}$ positioned at the vertices of the graph synchronize, in the sense that

*Speaker

they converge almost surely to a common random limit Z_∞ for any irreducible weighted adjacency matrix W. In addition, we provide some CLTs in the sense of stable convergence that establish the convergence rates and the asymptotic distributions for both convergence to the common limit $(Z_{n,j} - Z_\infty)$ and synchronization $(Z_{n,j} - Z_{n,k})$. Specifically, we are able to show explicitly how the convergence rates and the asymptotic variances depend on the behavior of the sequence $(r_n)_n$ and on the eigen-structure of the weighted adjacency matrix W. These theoretical results allow us to use the observations of the reinforced stochastic processes $\{(Z_{n,j})_n : 1 \leq j \leq N\}$ positioned at the vertices to construct asymptotic confidence intervals for the common random limit Z_∞ and asymptotic critical regions for the inference on the topology of the interaction network.

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A continuous-time random walk on a star graph and its diffusion approximation

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We consider a continuous-time random walk $\mathcal{N}$, that is an extended birth-death-immigration process defined on a lattice formed by the integers of d semiaxes joined at the origin., i.e. a star

*Speaker

graph. When the process reaches the origin, then it may jumps toward any semiaxis with the same rate. The dynamic on each ray evolves according to a one-dimensional linear birth-death process with immigration.

The linear birth-death process with immigration is often employed as a stochastic model for population processes in biology and ecology (see, for instance, Section 5 of Ricciardi [8]), and for capacity expansion problems in queueing theory (cf. Nucho [6]). A birth-death-immigration process including the possibility of multiple immigrations has been discussed recently by Jakeman and Hopcraft [5]. Random walks on graphs are often described by Markov processes and deserve interest in many applied fields (see, e.g., the review by Volchenkov [9] and references therein). We recall for instance the application of birth-death processes on graphs to evolutionary models of spatially structured populations. See Allen and Tarnita [1] for a comprehensive investigation on state-dependent birth-death population models with fixed population size and structure, and Broom and Rychtář [2] for evolutionary dynamics of populations on graphs.

We investigate the transient and asymptotic behavior of the process via its probability generating function. In particular, the adopted technique is based on the coupling of the homotopy perturbation method and the expansion in Taylor series. Moreover, we use the Laplace transform to disclose the asymptotic expression of the state probabilities, which involves a zero-modified negative binomial distribution. A formal expression for the probability that process $\mathcal{N}$ is located at the origin is also obtained.

A further object of our investigation is the diffusive approximation of $\mathcal{N}$ on the star graph, which involves a diffusion process with linear drift and infinitesimal variance. Diffusion processes on graphs have been studied by several authors. We recall for instance Freidlin and Wentzell [3], that is one of the first contributions on this topic. See also Weber [10] for occupation time functionals for diffusion processes and birth-and-death processes on graphs. Recently, some results for Brownian motion on a general oriented metric graph have been given in Hajri and Raimond [4]. An investigation involving a diffusion process on star graph has been performed in Papanicolaou *et al.* [7], where the authors obtain exit probabilities and certain other quantities involving exit and occupation times for a Brownian Motion on star graph.

In the present contribution, we adopt a customary scaling leading to a time-homogeneous diffusion process on the star graph, characterized by linear infinitesimal moments. A gamma-type stationary density is also obtained under suitable assumptions.

It is worth pointing out that, as a byproduct of our investigations, we obtain some new results of interest, such as a closed form of the number of permutations of $\{1, \ldots, n\}$ with k components and a new series form of the polylogarithm function expressed in terms of the Gauss hypergeometric function.

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5.4 Wednesday 21, 9.30-10.30 Room 2I

Prohorov-type local limit theorems on Gaussian spaces

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The classical one-dimensional central limit theorem asserts that, for a given sequence $\{X_n\}_{n\geq 1}$ of independent and identically distributed random variables, the sequence $\frac{X_1+\cdots+X_n-nE[X_1]}{\sqrt{nVar(X_1)}}$ converges in distribution, as n goes to infinity, to the standard normal law. One may wonder whether, under more restrictive assumptions, the previously mentioned convergence holds in some stronger sense. One can, for instance, be interested in the convergence of the density (with respect to the Lebesgue measure) of the law of $\frac{X_1+\cdots+X_n-nE[X_1]}{\sqrt{nVar(X_1)}}$ toward the function $\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{x^2}{2}\right\}$. This type of results, called *local limit theorems*, has attracted the attention of several authors: we recall the classical papers by Prohorov [6], which is concerned with convergence in $\mathcal{L}^1$, Gnedenko [2], who studies uniform convergence and Ranga Rao and Varadarajan [7], where point-wise convergence is investigated. More recently, Barron [1] proved convergence under the Kullback-Leibler divergence, thus improving Prohorov's result.

The aim of the present talk is to propose a new approach to Prohorov-type local limit theorems. We consider densities with respect to the Gaussian rather than Lebesgue measure and prove convergence results in the corresponding $\mathcal{L}^1$ space. This choice carries several important advantages: the validity of our results on infinite dimensional spaces, for instance the classical Wiener space, and the connection of the problem under investigation with some key tools from the Malliavin calculus. In fact, we will show that the roles of scaling operator and convolution product are naturally played by the Ornstein-Uhlenbeck semigroup and Wick product, respectively.

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Central limit theorem on $\otimes\mathbb{M}_2$ with the Jordan–Wigner embedding

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We study the limit

$$u_m(\varepsilon, c) := \lim_{N\to\infty} \left\langle \Phi_N, S_N^{(\varepsilon(1))}(c) \cdot \ldots \cdot S_N^{(\varepsilon(m))}(c)\, \Phi_N \right\rangle$$

where $m \in \mathbb{N}$, $c \in \mathbb{C}$, $\varepsilon = (\varepsilon(1), \ldots, \varepsilon(m)) \in \{-1, +1\}^m$ and for any $N \in \mathbb{N}$

- $S_N^{(-1)}(c) := \frac{1}{\sqrt{N}} \sum_{k=1}^N \begin{pmatrix} 1 & 0 \\ 0 & c \end{pmatrix}^{\otimes(k-1)} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}^{\otimes(N-k)}$;
- $S_N^{(+1)}(c) := \left(S_N^{(-1)}(c)\right)^*$; $\Phi_N := \binom{1}{0}^{\otimes N}$.

The main results are:

1. the limit above exists for any $m \in \mathbb{N}$, $\varepsilon \in \{-1, +1\}^m$ and $c \in \mathbb{C}$;
2. $u_m(\varepsilon, c) = 0$ whenever m is odd; $u_{2n}(\varepsilon, c)$ is also explicitly computed for any $n \in \mathbb{N}$, $\varepsilon \in \{-1, +1\}^{2n}$ and $c \in \mathbb{C}$;
3. for any real c, if one denotes $u_{2n}(c) := \sum_{\varepsilon\in\{-1.+1\}^{2n}} u_{2n}(\varepsilon, c)$, it follows
$$u_{2n}(c) = \frac{1}{n!} \prod_{k=1}^{n} \frac{c^{2k} - 1 - 2k(c-1)}{(c-1)^2}$$
i.e. $u_m(c)$ is a generalization of the m-th moment of $X_c(t)/\sqrt{t}$, where $\{X_c(t)\}_{t\geq 0}$ is the Azéma martingale with parameter c;
4. the following affirmations are equivalent:

- there is a certain one mode interacting Fock space such that for any $m \in \mathbb{N}$ and $\varepsilon \in \{-1,+1\}^m$
$$u_m(\varepsilon, c) = \left\langle \Phi, a^{(\varepsilon(1))} \cdot \ldots \cdot a^{(\varepsilon(m))}\Phi \right\rangle$$
where, $a^{(-1)}$ (respectively, $a^{(+1)}$) is the annihilation (respectively, creation) operator and Φ is the vacuum vector;
- $c \in \{-1, 1\}$.

A PDE approach to a 2-dimensional matching problem

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We prove asymptotic results for 2-dimensional random matching problems. In particular, we obtain the leading term in the asymptotic expansion of the expected quadratic transportation cost for empirical measures of two samples of independent uniform random variables in the square. Our technique is based on a rigorous formulation of the challenging PDE ansatz by S. Caracciolo et al. (Phys. Rev. E, 90, 012118, 2014) that linearizes the Monge-Ampère equation. Moreover, it provides a new approach to classical bounds due to Ajtai et al. (Combinatorica, 4, 1984). Joint work with L. Ambrosio and F. Stra.

5.5 Wednesday 21, 16.00-17.00 Room 1I

Optimal Investment in Markets with Over and Under-Reaction to Information

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In this paper we introduce a jump-diffusion model of shot-noise type for stock prices, taking into account over and under-reaction of the market to incoming news. We work in a partial information setting, by supposing that standard investors do not have access to the market direction, the drift, (modeled via a random variable) after a jump. We focus on the expected (logarithmic) utility maximization problem by providing the optimal investment strategy in explicit form, both under full (i.e., from the insider point of view, aware of the right kind of market reaction at any time) and under partial information (i.e., from the standard investor viewpoint, who needs to infer the kind of market reaction from data). We test our results on market data relative to Enron and Ahold. The

three main contributions of this paper are: the introduction of a new market model dealing with over and under-reaction to news, the explicit computation of the optimal filter dynamics using an original approach combining enlargement of filtrations with Innovation Theory and the application of the optimal portfolio allocation rule to market data.

Keywords: Jump-Diffusion Models; Portfolio Optimization; Nonlinear Filtering; Enlargement of Filtrations; Over and Under-Reaction.

Martingale representations in progressive enlargement by the reference filtration of a semi-martingale

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We state a general representation
result for the family of $\mathbb{F}\vee\mathbb{H}$-martingales, where $\mathbb{F}$ is the reference filtration of a semi-martingale $\mathbf{X}$ with values in $\mathbb{R}^m$ and $\mathbb{H}$ is the reference filtration of a semi-martingale $\mathbf{Y}$ with values in $\mathbb{R}^n$. In the context of mathematical finance $\mathbb{F}$ models the information on a market with risky asset prices $\mathbf{X}$ and $\mathbb{H}$ plays the role of the insider's information.
In our theorem $\mathbf{X}$ and $\mathbf{Y}$ are square integrable semi-martingales, $\mathbf{X}$ enjoys the $\mathbb{F}$-predictable representation property (p.r.p.) and $\mathbf{Y}$ enjoys the $\mathbb{H}$-p.r.p., $\mathbf{M}$ and $\mathbf{N}$ denote the martingale parts of $\mathbf{X}$ and $\mathbf{Y}$ respectively. We fix conditions under which every triplet of vector martingales given by $\mathbf{M}$, $\mathbf{N}$ and any process obtained ordering the family $([M^i, N^j],\ i = 1,\dots,m, j = 1,\dots,n)$ is an $\mathbb{F}\vee\mathbb{H}$-basis of martingales. We stress that in general, since neither $\mathbf{M}$ nor $\mathbf{N}$ are required to have pairwise $\mathbb{F}\vee\mathbb{H}$-strongly orthogonal components, the integrals involved in the representation formula are vector stochastic integrals, which generalize componentwise stochastic integrals.

After giving an idea of the proof, we present an application. We identify a basis of real orthogonal martingales for the filtration $\bigvee_{i=1}^d \mathbb{F}^i$, where, for i in $(1,\dots,d)$, $\mathbb{F}^i$ is the reference filtration of a real martingale M^i enjoying the $\mathbb{F}^i$-p.r.p..

Finally using last result we deal with a particular representation problem, suggested by recent studies in credit risk modeling. We get an $\mathbb{F}\vee\mathbb{H}$-martingale representation up to the finite time horizon T when $\mathbb{F}$ is the natural filtration of a Poisson process $\mathcal{N}$ and $\mathbb{H}$ is the natural filtration of the process defined at time t by $\mathbb{I}_{\tau\leq t}$. Here τ is a random time, which coincides with positive probability with one of the jump times of $\mathcal{N}$.

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Martingale representations in markets driven by processes sharing accessible jump times

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Completeness of financial markets corresponds to the predictable representation property of the semi-martingales modeling the risky assets prices. We underline the role of common accessible jump times of such semi-martingales in the framework of martingale representations when the market filtration is progressively enlarged that is when new dynamic information $\mathbb{H}$ is added to the current information $\mathbb{F}$.

The first part of the talk is devoted to the case when $\mathbb{F}$ and $\mathbb{H}$ are reference filtrations of two independent square-integrable martingales, M and N respectively, both enjoying the predictable representation property (p.r.p.). We prove that, according to the behavior of the random measures induced by the sharp bracket processes $\langle M \rangle$ and $\langle N \rangle$, the multiplicity in the sense of Davis and Varaiya of the filtration $\mathbb{F} \vee \mathbb{H}$ is 1, 2 or 3.

The second part concerns the case when $\mathbb{F}$ is the reference filtration of a multidimensional semi-martingale $\mathbf{X}$ enjoying the p.r.p. and $\mathbb{H}$ is the natural filtration of the default process associated to a general random time τ. First we prove that the compensated default process enjoys the $\mathbb{H}$-p.r.p.. Then, under the hypothesis of the existence of an equivalent decoupling measure for $\mathbb{F}$ and $\mathbb{H}$, we propose an extension of the classical Kusuoka's theorem.

*Speaker

5.6 Wednesday 21, 16.00-17.00 Room 3I

Optimal stopping and Skorokhod embedding

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The Skorokhod embedding problem (SEP) is one of finding a stopping time σ_* for a standard Brownian motion W such that W_{σ_*} is distributed according to a given probability law μ. In this talk I describe a connection between a family of optimal stopping problems and a solution of the SEP (in particular I focus on the so-called Rost's solution).

It turns out that σ_* may be characterised as the hitting time to a suitable time reversal of stopping sets for optimal stopping problems parametrised by their time horizon. I use a new approach entirely based on stochastic calculus and probability theory with specific emphasis on the role of the optimal stopping boundaries and on the regularity of the value functions. Other existing results of such connection rely instead upon PDE theory and/or viscosity solutions of variational problems.

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A study of inactivity times of coherent systems with dependent components*

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Consider a coherent system with possibly dependent components having lifetime T. Its inactivity time $t - T$, assuming that it failed before a given time $t > 0$, can be evaluated under different conditional events. For all the cases, a representation of the reliability function of system inactivity time based on distortion functions is obtained, including a description of the structure of dependence between components through the copula of the vector of components' lifetimes. Thanks to these representations, new stochastic comparison results for inactivity times under the different conditional events are provided. These results are also applied to order statistics, that are particular cases of coherent systems (k-out-of-n systems).

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Introduction

In reliability theory, the study of coherent systems and their residual lifetime under different assumptions is a relevant topic (see [2] and [4]).

However, in some situations, the interest may be not only on the future, but on the past lifetime of a system, i.e., on its inactivity time, having observed that the system is failed at a given time t.

Let T be the lifetime of the system, and let $X_i,\ i = 1, \ldots, n$, be the lifetimes of its components. Dealing with inactivity times, different conditions can be assumed observing that the system has failed at a time $t > 0$. In a particular case, one can affirm that the stochastic inequality

$$(t - T|T \leq t) \leq_{ST} (t - T|X_1 \leq t, \ldots, X_n \leq t) \quad \forall t \geq 0, \tag{1}$$

holds true for every coherent system. However, this assertion is not always satisfied.

Main results

Motivated by the fact that (**??**) is not always true, Navarro et al [5] provides a study on the inactivity time of coherent systems formed by a number n of components with possibly dependent lifetimes, considering different conditioning events on the failed components in the system.

For all of them new representations for the reliability functions of the corresponding inactivity times are given and are used to prove simple conditions for comparisons of inactivity times according to the most important stochastic orders. The representations of the reliability function of inactivity times of coherent systems based on distortion functions (recently introduced in the literature in order to formally describe the influence of the dependence among components on the lifetime of a system) are provided.(see [1], for details on distortion functions).

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A second order ODE in Wasserstein space for the Schrödinger bridge*

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The Schrödinger bridge between two probability distributions is a way to interpolate between them while minimizing the relative entropy with respect to a reference law on the path space. In this talk, we prove that the Schrödinger bridge solves a second order ODE in the Riemannian-like structure of optimal transport. We then apply this result to obtain an estimate for the evolution of the marginal entropy and the Fisher information along a Schrödinger bridge.

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5.7 Wednesday 21, 16.00-17.00 Room 5I

Reconstruction trees*

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We propose and study an algorithm to learn a succinct representation from data coming from a fixed unknown probability distribution. The proposed approach can be seen as a geometric extension of decision trees for unsupervised learning. Alternatively, it can also be seen as a form of multi-scale vector quantization for data and probability distributions. Our main contribution is a finite sample analysis on the expected reconstruction error. While our approach is general, a special case of interest is the one where the data distribution is supported on a low dimensional manifold embedded in a high-dimensional space.

Main result

The statistical model is given by a family of independent random vectors $X_1, \ldots, X_n, X$ identically distributed with probability distribution ρ and taking value in a bounded subset $\mathcal{X} \subset \mathbb{R}^D$. The goal is to find a non-linear projection $\hat{P}_n : \mathcal{X} \to \mathbb{R}^D$, only depending on the training set $X_1, \ldots, X_n$, such that its mean square error $\mathcal{E}[\hat{P}_n] = \mathbb{E}\left[\|X - \hat{P}_n(X)\|^2 \mid X_1, \ldots, X_n\right]$ is small, with high probability.

Following [1], the main ingredient to define the estimator $\hat{P}_n$ is a (deterministic) partition tree $\mathbb{T}$, *i.e.* a family $\{\Lambda_j\}_{j\in\mathbb{N}}$ of partions on of the ambient space $\mathcal{X}$ such that for each $I \in \Lambda_j$, there exists $\mathcal{C}(I) \subseteq \Lambda_{j+1}$ and $I = \bigcup_{J\in\mathcal{C}(I)} J$. To define the estimator, we set

$$n_I = \sum_{i=1}^{n} 1_I(X_i) \qquad \hat{c}_I = \frac{1}{n_I}\sum_{i=1}^{n} X_i 1_I(X_i)$$

$$\hat{E}_I = \frac{1}{n}\sum_{i=1}^{n} \|X_i - \hat{c}_I\|^2 1_I(X_i) \qquad \hat{\epsilon}_I^2 = \hat{E}_I - \sum_{J\in\mathcal{C}(I)} \hat{E}_J,$$

where $1_I(x) = 1$ if $x \in I$ and $\text{i}1_I(x) = 0$ otherwise. We define the adaptive data-dependent subtree

$$\hat{\mathcal{T}}_n = \begin{cases} \{\mathcal{X}\} & \text{if } \hat{\epsilon}_I < \eta_n \quad \forall I \in \bigcup_{j\le j_n} \Lambda_j \\ \{I \in \mathbb{T} \mid \exists j \le j_n,\, J \in \Lambda_j,\, J \subset I,\, \hat{\epsilon}_J \ge \eta\} & \text{otherwise} \end{cases}$$

where j_n is a suitable threshold and η behaviours as $\sqrt{\frac{\ln n}{n}}$. We consider the estimator given by

$$\hat{P}_n(x) = \sum_{I\in\Lambda} \hat{c}_I 1_I(x)$$

*This work is supported by Project No., Support Foundation.
†Speaker

where $\hat{\Lambda}_\eta$ is the partition defined by the outer leaves of $\hat{\mathcal{T}}_\eta$.

Under the assumptions that, for all $i, k = 1, \dots, n$

$$\mathbb{P}[\|X_i - X_k\| \lesssim \rho_I^s \mid X_i, X_k \in I] = 1 \qquad \text{for all } I \in \mathbb{T}$$
$$\mathbb{P}[\|X_i - X_k\| \lesssim b^{-j} \mid X_i, X_k \in I] = 1 \qquad \text{for all } I \in \Lambda_j,\ j \in \mathbb{N},$$

for some $s > 0$ and $b > 1$, we prove that, with probability greater than $1 - \delta$,

$$\mathcal{E}[\hat{P}_{\hat{\Lambda}_\eta}] \lesssim \left(\frac{\ln n}{n}\right)^{\frac{2\sigma}{2\sigma+1}} \ln \frac{C}{\delta}$$

where C is a suitable constant and $\sigma < s$.

Discussion

We discuss how our result interacts with other works in the same framework. For sake of simplicity, we restrict the analysis to the case that the support of ρ is a d-dimensional compact manifold $\mathcal{M}$ and ρ is proportional the the Riemannian volume of $\mathcal{M}$. In this setting it is possible to prove that $s = 1/d$, provided that the cells of the partition tree $\mathcal{T}$ satisfy some regularity property. Hence our estimator has a convergence rate of the order $\left(\frac{\ln n}{n}\right)^{\frac{2}{2+d}}$ In [3], the k-means algorithm is analysed, when applied to learning manifolds. The representation built this way is, just like ours, a set of vectors equipped with a corresponding partition. The main differences include: the partition is built as a Voronoi tailing of the ambient space based on the set of vectors and the number of vectors is fixed at the beginning. The learning rate found for k-flats, referring to the best choice of k, is $O(n^{\frac{1}{d+2}})$, while our analysis in the optimal scenario estimates the rate $O(n^{\frac{2}{d+2}})$.

The other comparison in order is that with the so called Geometric Multiresolution Analysis [4]. This algorithm still learns a manifold through a partition having a tree-structure, but now in each cell the set reconstructing the manifold is an affine subspace rather than a single vector, and therefore the overall estimator is considered piecewise linear rather than piecewise constant. The GRLM algorithm has a convergence rate of the order $\left(\frac{1}{n}\right)^{\frac{2}{2+d}}$, too. To be complete in this scenario, we also mention here the other algorithm called k-flats, which works like k-means but using subspaces instead of vectors, whose analysis is in [3]. In general a reconstruction through subspaces rather than single vectors, even though requiring more assumptions, is expected to be more accurate. Here the rate is the order $\left(\frac{1}{n}\right)^{\frac{2}{4+d}}$

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Test statistics for stochastic differential equations sampled at discrete times

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Let $(\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\geq 0}, P)$ be a filtered complete probability space. Let us consider a one-dimensional diffusion process $X = (X_t)_{t\geq 0}$ solution to the following stochastic differential equation

$$\mathrm{d}X_t = b(\alpha, X_t)\mathrm{d}t + \sigma(\beta, X_t)\mathrm{d}W_t, \quad X_0 = x_0,$$

where x_0 is a deterministic initial point. We assume that $b : \Theta_\alpha \times \mathbb{R} \to \mathbb{R}$ and $\sigma : \Theta_\beta \times \mathbb{R} \to \mathbb{R}$ are Borel known functions (up to θ), while $(W_t)_{t\geq 0}$ is a one-dimensional standard $\mathcal{F}_t$-Brownian motion. Furthermore, $\alpha \in \Theta_\alpha \subset \mathbb{R}$ and $\beta \in \Theta_\beta \subset \mathbb{R}$, are unknown parameters. Let $\theta := (\alpha, \beta) \in \Theta$ where Θ is a compact subset of $\mathbb{R}^2$ and denote by $\theta_0 := (\alpha_0, \beta_0)$ the true value of θ. The asymptotic scheme adopted in this talk is the following: $n\Delta_n \to \infty$, $\Delta_n \to 0$ and $n\Delta_n^p \to 0, p \in \mathbb{N}$, as $n \to \infty$.

We assume that the process X is discretely observed at n equidistant times $t_i^n = i\Delta_n$, $1 \leq i \leq n$. In this talk we introduce a family of test statistics for the hypothesis testing problem $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$. Our proposal was inspired by the ϕ-divergence theory (see [4]); that is we deal with a suitable measure of discrepancy between to parametric models. Furthermore, the test statistics are built by taking into account a Gaussian approximation of the transition functions of X (see [2]). We prove that the proposed test statistics weakly converge to a chi-squared random variable either under the null hypothesis or in the case of local alternatives. Hypothesis testing problems for stochastic differential equations were also studied in [1] and [3].

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*Speaker

Compounds of conditionals and iterated conditioning under coherence

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We discuss the problem of defining logical operations among conditional events. Differently from many authors, we define the conjunction and disjunction in the setting of conditional random quantities. In probability theory and in probability logic a relevant problem, largely discussed by many authors (see, e.g., [1, 2, 9, 10]), is that of defining logical operations among conditional events. In the many works concerning these operations, the conjunction and disjunction have been usually defined as suitable conditional events. In [11] it has been proposed a theory for the compounds of conditionals which has been framed in the setting of coherence in [6, 7, 8]. In this framework, which is quantitative rather than a logical one, conjunction and disjunction of conditional events are interpreted as conditional random quantities (c.r.q.'s), which can sometimes reduce to conditional events given logical dependencies. In our approach, given a random quantity X and a non impossible event H (we use the same symbols to refer to the event H and its indicator), we look at a c.r.q. $X|H$ as an extended quantity $XH + \mu\overline{H}$, where μ is the conditional prevision $\mathbb{P}(X|H)$ which is subjectively evaluated. In particular (the indicator of) a conditional event $A|H$ is looked at as $AH + P(A|H)\overline{H}$. In this way, based on the betting scheme of de Finetti and its generalizations, the c.r.q. $X|H$ may be interpreted as the amount that you receive (resp., pay) in a bet on X conditional on H, if you agree to pay (resp., to receive) $\mathbb{P}(X|H)$. This extended notion allows algebraic developments among c.r.q.'s also when the conditioning events are different. Then, among other things, we can give a meaning to the conjunction and disjunction of conditional events and we can define the notion of iterated conditionals; in particular we obtain that the usual probabilistic properties are preserved. Therefore, we consider the coherent prevision of these suitable random quantities as the generalization of the corresponding coherent probabilistic versions of unconditional events. These notions are relevant in natural language for a probabilistic interpretation of the conjunction, disjunction and iteration of conditionals. In particular they are useful for the probabilistic approaches in the psychology of reasoning ([3, 4, 14]) and in the conditional syllogisms ([12, 13]).

Conjunction and disjunction of two conditional events ([6, 7, 8]).

Definition 1 *Given any pair of conditional events $A|H$ and $B|K$, with $P(A|H) = x$, $P(B|K) = y$, we define their conjunction as the c.r.q. $(A|H) \wedge (B|K) = Z \,|\, (H \vee K)$, where $Z = \min\{A|H, B|K\}$, and their disjunction as $(A|H) \vee (B|K) = W \,|\, (H \vee K)$, where $W = \max\{A|H, B|K\}$.*

Based on the betting scheme the quantity $(A|H) \wedge (B|K)$ coincides with $1 \cdot AHBK + x \cdot \overline{H}BK + y \cdot AH\overline{K} + z \cdot \overline{H}\,\overline{K}$, where z is the *prevision* $\mathbb{P}[(A|H) \wedge (B|K)]$. Notice that z represents the amount you agree to pay (resp., to receive), with the proviso that you will receive (resp., will pay) the quantity $(A|H)\wedge(B|K)$. For examples see [7]. In particular, by linearity of prevision, when $P(H \vee K) > 0$, we find the formula given in [11]: $\mathbb{P}[(A|H) \wedge (B|K)] = \frac{P(AHBK)+P(A|H)P(\overline{H}BK)+P(B|K)P(AH\overline{K})}{P(H\vee K)}$. With our approach we obtain the previous result in a direct and simpler way in the setting of coherence. Moreover, if we only assess $P(A|H) = x, P(B|K) = y$, we can study the basic aspect of checking coherence of the extension $z = \mathbb{P}[(A|H) \wedge (B|K)]$. Indeed, assuming A, H, B, K logically independent, in [8] it has been proved that z is a coherent extension of (x, y) if and only if the Fréchet-Hoeffding bounds are satisfied: $max\{x + y - 1, 0\} = z' \;\leq\; z \;\leq\; z'' = min\{x, y\}$. Then $0 \leq z' \leq z'' \leq 1$ for every coherent assessment (x, y), so that $(A|H)\wedge(B|K) \in [0, 1]$. As the negation is usually defined as $\overline{A|H} = \overline{A}|H = (1-A)|H$, in our approach we have $(1-A)|H = 1-A|H$; hence $\overline{A|H} = 1-A|H$. Moreover, the negation of the conjunction $(A|H)\wedge(B|K)$ is defined as $\overline{(A|H) \wedge (B|K)} = 1-(A|H)\wedge(B|K)$.

A similar analysis can also be done for the disjunction of two conditional events. Finally, De Morgan's laws and the sum rule still hold for two conditional events: $\overline{(A|H) \wedge (B|K)} = (\bar{A}|H) \vee (\bar{B}|K)$, $\overline{(A|H) \vee (B|K)} = (\bar{A}|H) \wedge (\bar{B}|K)$, and $\mathbb{P}[(A|H) \vee (B|K)] = \mathbb{P}(A|H) + \mathbb{P}(B|K) - \mathbb{P}[(A|H) \wedge (B|K)]$.

Iterated conditioning ([3, 7]).

Definition 2 *Given any pair of conditional events $A|H$ and $B|K$, the iterated conditional $(B|K)|(A|H)$ is the c.r.q. $(B|K)|(A|H) = (B|K) \wedge (A|H) + \mu \bar{A}|H$, where $\mu = \mathbb{P}[(B|K)|(A|H)]$.*

Coherence requires the following product formula ([7]): $\mathbb{P}[(B|K) \wedge (A|H)] = \mathbb{P}[(B|K)|(A|H)]P(A|H)$. If $P(A|H) > 0$, it follows that $\mathbb{P}[(B|K)|(A|H)] = \frac{\mathbb{P}[(B|K) \wedge (A|H)]}{P(A|H)}$. Concerning the lower and upper bounds for the prevision of the iterated conditioning ([3]), given any coherent assessment (x, y) on $\{A|H, B|K\}$, it holds that the extension $\mu = \mathbb{P}((B|K)|(A|H))$ is coherent if and only if $\mu \in [\mu', \mu'']$, where

$$\mu' = \begin{cases} \max\left\{0, \frac{x+y-1}{x}\right\}, & \text{if } x > 0; \\ 0, & \text{if } x = 0; \end{cases} \quad \mu'' = \begin{cases} \min\left\{1, \frac{y}{x}\right\}, & \text{if } x > 0; \\ 1, & \text{if } x = 0. \end{cases}$$

Future work
Working in progress concerns the definitions of conjunction and disjunction for n conditional events and the study of coherence for lower and upper bounds in the case $n = 3$ ([5]). We will also consider the generalization of results to the case of imprecise prevision assessments.

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5.8 Thursday 22, 11.00-12.40 Room 3I

Probabilistic approach to finite state mean field games*

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Mean field games were introduced about ten years ago in [1] and [2] as limit models for symmetric non-zero sum dynamic games, when the number N of players tend to infinity. We consider here games in continuous time where the position of each agent is in a finite state space $\Sigma = \{1, \dots, d\}$. Such games and their mean field limit were studied in [3] and [4] through the associated infinitesimal generator. In this paper we adopt a different approach introducing a probabilistic representation of the system.

We write the dynamics of any player as a stochastic differential equation with respect to a Poisson random measure:

$$X_i^N(t) = \xi_i + \int_0^t \int_U f(s^-, X_i^N(s^-), u, \alpha_i^N(s), \mu^N(s^-))\mathcal{N}_i(ds, du) \qquad i = 1, \dots, N \tag{1}$$

where α_i^N is the control of player i (here in open loop form) and $\mu^N(s^-)$ is the empirical measure of the system immediately before time s. This description allows to write easily the mean field limiting dynamics in which a single players evolves according to

$$X(t) = \xi + \int_0^t \int_U f(s^-, X(s^-), u, \alpha(s), m(s))\mathcal{N}(ds, du), \tag{2}$$

where the state depends on a control α and a deterministic flow of probability measures $m : [0, T] \to \mathcal{P}(\Sigma)$, which takes the place of μ^N.

We will consider several types of controls: open loop, feedback, relaxed and relaxed feedback. Each player wants to optimize his reward over a finite time horizon T. The running cost and the terminal cost are the same for all agents, may depend on μ^N and are used also in the cost of the limiting problem. There is a natural definition of solution of the mean field game: fixed m find a control α_m optimal and then impose that the corresponding solution X is such that $Law(X(t)) = m(t)$ for any t. Such definition provides approximate Nash equilibria for the N players game.

We firstly study the mean field game and show that it admits a solution in relaxed controls. We require only the continuity of the functions involved and prove the existence using a general fixed point theorem for point-to-set maps. In order to write the dynamics when using a relaxed control ρ, we need to define a proper generalization of $\mathcal{N}$ which is called relaxed Poisson measure $\mathcal{N}_\rho$ and was introduced in [5]. The same assumptions give also the existence of a relaxed feedback solution of the mean field game, applying an idea of [6]. Relaxed controls are used only for the limiting dynamics.

Then we show that these mean field game solutions provide ϵ_N-Nash equilibria for the N players game, for any horizon T, both in open loop and in feedback strategies (not relaxed). We approximate a limiting optimal relaxed control by an ordinary one, using a suitable version of the chattering lemma which works also in the feedback setting. Then we show that the control we have found provides a

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symmetric ϵ_N-Nash equilibrium, with $\epsilon_N \leq \frac{C}{\sqrt{N}}$, decentralized when considering feedback strategies. The proof relies on the probabilistic representation of the system and on a coupling argument.

We also study the problem of finding solutions of the mean field game in (ordinary) feedback controls. We need stronger assumptions in order to guarantee the uniqueness of an optimal feedback control for any fixed m (existence always holds): these are linearity of the dynamics in the control and strict convexity of the costs. Moreover we prove, under additional assumptions, that the feedback mean field game solution is unique for small times by showing that the fixed point map is a contraction.

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Verification theorem for stochastic impulse non-zero sum games and applications.

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In this talk, we provide a general framework for impulse stochastic differential nonzero sum games. Within this setting, we investigate the notion of Nash equilibrium through the corresponding quasi-variational inequalities (QVI). Related former works address the case of non-zero sum optimal stopping games (Bensoussan-Friedman 1977) and zero-sum stochastic differential games with impulse control (Cosso 2013), but to our knowledge, the class of non-zero sum stochastic impulse games have

[*]Speaker

not yet been addressed in the literature in its generality. We fill this gap by providing the right system of QVI with the right smoothness required to have both classical solutions and working examples. Finally, we present some applications.

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Verification theorems for stochastic optimal control problems in Hilbert spaces by means of a generalized Dynkin formula

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Verification theorems are key results to successfully employ the dynamic programming approach to optimal control problems. In this paper we introduce a new method to prove verification theorems for infinite dimensional stochastic optimal control problems. The method applies in the case of additively controlled Ornstein-Uhlenbeck processes, when the associated Hamilton-Jacobi-Bellman (HJB) equation admits a *mild solution*. The main methodological novelty of our result relies on the fact that it is not needed to prove, as in previous literature, that the mild solution is a *strong solution*, i.e. a suitable limit of classical solutions of the HJB equation. To achieve our goal we prove a new type of Dynkin formula, which is the key tool for the proof of our main result.

*Speaker

Filtering and control of time-homogeneous pure jump Markov processes with noise-free observation

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In this talk I am going to address a stochastic filtering and optimal control problem with partial observation, mainly characterized by observations not corrupted by noise. Let X and Y be a given couple of stochastic processes, with values in two Borel spaces $(I, \mathcal{I})$ and $(O, \mathcal{O})$ respectively. The unobserved (or signal) process X is a time-homogeneous pure jump Markov process, whose rate transition measure is known. The observed process Y is defined as $Y_t = h(X_t)$, $t \geq 0$, where $h\colon I \to O$ is a known surjective and measurable function.

The first aim is to provide an explicit SDE for the filtering process $(\pi_t)_{t \geq 0}$, satisfying

$$\pi_t(\varphi) = \mathbb{E}\big[\varphi(X_t) \mid \mathcal{Y}_t\big] \tag{1}$$

for all $t \geq 0$ and all $\varphi\colon I \to \mathbb{R}$ bounded and measurable functions; here $\mathcal{Y}_t = \sigma(Y_s\colon 0 \leq s \leq t)$ denotes the natural filtration of Y. The problem is tackled with the aid of known results from marked point processes theory and a martingale approach (see e.g. [1]). The filtering process is also characterized as a *Piecewise Deterministic Markov Process*, in the sense of Davis [2].

The second goal is to solve an infinite-horizon optimal control problem. The aim is to minimize a discounted cost functional by controlling the rate transition measure of the unobserved process via the information provided by the observed process. The problem is reformulated into a PDP optimal control problem for the filtering process and subsequently into a deterministic optimal control problem. In the case of a finite-state controlled Markov chain (i.e. when the space I is of finite cardinality), the value function can be characterized as the unique fixed point of a suitably defined operator. In addition, a HJB equation can be explicitly written and the value function is proved to be its unique constrained viscosity solution, in the sense of Soner [3]. Finally, the existence of an optimal control is shown.

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Regress Later Monte Carlo for Controlled Markov Processes

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Consider the problem of controlling a general discrete-time Markov process

$$X_{t+1} = \varphi(n, X_t, \xi_t, u_t)\,, \;\; X_0 = x, \tag{1}$$

where (X_n) is a Markov process on $\mathbb{R}^p$, φ is a Borel-measurable function and $\{\xi_t\}$ is a collection of i.i.d. standard uniform random variables. The set of admissible controls $\mathcal{U}$ contains stochastic processes $(u_t)_{t=0,\dots,T-1}$ adapted to the filtration generated by (X_t), such that u_t takes values in a compact set U and such that the controlled process $X_t \in \mathcal{D}_t$ for all t. The objective is to maximise the following criterion

$$\mathbb{E}\Big[\sum_{s=t}^{T-1} f(s, X_s, u_s) + g(X_T)|X_t^u = x\Big] \tag{2}$$

for measurable functions f and g.

In this talk I will present a method to avoid the curse of dimensionality constraining the use of numerical methods based on Markov chain approximations (i.e. we avoid the discretisation of the state space used in [4]). We build on the method introduced in [3] without however using any control randomization; rather, we use regression to decouple the value of the controlled process in one time step to the following one allowing the backward decision on the control to be dependent on the present state only and the future information to be used solely for training.

The key to our solution is the regress later approach, mostly used for American option pricing and studied by [2] and [1]. It consists of projecting the value function $V(t+1, X_{t+1})$, or another $\mathcal{F}_T$-measurable random variable (when policy iteration is used), over the space generated by a set of basis functions $\{\phi_k(t+1, X_{t+1})\}_{k=1}^K$, then computing, possibly analytically,

$$\mathbb{E}_{u_t}[V(t+1, X_{t+1})|X_t] \approx \sum_{k=1}^{K} \alpha_k^{t+1} \mathbb{E}_{u_t}[\phi_k(t+1, X_{t+1})|X_t] \tag{3}$$

from which the control at time t is computed. The procedure is iterated backward in time over the time horizon of the problem.

The Regress Later approach allow us to:

1. avoid randomisation of the control improving speed and accuracy;

2. place training points freely in the state space, leading to faster convergence;

3. use policy iteration without resimulation, which is not possible when the control is randomised, saving computational time.

The latter, a Longstaff Schwartz-type approach [5], is implemented either using an innovative algorithm based on a coupling argument (if the controlled process evolves in continuous time) or solving iteratively fixed point problems.

The performance of different Regression Monte Carlo algorithms have been compared numerically and the results will be shown during the talk.

*Presenting

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5.9 Thursday 22, 11.00-12.40 Room 5I

High order heat-type equations and random walks on the complex plane

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The celebrated Feynman-Kac formula for the solution of the heat equation is the first (and most famous) example of an extensively developed theory connecting stochastic processes with the solution of parabolic equations associated to second order elliptic operators. However, this theory cannot be applied to higher order PDEs such as, for instance, high-order heat type equations of the form:

$$\begin{aligned} \frac{\partial}{\partial t}u(t,x) &= a\frac{\partial^N}{\partial x^N}u(t,x), \\ u(0,x) &= f(x) \end{aligned} \tag{1}$$

where $t \in \mathbb{R}^+, x \in \mathbb{R}, a \in \mathbb{R}$ and $N \in \mathbb{N}$, $N > 2$. In fact, the lack of a maximum principle for Eq. (1) forbids a probabilistic representation of its solution of the form

$$u(t,x) = \mathbb{E}[f(x + X_t)],$$

in terms of the expectation with respect to the distribution of a *real valued* stochastic process $\{X_t\}_{t\in\mathbb{R}^+}$. This problem has been extensively studied, e.g. by Krylov (1960), Hochberg(1978), Funaki (1979), Burzdy (1995), Orsingher (1999), Levin and Lyons (2009).

Recently an alternative technique has been proposed in [1]. A sequence $\{W_n^N(t)\}$ of scaled random walks on the complex plane is constructed as

$$W_n^N(t) = \frac{1}{n^{1/N}}\sum_{j=1}^{\lfloor nt\rfloor}\xi_j, \tag{2}$$

where $\{\xi_j\}_{j\in\mathbb{N}}$ are independent identically distributed complex random variables, uniformly distributed on the set of $N-th$ roots of unit. If $N>2$, the particular scaling exponent $1/N$ appearing in (2) does not allow the weak convergence of $W_n^N(t)$. Nevertheless, the expectation of particular functionals admit a limit for $n\to\infty$, in particular the following result holds

$$\lim_{n\to+\infty}\mathbb{E}[\exp(i\lambda W_n^N(t))]=\exp\left(\frac{i^n}{N!}\lambda^N t\right) \tag{3}$$

and allows to interpret in a weak sense the limit of $W_n^N(t)$ as an N-stable random variable. The convergence in Eq (**??**) allows the proof of the following probabilistic representation formula for the solution of (1)

$$u(t,x)=\lim_{n\to+\infty}\mathbb{E}[f(x+W_n^N(t))],$$

for a suitable class of analytic initial data f. Furthermore in [2] a stochastic calculus for the sequence of processes $\{W_n^N(t)\}_{n\in\mathbb{N}}$ has been developed and applied to the construction of a Feynman-Kac type formula

$$u(t,x)=\lim_{n\to+\infty}\mathbb{E}[f(x+W_n^N(t))e^{\int_0^t V(x+W_n^N(s))ds})],$$

for the probabilistic representation of the perturbed Cauchy problem

$$\begin{aligned}\frac{\partial}{\partial t}u(t,x) &= a\frac{\partial^N}{\partial x^N}u(t,x)+V(x)u(t,x),\\ u(0,x) &= f(x)\end{aligned} \tag{4}$$

Recently in[3] these results are generalized to the case of high-order heat-type equations with fractional derivatives (either in the space or in the time variable) by means of Bochner's subordination techniques.

The talk will provide an overview of these results, obtained in collaboration with S. Bonaccorsi, C. Calcaterra and M. D'Ovidio.

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Ergodicity of a system of interacting random walks with asymmetric interaction

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In this talk we aim to present a model of interacting random walks on $\mathbb{N}$ introduced in [1], where an asymmetric mean-field interaction may ensure ergodicity. For a fixed $N \geq 2$, we consider N particles evolving in $\mathbb{N}$. We provide every particle with an intrinsic dynamics given by a biased random walk reflected in zero, which gives clearly a transient process. Therefore, we add an asymmetric interaction that pushes each particle towards the origin and depends only on the fraction of particles *below* its position. We focused on the critical interaction strength above which the N particle system and its corresponding nonlinear limit have a stationary measure, balancing the tendency of the biased random walks to escape to infinity. A similar model has been studied in the continuous with diffusive dynamics [2], where the authors consider a system of particles interacting through their *cumulative distribution function*. The discrete model we consider displays a peculiar difference: the particles can form large clusters on a single site and, according to our description, they cannot interact. This gives rise to non-trivial expression for the critical interaction strength, unexpected from the analysis of the continuum model. This talk is based on a joint work with Amine Asselah and Paolo Dai Pra.

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Collective periodic behavior in spin-flip models with dissipation

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Interacting particle systems constitute a wide class of models, originally motivated by Statistical Mechanics, which in the last decades have become more and more popular, extending their applications to various fields of research such as biology and social sciences. These models are important tools which may be used to study macroscopic behaviours observed in complex systems. Among these phenomena, a very interesting one is collective periodic behaviour, in which the system exhibits the emergence of macroscopic rhythmic oscillations even though single components have no natural tendency to behave periodically.
Self sustained rhythm may appear in complex systems in which single units tend to cooperate with

each other but their interaction energy is dissipated over time: this mechanism is well described in [1], where a Curie-Weiss model with dissipation is introduced. This spin-flip model is subject to a mean-field interaction and, as the number of particle N goes to infinity, its limiting dynamics can be described by a two-dimensional ODE. This dynamical system exhibits a phase transition through a Hopf bifurcation: hence, for sufficiently low temperature, the macroscopic magnetization fluctuates periodically in time.
An interesting question is whether the mean-field interaction is necessary to produce stable oscillations: in this talk, we introduce an Ising model with dissipation, in which each spin only interacts with its neighbours and we present some result concerning macroscopic oscillations in the zero-temperature limit. In this case, the limiting dynamics can not be described by a finite set of equations, but we proved that a time-scaled version of the macroscopic magnetization converges to an oscillating process. In this presentation, we focus on role of the correct choice for the time scales. This is a joint work with Raphaël Cerf, Paolo Dai Pra and Marco Formentin.

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Entropy chaos and Bose-Einstein Condensation

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(joint work with S. ALBEVERIO and F. C. DE VECCHI)

The quantum phenomenon of Bose-Einstein Condensation (BEC) was predicted by Einstein in 1925 by theoretical arguments using some statistical ideas due to Bose. It was experimentally verified only in 1995 by E. Cornell and C. Wieman, who obtained the Nobel Prize in Physics in 2001. Qualitatively, when a localized system of many particles of boson type with a certain degree of dilution is cooled under a critical very low temperature it falls into a very peculiar quantum state: (almost) all interacting particles assume the lowest energy state and therefore they can be described by the same ground state wave function. This quantum state is known as a Bose Einstein condensate and the behavior of the particles in this system is commonly described by a one-particle model of non linear type, denominated GP model. The problem of giving a mathematical (and physical) justification of the GP model of the Bose Einstein Condensate starting from first principles was immediately tackled. Starting from the N body Hamiltonian, which describes a system of N Bose particles in a suitable trapping potential V which interact through a pairwise potential v_N, Lieb, Seiringer and Yngvason ([7],[8]) proved that the GP model can be rigorously obtained from the N body Hamiltonian by performing a suitable limit of infinitely many particles together with a well-defined re-scaling of the interaction potential v_N. In particular they showed that the one-particle quantum mechanical energy converges to the minimum of the GP functional. The GP scaling is more complex than the mean-field one: the interaction potential converges to the Dirac delta function and the limit intensity is expressed in term of the scattering length of the potential because of the existence of a non trivial short range correlation structure in the ground state wave function.

A well-posed probabilistic way of looking at the Bose-Einstein condensation consists in introducing an appropriate probability measure on the path space $C([0,T],\mathbb{R}^{3N})$, by rigorously associating a diffusion process to the ground state eigenfunction of the N-body Hamiltonian H_N through Nelson's map [12] plus relevant results due to Carlen [3]. Nelson's map has been applied to BEC phenomenon for the first time in 2007 [9]. In 2011 ([10]) it has been shown that in the GP scaling limit, by using a relative entropy approach, the suitably stopped one-particle process converges in the total variation sense. The limit diffusion process has a drift which is uniquely determined by the minimizer of the GP functional. Successively the phenomenon of the asymptotic localization of relative entropy has been investigated ([11]). In [13] Kac's chaos for the same N interacting diffusions system under the GP scaling limit was proved. Since the Nelson map cannot be applied to a non linear Hamiltonian, the problem of correctly individuating the process corresponding to the minimizer of the (non linear) GP functional had to be faced in [2], deriving an asymptotic non linear diffusion generator with a killing rate governed by the wave function of the condensate. With the introduction of a proper one-particle relative entropy an existence theorem for the probability measure associated to the minimizer of the GP functional was proved in [5].

The entropy chaos property, according with Carlen et al ([4]) and Hauray and Mischler ([6]), is a stronger chaotic property than the well-known Kac's chaos. We show the convergence of the N-particle process $\hat{Y}_t^{(N)}$ at fixed time $t \in \mathbb{R}_+$ under the same technical hypothesis as in the standard quantum mechanical approach.

1. When the confining potential satisfies $V(\mathbf{r}) \geq \alpha r^{1+\epsilon} + \beta$ for some $\alpha, \epsilon > 0$ and $\beta \in \mathbb{R}$, $r = |\mathbf{r}|$, the process $\hat{Y}_t^{(N)}$ is ρ_{GP}-entropy-chaotic, where ρ_{GP} the probability density associated with the minimizer of the GP functional. Moreover the Lebesgue densities $\rho_N^{(1)}$ of the one-particle process $\hat{Y}_t^{(N)}$ at any fixed time t strongly converge, as N tends to infinity, to the limit density ρ_{GP} in L^1. The total variation convergence is implied.

Since the interaction between the particles asymptotically concentrates on a random region having Lebesgue measure zero (see [10]) but it does not disappear, the convergence problem of the one-particle probability measure $\mathbb{P}_N^{(1)}$ on the path space $C([0,T], \mathbb{R}^3)$ is not trivial and one cannot hope to find a convergence result as strong as the total variation one.

2. On the path space $C([0,T], \mathbb{R}^3)$ the one-particle probability measure $\mathbb{P}_N^{(1)}$ weakly converges, as N tends to infinity, to the measure $\mathbb{P}_{GP}$ of the limit process X^{GP}.

Since it is well-known that Nelson's Stochastic Mechanics is not a phenomenological model of quantum phenomena, but an alternative stochastic version of Quantum Mechanics able to reproduce many important properties of quantum systems and in particular the same configurations probabilities predictions, our weak convergence result yields a probabilistic justification of the GP quantum model for the Bose-Einstein condensate.

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Skew diffusions across Koch interfaces

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We consider planar skew Brownian motion across pre-fractal Koch interfaces $\partial\Omega^n$ and moving on $\overline{\Omega^n} \cup \Sigma^n = \Omega^n_\varepsilon$ where Σ^n is a suitable neighbourhood of $\partial\Omega^n$. We study the asymptotic behaviour of the corresponding multiplicative functionals when thickness of Σ^n and skewness coefficients vanish with different rates. Thus, we provide a probabilistic framework for studying diffusions across semi-permeable pre-fractal (and fractal) layers and the asymptotic analysis concerning the insulating fractal layer case. Then, we consider time-changes and fractional (non-local) Cauchy problems on random Koch domains with Robin condition on the pre-fractal boundary and obtain asymptotic results. Random Koch domains are obtained as mixtures of Koch curves with random scales.

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5.10 Thursday 22, 11.00-12.40 Room 2I

Semi-Markov processes and their Kolmogorov's equations*

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A process $X(t)$, $t \geq 0$, is said to be a semi-Markov process in the sense of Gihman and Skorohod [1] if the couple $(X(t), \gamma(t))$, where $\gamma(t)$ is the sojourn time in the current position, is a strong (homogeneous) Markov process. The Kolmogorov's equations of such processes are investigated. An evolutionary form (integro-differential) for such equations is derived by using the construction of a semi-Markov processes as time-changed Markov processes [2, 3, 4, 5, 6]. The time-fractional equation is an interesting particular case, obtained when the waiting times between the jumps are i.i.d. r.v.'s with Mittag-Leffler distribution.

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Non-homogeneous subordinators and their connection with semi-Markov processes

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Recent papers deal with some additive processes (i.e. right-continuous processes with independent increments) which reduce to the well-known Levy processes under the further condition of stationarity of the increments.

For example, the class of multistable processes was studied in [7], [14] [6], [10], [11]. These processes extend the well-known stable processes by letting the stability index α evolve autonomously in time. Such models are useful in various applications where the data display jumps with varying intensity, such as financial records or natural terrains: indeed, multistability is one practical way to deal with non- stationarities observed in various real-world phenomena. One of the most important feature of stable processes is self-similarity. This property is not shared by their time-inhomogeneous extensions, even if they are "locally" self-similar.

Moreover, a class of non decreasing jump processes is considered in literature, which generalizes the classical subordinators. In [16], the main aspects of the theory of classical subordinators are generalized to the time-inhomogeneous case. This leads to an interesting class of additive processes which may well turn out to be applicable to problems in finance and statistical physics.

We recall that classical subordinators, their inverses, time-changed diffusion processes and their governing equations are all interrelated, according to a well-established theory (see for example [9]). In [16] the authors generalize the above objects to the non-homogeneous case. The generalized subordinator is easily defined: in few words, its construction is based on a time dependent Levy measure. However, the intricate connection with Bernstein functions, hitting times and their fractional governing equations requires a deep analysis which is expounded in [16]. Because of time-inhomogeneity, two-parameter semigroups (propagators) arise and the Laplace exponents of the finite-dimensional distributions are possibly different Bernstein functions for each time t. By means of these processes, a generalization of subordinate semigroups in the sense of Bochner is proposed. To this regard, the authors in [16] provide a generalization of the Phillips formula, which leads to abstract Cauchy problems related to time dependent generators.

The inverse processes (i.e. the right continuous hitting times) of inhomogeneous subordinators have also been studied. In [3] the inverse of a multistable subordinator is used to give a generalization of the fractional Poisson process already constructed in [12].

The hitting times of inhomogeneous subordinators are also used in [15] to construct a large class of stepped semi-Markov processes. In few words, let M be a stepped Markov process and let L be

*Speaker

the inverse of an inhomogeneous subordinator. Suppose that M and L are dependent processes in the sense of [15]. Then $M(L(t))$ is a stepped semi-Markov process which is governed by a generalized fractional integro-differential equation. Such equation is obtained by replacing the time-derivative of the Kolmogorov backward equation of M with the generalized fractional derivative. This result completes the analysis given in Feller [8].

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A compound Poisson approximation to estimate the Lévy density*

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We construct an estimator of the Lévy density, with respect to the Lebesgue measure, of a pure jump Lévy process from high frequency observations. Suppose we observe one trajectory of the Lévy process over $[0, T]$ at the sampling rate Δ, where $\Delta \to 0$ as $T \to \infty$. The main novelty of our result is that we directly estimate the Lévy density in cases where the process may present infinite activity and that we study the risk of our estimator with respect to L_p loss functions, $p \geq 1$. The idea of the procedure is to use that "*every infinitely divisible distribution is the limit of a sequence of compound Poisson distributions*" (see e.g. Corollary 8.8 in Sato [2]) and to take advantage of the fact that the estimation of the Lévy density of a compound Poisson process is well known in the high frequency setting. We consider linear wavelet estimators and the performance of our procedure is studied in term of L_p loss functions, $p \geq 1$, over Besov balls. Our result is illustrated for several examples.

Contents

From a mathematical point of view the jump dynamics of a Lévy process X is dictated by its Lévy density, that is denoted by f in the sequel. If f is continuous, its value at a point x_0 determines how frequent jumps of size close to x_0 are to occur per unit time. Thus, to understand the jumps behavior of X, it is of crucial importance to estimate its Lévy density. A problem that is well understood is the estimation of the Lévy density of a compound Poisson process, that is a pure jumps Lévy process with a finite Lévy measure. Building an estimator of the Lévy density for a Lévy process X with infinite Lévy measure is a more demanding task since, for instance, for any time interval $[0, t]$, the process X will almost certainly jump infinitely many times. In particular, the Lévy density f, that is to be estimated, is unbounded near 0. This implies that the techniques used for compound Poisson processes do not generalize immediately. The essential problem is that the knowledge that an increment $X_{t+\Delta} - X_t$ is larger than some $\varepsilon > 0$ does not give any insight on the size of the largest jump that has occurred between t and $t + \Delta$. Some results are nevertheless already present in the literature concerning the estimation of the Lévy density from discrete data without the hypothesis of finiteness, i.e. when $\int_{\mathbb{R}} f(x)dx = \infty$. A number of different techniques has been employed to attack this problem:

- To limit the estimation of f on a compact set outside of 0;
- To study a functional of the Lévy density, such as $x^2 f(x)$.

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In these results, the analysis systematically relies on a spectral approach, based on the use of the Lévy Kintchine formula, that allows estimates for L_2 and L_∞ loss functions, but does not generalize easily to L_p for $p \notin \{2, \infty\}$. A review is available in the textbook [1]. The difference in purpose between the present work and those in the literature is that here we aim to construct a direct estimator $\hat{f}$ of f in order to study

$$\mathbb{E}\bigg[\int_{A(\varepsilon)} |\hat{f}(x) - f(x)|^p dx\bigg],$$

where, $\forall \varepsilon > 0$, $A(\varepsilon)$ is an interval bounded away from 0 of the form $\mathbb{R} \setminus [-a(\varepsilon), a(\varepsilon)]$ where $a(\varepsilon) \geq \varepsilon$ such that $a(\varepsilon) \to 0$ as $\varepsilon \to 0$.

Formally, we consider the class of Lévy processes of Lévy triplet $(0, 0, \nu)$, that is càdlàg processes of the form

$$X_t = \lim_{\eta \to 0} \bigg(\sum_{s \leq t} \Delta X_s \mathbb{I}_{\eta < |\Delta X_s| < 1} - t \int_{\eta < |x| < 1} x\nu(dx) \bigg) + \sum_{s \leq t} \Delta X_s \mathbb{I}_{|\Delta X_s| \geq 1};$$

where $\Delta X_s = X_s - \lim_{r \uparrow s} X_r$ denotes the jump of X at time s. We consider a high frequency setting: the observations at our disposal are of the form

$$\big(X_{i\Delta} - X_{(i-1)\Delta},\ i = 1, \ldots, n\big) \quad \text{with } \Delta \to 0 \text{ as } n \to \infty. \tag{1}$$

We want to estimate the density of the Lévy measure ν with respect to the Lebesgue measure, $f(x) := \frac{\nu(dx)}{dx}$, from the observations (**??**) on the sets $A(\varepsilon)$ as $\varepsilon \to 0$. The Lévy measure ν may have infinite variation, i.e.

$$\nu : \int_{\mathbb{R}} (x^2 \wedge 1)\nu(dx) < \infty \text{ but possibly } \int_{|x| \leq 1} |x|\nu(dx) = \infty.$$

The starting point of this investigation is to look for a translation from a probabilistic to a statistical setting of Corollary 8.8 in [2]: "*Every infinitely divisible distribution is the limit of a sequence of compound Poisson distributions*" and to take advantage of the fact that estimation of the Lévy density of a compound Poisson process is well known. We may formalize the latter statement as follows. For all $\varepsilon \in [0, 1]$, let $Z(\varepsilon)$ be a compound Poisson process of intensity $\lambda_\varepsilon = \nu(\mathbb{R} \setminus [-\varepsilon, \varepsilon])$ and jump density $h_\varepsilon = \frac{f}{\lambda_\varepsilon} \mathbb{I}_{(-\infty, -\varepsilon) \cup (\varepsilon, \infty)}$. It follows that $f(x) = \lim_{\varepsilon \to 0} \lambda_\varepsilon h_\varepsilon(x) \mathbb{I}_{|x| > \varepsilon}$, for all $x > 0$. Therefore, one can construct an estimator of f on $A(\varepsilon)$ by constructing estimators for λ_ε and h_ε separately. The properties of this estimator will be illustrated during the talk.

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On a Class of Gauss-Markov processes for neuronal models with jumps

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The theory of Gauss-Markov (GM) processes ([10]) and the First Passage Time (FPT) problem through specified boundaries turns out especially useful in the stochastic modeling of neuronal firing ([1],[2], [3], [9],[14]). The membrane voltage of a neuron fluctuates in response to synaptic input signals and an internal noise. As soon as a threshold voltage is crossed, the neuron fires a spike (action potential). Therefore, the generation of the action potential corresponds to the first passage of the fluctuating membrane voltage through the threshold. Pioneering models as that of Gerstein and Mandelbrot until to the widely used Leaky Integrate-and-Fire (LIF) model ([1]), and some others more recent models ([13],[16], [18], [19]), are essentially based on the stochastic dynamics of diffusion processes for describing the membrane voltage. This kind of models often relies on the use of the Ornstein-Uhlenbeck (OU) process ([15]), although sometimes shortcomings have been highlighted (for instance, [17]). The class of GM processes, including also time-inhomogeneous OU-type processes, appears more suitable to model several complex aspects of neuronal activity. We remark that for evaluations of FPT densities of GM processes the Volterra-integral approach is available. Hence, not only reliable simulations can be performed but also numerical estimations of firing densities can be obtained.

In neuronal coding the interest is mainly focused on the rate codes, i.e. the average number of spikes per unit of time, and on the temporal codes, in which the timing of action potentials is related to the information transmission. The latter codes are of primary interest when the neuron is subject to a time-varying input. In this context, for the adaptation phenomenon ([9],[13]), in [7] a LIF model, essentially based on time-inhomogeneous GM processes, was proposed for providing estimations of firing densities and firing rates.

Special stochastic processes subject to jumps, random in time and in amplitude, are required to model successive spike times. In [11] and [12], we model the second spike time by means of the FPT of a process that is subject to a jump when the first spike occurs. From it a new GM process is constructed *ad hoc* for embodying the memory of a spike already occurred while taking into account the ongoing action of the input signal. For the general successive spike times the results can be accordingly generalized.

Again, the GM processes are a valid tool to model the membrane voltage of a neuron embedded in a network, i.e. when it is subject to random jumps due to interactions with the surrounding neurons. For this case, linked GM processes are proposed to describe the neuronal dynamics and to provide estimations of firing probability densities. Asymptotic approximations of firing densities of surrounding neurons are used to obtain closed-form expressions for the mean of involved processes.

Finally, the need of describing several phenomena, such as special interactions between neurons ([4]), effects of input currents ([8]), adaptation of the firing activity ([7],[13]), occurrence of spike trains ([11],[12]) and networks of neurons have led us to consider suitable GM processes, by which to design specialized neuronal models, and also to obtain new theoretical results as, for instance, those in [5] and [6].

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On the reference frame invariance of stimulus-specific information measures*

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The neural coding problem is often approached by virtue of the stimulus-response model, in which the neuronal response r is related to the stimulus parameter s. Any particular stimulus intensity, as a physical quantity, can be equivalently described in different unit systems. It would be desiderable that the methodology and the inference obtained about the neural coding precision to be independent from such a subjective choice. However it has been demonstrated in a series of recent papers (see for instance [1] and [2]) that the choice of stimulus units has a profound impact on the inference about the neural decoding accuracy. The reference frame invariance is one of the cornerstones of modern physics, however, it is rarely considered in the field of computational neuroscience. In fact, the individual specific information measures may depend on the frame of reference even though the mutual information is an invariant quantity. We argue that the invariance principle is a logical necessity, which helps us to resolve the ambiguity in the choice of possible specific information measures ([3]). We will show that the only stimulus-specific information measure that is invariant under regular transformations of the reference frame is the one introduced by Kullback and Leibler ([4]). We will show also how some other stimulus-specific information measures fail to be consistent even in cases of interest in application areas, like the classical model of a cat auditory nerve fiber proposed by Winslow and Sachs ([5]).

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6 Posters

1

Some remarks on the Joint Distribution of First-passage Time and First-passage Area of Drifted Brownian Motion

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For drifted Brownian motion $X(t) = x - \mu t + B_t$ $(\mu > 0)$ starting from $x > 0$, we study the joint distribution of the first-passage time below zero, $\tau(x)$, and the first-passage area, $A(x)$, swept out by X till the time $\tau(x)$. In particular, we establish differential equations with boundary conditions for the joint moments $E[\tau(x)^m A(x)^n]$, and we present an algorithm to find recursively them, for any m and n. Finally, the expected value of the time average of X till the time $\tau(x)$ is obtained.

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Strong approximations for reaction models and density dependent families of Markov Chains in bounded domains*

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Density dependent families of Markov chains, such as the stochastic models of mass-action chemical kinetics, converge for large values of the indexing parameter N to deterministic systems of differential equations (Kurtz, 70). Moreover for moderate N they can be approximated trajectory by trajectory by paths of a diffusion process (Kurtz, 76). Such an approximation however fails if the state space is bounded (at zero or at a constant maximum level due to conservation of mass) and if the process visits the boundaries with non negligible probability. We present a strong approximation by jump-diffusion processes which is robust to this event.

Asymptotics of the Empirical Identity Process and its integral*

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Let $\{U_i\}_{i=1\cdots n}$ be an iid sample, and let the random variables $U_{n,1},\cdots,U_{n,n}$ be the corresponding order statistics. Let $\mathbb{F}_n(t)=\frac{1}{n}\sum_{i=1}^{n}X_i$ be the empirical distribution function and $\mathbb{Q}_n(u)$ the so called empirical quantile function, its (left-continuous) generalized inverse. We define the empirical identity function

$$R_n^L(t)=U_{n,n\mathbb{F}_n(t)}=\begin{cases}0 & \text{if } 0\le t<U_{n,1}\\ \mathbb{Q}_n(\mathbb{F}_n(t)) & \text{if } U_{n,1}\le t\le 1\end{cases} \quad (1)$$

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which converges to the identity function for $n \to \infty$ and study the how the scaled difference $Y_n(t) = n(R_n^L(t) - t)$, which we call the *Empirical identity process* behaves asymptotically for $n \to \infty$. The limiting process turns out to be highly irregular, its trajectories not being right continuous. However, an appropriately scaled and centered version of the integral of the empirical identity process turns out to converge weakly to a Brownian motion which at time 1 is pinned at random with a given Gaussian distribution. Two new goodness of fit statistics based on the EIP and on the integrated EIP are introduced and compared with other more classical test.

Some Properties of Cumulative Tsallis Entropy*

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Tsallis entropy plays an important role in the measurement uncertainty of random variables. In this poster, we introduce a new measure of cumulative Tsallis entropy and its dynamic version and we study their properties. The proposed measures have relationships with other important informations, reliability measures and some features related to stochastic orders.

Definition and properties

Tsallis entropy was introduced by Tsallis [3] in 1988 and it is a generalization of Boltzmann-Gibbs statistics. For a continuous random variable X with pdf $f(x)$, Tsallis entropy of order α is defined by

$$T_\alpha(X) = \frac{1}{\alpha - 1}\left(1 - \int_0^{+\infty} f^\alpha(x)dx\right); \qquad \alpha \neq 1, \quad \alpha > 0. \tag{1}$$

Rajesh and Sunoj [2] introduced an alternative measure of cumulative residual Tsallis entropy (CRTE) of order α

$$\xi_\alpha(X) = \frac{1}{\alpha - 1}\int_0^{+\infty} \left(\bar{F}(x) - \bar{F}^\alpha(x)\right) dx; \qquad \alpha \neq 1, \quad \alpha > 0, \tag{2}$$

where $\bar{F}(x)$ is the survival function of X.
Recall the definition of cumulative entropy (CE), given by Di Crescenzo and Longobardi [1]

$$\mathcal{CE}(X) = -\int_{-\infty}^{+\infty} F(x)\log F(x)dx. \tag{3}$$

Motivated by (1)-(3), we propose the cumulative Tsallis entropy (CTE), as

$$\mathcal{C}\psi_\alpha(X) = \frac{1}{\alpha - 1}\left(\int_0^{+\infty} \left(F(x) - F^\alpha(x)\right) dx\right); \qquad \alpha \neq 1, \quad \alpha > 0. \tag{4}$$

We study the relation between the proposed CTE and mean inactivity time, defined as $\tilde{\mu}_X(t) = E[t - X \mid X \leq t] = \frac{1}{F(t)}\int_0^t F(x)dx$ and the relation with the cumulative entropy. We discuss the

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effect of increasing transformation on the CTE. Then we compare $\mathcal{C}\psi_\alpha(X)$ and $\mathcal{C}\psi_\alpha(Y)$ when X and Y are related in some stochastic orders.

If we consider $X_1, ..., X_n$, iid non-negative continuous random variables with common cdf F, we define $X_{1:n}, ..., X_{n:n}$ as the corresponding order statistics. We study bounds for $\mathcal{C}\psi_\alpha(X_{1:n})$ and $\mathcal{C}\psi_\alpha(X_{n:n})$. These inequalities are useful to analyze, for example, lifetimes of series and parallel systems.

Dynamic cumulative Tsallis entropy

Rajesh and Sunoj [2] proposed the dynamic cumulative residual Tsallis entropy as

$$\psi_\alpha(X;t) = \frac{1}{\alpha-1}\int_t^{+\infty}\left[\bar{F}_{X_t}(x)-\bar{F}^{\alpha}_{X_t}(x)\right]dx,$$

where $X_t = [X-t|X>t]$ is the random variable describes the residual lifetime and $\bar{F}_{X_t}(x)$ is the survival function of X_t. We define the dynamic cumulative Tsallis entropy (DCTE) of a nonnegative absolutely continuous random variable X as

$$\mathcal{C}\psi_\alpha(X;t) = \frac{1}{\alpha-1}\int_0^{+\infty}\left[F_{X_{(t)}}(x)-F^{\alpha}_{X_{(t)}}(x)\right]dx,$$

where $X_{(t)} = [X \mid X \le t]$ is the random variable describes the past lifetime of system at age t and $F_{X_{(t)}}$ is the distribution function of $X_{(t)}$. It can be rewritten as:

$$\mathcal{C}\psi_\alpha(X;t) = \frac{1}{\alpha-1}\left[\tilde{\mu}(t)-\frac{\int_0^t F^{\alpha}(x)dx}{F^{\alpha}(t)}\right].$$

Also for dynamic version we prove that it can be expressed as a function of mean inactivity time. We find a relation between DCTE and the dynamic version of CE. We analyze when DCTE is increasing or decreasing and, noting an analogy with the behaviour of DCTE when X is IMIT or DMIT (Increasing in Mean Inactivity Time or Decreasing in Mean Inactivity Time), we can define a new nonparametric classes of life distributions.

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Invariance properties of SDEs with application to stochastic calculus

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The study of invariance and symmetries properties of ordinary differential equations (ODEs) and partial differential equations (PDEs) was introduced by Sophus Lie and nowaday constitutes fundamental ideas and methods in the modern theory of integrable systems for both the explicit computation of solutions to the equations and for a deep study of their qualitative behavior (see [1, 3, 2]). In this talk we plan to describe some recent generalizations of Lie techniques to the case of Brownian-motion-driven stochastic differential equations (SDEs) ([4, 5]).
The first step to reach this generalization is the introduction of a group of stochastic transformations which act both on the weak solution (X, W), where X is a continuous semimartingale in $M \subset \mathbb{R}^n$ and W is a m dimensional Brownian motion, of a SDE and on the coefficients (μ, σ) of the SDE $dX_t = \mu(X_t)dt + \sigma(X_t) \cdot dW_t$. Any transformation in this group can be characterized by a triple (Φ, B, η) composed by a diffeomorphism Φ of M, a stochastic rotation $B_t(\omega) \in O(m)$ (where $O(m)$ is the group of $m \times m$ orthogonal matrices) of the Brownian motion W and a stochastic time change η. These stochastic transformations T act both on the solution $P_T(X, W)$ and on the coefficients $E_T(\mu, \sigma)$ of the considered SDE and they have the important property of preserving the markovian property of the SDE.
Once the group of stochastic transformations is introduced, we say that T is a symmetry of the SDE (μ, σ) if it transforms any solution (X, W) of (μ, σ) into a new process $P_T(X, W)$ solution of the same SDE (μ, σ). The symmetry group of a given SDE is solution of a overdetermined first order PDE which usually can be explicitly solved ([4]).
When a SDE admits a continuous Lie group of symmetries it is possible to reduce the SDE to a new SDE with a dimension smaller then the original one. Furthermore when the Lie group is solvable it is possible to reconstruct the solution of the complete SDE from the reduced one using only composition with smooth functions and integration with respect to the time and the Brownian motion W. In other words the SDE can be reconstructed by quadratures ([5]). These kinds of reduction and reconstruction techniques have interesting applications in providing more efficient numerical methods for integrating SDEs via symmetries (see [6]).
Finally we propose some possible generalizations of the invariance properties above described to the case of Levy-driven SDEs and to infinite dimensional SDEs.

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A symbolic approach to multivariate polynomial Lévy processes

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By means of a symbolic method, a family of multivariate polynomials is introduced allowing us to deal with multivariate Lévy processes. In the univariate case, these polynomials are known as time-space harmonic polynomials. Simple closed-form expressions of some multivariate classical families of polynomials are given as well as their generalizations, the multivariate Lévy-Sheffer systems. The main advantage of this symbolic representation is the plainness of the setting which reduces to few fundamental statements but also of its implementation in any symbolic software.

Polynomial processes

Given a Lévy process $\boldsymbol{X}_t$ in $\mathbb{R}^d$, the corrisponding *polynomial process* is a family of martingales $\{Q(\boldsymbol{X}_t, t)\}$

$$E[Q(\boldsymbol{X}_t, t)|\boldsymbol{X}_s] = Q(\boldsymbol{X}_s, s) \text{ for } s \leq t$$

with $Q(\boldsymbol{x}, t)$ a multivariate polynomial such that

$$Q(\boldsymbol{x}, t) = \sum_{|\boldsymbol{k}|=0}^{m} c_{\boldsymbol{k}} \boldsymbol{x}^{\boldsymbol{k}} \text{ for } \boldsymbol{x} \in \mathbb{R}^d, c_{\boldsymbol{k}} \in \mathbb{R}. \tag{1}$$

In the univariate case, the polynomials (1) are called *time-space harmonic* polynomials and have been deeply analyzed by different authors, see [5] and references therein. For the multivariate setting, this class is included in a more general class of Markov processes, called m-polynomial processes, and introduced in [1]. Many popular models such as exponential Lévy models or affine models are covered by this setting and in [1] applications to pricing and hedging in mathematical finance are given. In particular, one looks for a computational efficient way to compute moments of all orders of $Q(\boldsymbol{X}_t, t)$ by using the martingale property. In [1], the coefficients of these polynomials are calculated by exponentiating a matrix.

A different approach is proposed in [3] where matrix-valued stochastic processes are referred to multivariate Hermite or Laguerre polynomials. In order to characterize these processes, Haar measure and zonal polynomials are involved. In particular, as coefficients of hypergeometric functions, zonal polynomials have manageable expressions only in some special case and a computational efficient way to deal with general zonal polynomials is not yet available.

*Speaker

The symbolic method

To compute the coefficients of $Q_{\boldsymbol{v}}(\boldsymbol{x},t)$, we show how to use a different method, essentially relied on the multivariate Faà di Bruno formula, a combinatorial tool to evaluate the m-th derivative of a composite function of two (or more) formal power series. The implementation of the multivariate Faà di Bruno formula has been recently optimized [2] by using a symbolic method particularly suited to manage sequence of polynomials, when the indeterminates are replaced by other polynomials. The basic device of this symbolic device is to represent a unital sequence of numbers by a symbol α, called umbra, i.e., to associate the sequence $1, a_1, a_2, \ldots$ to the sequence $1, \alpha, \alpha^2, \ldots$ of powers of α via an operator E that looks like the expectation of random variables.

The main theorem is the following closed form formula for $Q_{\boldsymbol{v}}(\boldsymbol{x},t)$: for all $\boldsymbol{v} \in \mathbb{N}_0^d$, the family of polynomials

$$Q_{\boldsymbol{v}}(\boldsymbol{x},t) = E[(\boldsymbol{x} - t.\boldsymbol{\mu})^{\boldsymbol{v}}] \in \mathbb{R}[\boldsymbol{x}] \tag{2}$$

is time-space harmonic with respect to $\{t.\boldsymbol{\mu}\}_{t\geq 0}$, a symbolic representation of the Lévy process $\boldsymbol{X}_t$. Here $\boldsymbol{\mu}$ denotes a d-tuple of umbral monomials. The coefficients of $Q_{\boldsymbol{v}}(\boldsymbol{x},t)$ can be recovered by a suitable generalization of the multinomial expansion. Then recurrence formulae are recovered easily implementable in any symbolic software. The polynomials in (**??**) form a bases for the vector space of the time-space harmonic polynomials with respect to $\{t.\boldsymbol{\mu}\}_{t\geq 0}$.

Special families of multivariate polynomials such as Hermite polynomials, Bernoulli polynomials and Euler polynomials are then recovered.

A generalization is given when $\boldsymbol{x}$ is replaced by a polynomial umbra, representing sequences of suitable multivariable polynomials. This family of polynomials $V_{\boldsymbol{v}}(\boldsymbol{x},t) = E[(\boldsymbol{\nu}_{\boldsymbol{x}} - t.\boldsymbol{\mu})^{\boldsymbol{v}}]$ is called Lévy-Sheffer system. The univariate family is studied in [4]. The associated Lévy process corresponds to a multivariate subordinator, offering a different way to study the stochastic properties of these processes.

An interesting application which deserves further deepening studies is the orthogonal property for some families of multivariate time-space harmonic polynomials. Some preliminarily results are given in [3] for Hermite and Laguerre polynomials.

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Two-boundary first exit time of Gauss-Markov processes: a biological model and asymptotics

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The first exit time (FET) problem for Gauss-Markov (GM) processes ([3]) plays a key role in the construction and development of models in a wide variety of fields, such as mathematical biology, financial markets, population dynamics and neuronal modeling (see, for instance, [6], [8], [9], [13]). In the context of mathematical biology, in [2] and recently in [5], GM processes, the first passage time (FPT) random variable and the FET through suitable boundaries were proposed to model the interaction between actin and myosin filaments responsible of the contraction in the skeletal muscles. Specifically, in [5], the steps of the myosin head are represented by the exit events from a strip. Starting from the stochastic differential equation for modeling the interaction between actin and myosin filaments, we use the time-inhomogeneous Ornstein-Uhlenbeck process for modeling phenomena subject to external time-dependent forces. By means of simulations of trajectories of the stochastic dynamics via an Euler discretization-based algorithm, we fit experimental data and determine the values of involved parameters. We propose a specific stochastic model based on the corresponding time-inhomogeneous Gauss-Markov and diffusion process evolving between two boundaries. The mean and covariance functions of the stochastic modeling process are provided taking into account time-dependent forces including the effect of an external load. We are also able to determine an accurate estimation of the probability density function (pdf) of FET from the strip by solving a system of two non singular second-type Volterra integral equations via a numerical quadrature. We provide numerical estimations of the mean of FET as approximations of the dwell-time of the proteins dynamics. The percentage of backward steps is given in agreement to experimental data.

We also briefly recall here some main results from [3], [4], [5], [10], [11] and we use them to refine theoretical and applicative aspects of GM processes in presence of two boundaries.

In [10] the two-boundary FET pdfs of GM processes are proved to be solutions of a system of continuous-kernel Volterra integral equations. The authors provided also a condition on the boundaries such that the system of two integral equations reduces to a single non-singular integral equation. In the latter case, closed form solutions for the FET density can be determined (see [4], [10]). In all other cases analytical results are rare (see for instance [1], [12], [14], [15], [16]) and the system has to be solved using numerical procedures. For this reason, informations on the asymptotic behaviour of the FET density can be useful not only for theoretical reasons but also for finite approximations in a specified validity range. We show the extension to the two-boundary case for GM processes of results on the asymptotic behaviour of FPT pdfs in the presence of single boundary obtained in [11] and of asymptotic FET pdfs of diffusion processes carried out in [7]. We determine the asymptotic behaviour of FET density in the presence of asymptotically constant boundaries and asymptotically periodic boundaries. In both cases we determine the results for the Ornstein-Uhlenbeck process and then we extend them to GM processes. Finally, we provide some application examples considering stochastic models based on the use of GM processes in the presence of two boundaries. We estimate parameters useful for approximations and we show the numerical results in tables and figures.

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Weighting approach for multiple mediators in survival analysis

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The statistical methodology based on mediation analysis can be employed in a series of application contexts. In particular in medical and epidemiological research once an exposure and an outcome of interest have been identified, the goal is often to understand the biological or mechanistic pathways that produce the effect of the exposure on the outcome. The aim of mediation analysis is to disentangle this effect as a whole into the indirect effect, i.e. the effect through intermediate variables (mediators), and the direct effect.

Mediation analysis can be performed in a series of instances by resorting to a counterfactual approach (cf. [1]) that gives more general definitions of the direct and indirect effects allowing the presence of nonlinearities and interactions between exposure and mediators in the models for the outcome.

As far as survival analysis is concerned, a mediation approach involving a single mediator was firstly proposed in [2] where an additive hazard model was employed to study the case when the outcome is in the form of time-to-event. In [3] several effect measures in survival analysis are discussed and the weighting approach in the presence of a single mediator is reported also in the survival framework. The purpose of the present work (cf. [4]) is to show how to extend to survival outcome the weighting approach for multiple mediators (cf. [3]). The main advantage of the method is its applicability in frameworks where mediators are dependent on each other. Furthermore it does not require specific models for the mediators thus reducing the problem of model incompatibility and, similarly to the other weighting approaches, it does not rely on the assumption of rare outcomes. We describe in details the mediation approach proposed stressing the underlying assumptions and

*Speaker

we describe how this method can be implemented in practice to estimate the effects. We consider two different classes of survival models: proportional hazards models and accelerated failure time models.

The results obtained in an examples of application in the epidemiological framework (cf. [5]) are then briefly examined. We first consider the role of abdominal adiposity (i.e., waist-to-hip ratio, WHR) as a potential mediator of the relationship between the Mediterranean diet (MD) and colorectal cancer i(CRC) n the Italian centres of the European Prospective Investigation into Cancer and Nutrition study. After evaluating the effect of the Italian Mediterranean Index (IMI) on WHR and of WHR on CRC risk we estimate the natural indirect effect (NIE, mediated by WHR) and the pure direct effect (PDE, unmediated) of IMI on CRC risk using mediation analyses, The results of our study suggest that the MD has a beneficial effect on CRC risk in a Mediterranean population, but its pathway is independent from abdominal obesity. Since this is the first study to investigate the mediating effect of abdominal obesity, other studies are needed to replicate this result.

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Parametrix technique for SDE solutions and Markov Chains.*

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The parametrix is a technique which comes from the theory of ODE. Now it reformulates as a continuity technique that provides a formal representation for the density of the SDE's solutions in terms of infinite series involving the density of another, simpler, Markov process. Although the method itself has been known for many years there are still many open problems. We are focused on both - technique's development and it's applications.

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Parametrix techique.

Consider the class of stochastic problems that can be solved with the parametrix method.

Suppose that on the interval $[0,1]$ there is a sequence of partitions $\Gamma_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \dots, 1\right\}$, $n = 1, 2, \dots$, and a sequence of Markov chains $X_t^{(n)}$ with discrete time and continuous state space. Chains $X_t^{(n)}$ are defined on the grid Γ_n, have initial distribution $\delta_{x_0}(\cdot)$, and the one-step transition probability has density

$$p^{(n)}\left(\frac{1}{n}, x, A\right) = P\left(X^{(n)}_{\frac{i+1}{n}} \in A | X^{(n)}_{\frac{i}{n}} = x\right) = \int\limits_A p^{(n)}_{\frac{i}{n},x}\left(\frac{1}{n}, x, z\right) dz$$

Under this condition, transition probability over n steps is also absolutely continuous with respect to the Lebesgue measure and has density $p^{(n)}(1, x_0, z)$. The problem is to find conditions under which the local limit theorem holds, i.e., conditions under which the density $p^{(n)}(1, x_0, z)$ can be approximated with a density of some diffusion process $p(1, x_0, z)$.

The Parametrix Method in the Form of McKean and Singer

A general form of SDE:

$$dX_t = b(X_t)dt + \sigma(X_t)dW_t, \quad X_0 = x; \tag{1}$$

where $b : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d$, and $\sigma : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}^d \otimes \mathbb{R}^d$.
Such equations appear in various fields, from Hamiltonian mechanics to finance.

Let us define the transition density as $\mathbb{P}(X_t \in dy | X_s = x) =: p(s, t, x, y)$. Under some appropriate conditions we assume that the existing solution $(X_t^{s,x})_{t \geq s}$ of (1) starting from x at time s has for all $t > s$ a smooth density $p(s, t, x, \cdot)$. For all $(s, x) \in [0, T] \times \mathbb{R}^d$, $t \geq s$ we introduce the following Gaussian inhomogeneous process with spatial variable frozen at given point $y \in \mathbb{R}^d$:

$$\tilde{X}_t^y = x + \int_s^t b(u, y)du + \int_s^t \sigma(u, y)dW_u.$$

Its density $\tilde{p}^y$ readily satisfies the Kolmogorov Backward equation:

$$\begin{cases} \partial_u \tilde{p}^y(u, t, z, y) + \tilde{L}_u^y \tilde{p}^y(u, t, z, y) = 0, \ s \leq u < t, z \in \mathbb{R}^d, \\ \tilde{p}^y(u, t, \cdot, y) \underset{u \uparrow t}{\to} \delta_y(\cdot), \end{cases} \tag{2}$$

where for all $\varphi \in C_0^2(\mathbb{R}^d, \mathbb{R}), z \in \mathbb{R}^d$:

$$\tilde{L}_u^y \varphi(z) = \frac{1}{2} \mathrm{Tr}\left(\sigma\sigma^*(u, y) D_z^2 \varphi(z)\right) + \langle b(u, y), D_z \varphi(z) \rangle,$$

stands for the generator of $\tilde{X}^y$ at time u. Since we have assumed the density of X to be smooth, it must satisfy the Kolmogorov forward equation for a given starting point $x \in \mathbb{R}^d$

$$\begin{cases} \partial_u p(s, u, x, z) = L_u^* p(s, u, x, z) = 0, \ s < u \leq t, z \in \mathbb{R}^d, \\ p(s, u, x, \cdot) \underset{u \downarrow s}{\to} \delta_x(\cdot), \end{cases} \tag{3}$$

where L_u^* stands for the *formal* adjoint (which is again well defined if the coefficients in (1) are smooth) of the generator of (1) which for all $\varphi \in C_0^2(\mathbb{R}^d, \mathbb{R}), z \in \mathbb{R}^d$ writes:

$$L_u \varphi(z) = \frac{1}{2} \mathrm{Tr}\left(\sigma\sigma^*(u, z) D_z^2 \varphi(z)\right) + \langle b(u, z), D_z \varphi(z) \rangle.$$

Equations (2), (3) yield the formal expansion below which is initially due to [1]

$$p(s,t,x,y) = \sum_{r=0}^{\infty} \tilde{p} \otimes H^{(r)}(s,t,x,y), \tag{4}$$

where

$$H(s,t,x,y) := (L_s - \tilde{L}_s)\tilde{p}(s,t,x,y) := (L_s - \tilde{L}_s^y)\tilde{p}^y(s,t,x,y)$$

is a "parametrix kernel",

$$f \otimes g(s,t,x,y) = \int_s^t du \int_{\mathbb{R}^d} dz f(s,u,x,z) g(u,t,z,y)$$

stands for the "convolution-type" operator with $\tilde{p} \otimes H^{(0)} = \tilde{p}, H^{(r)} = H \otimes H^{(r-1)},\ r \geq 1$ for $\tilde{p}(s,t,x,y) := \tilde{p}^y(s,t,x,y)$ - the density of the frozen process at the final point and observe it at *that specific* point.

The Parametrix Method. Discrete case

Consider a sequence of partitions $\Gamma_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \ldots, 1\right\}$, $n = 1, 2, \ldots$, and a sequence of Markov chains $X_t^{(n)}$ with discrete time and continuous state space. Chains $X_t^{(n)}$ are defined on the grid Γ_n, have initial distribution $\delta_{x_0}(\cdot)$, and the chain's dynamics is defined by recurrent relation

$$X_{\frac{i+1}{n}}^{(n)} = X_{\frac{i}{n}}^{(n)} + \frac{1}{n} b\left(X_{\frac{i}{n}}^{(n)}\right) + \frac{1}{\sqrt{n}} \varepsilon_{\frac{i+1}{n}}^{(n)}, \quad 0 \leq i \leq n-1, \quad X_0^{(n)} = x. \tag{5}$$

For each $0 > s = \frac{j}{n} < 1$ and $x, y \in \mathrm{R}^{\mathrm{d}}$ we define a Markov chain $\tilde{X}_t^{(n)} = \tilde{X}_{s,x,y}^{(n)}$. This chain is defined on the grid $\left\{\frac{j}{n}, \frac{j+1}{n}, ..., 1\right\}$ with recurrent relation

$$\tilde{X}_{\frac{i+1}{n}}^{(n)} = \tilde{X}_{\frac{i}{n}}^{(n)} + \frac{1}{n} b\,(y) + \frac{1}{\sqrt{n}} \tilde{\varepsilon}_{\frac{i+1}{n}}^{(n)}, \qquad j \leq i \leq n-1, \quad \tilde{X}_j^{(n)} = x. \tag{6}$$

We introduce discrete counterparts $H_n(t,s,x,y)$ and $\otimes_n$, of the "parametrix kernel" $H(t,s,x,y)$ and the "convolution-type" operator $\otimes$:

$$\begin{aligned} &H_n\left(\tfrac{j'}{n}, \tfrac{j}{n}, x, y\right) = \left(L^n - \tilde{L}^{n,y}\right) \tilde{p}_n^y\left(\tfrac{j'}{n}, \tfrac{j+1}{n}, x, y\right), \\ &(f \otimes_n g)\left(\tfrac{j'}{n}, \tfrac{j}{n}, x, y\right) = \textstyle\sum_{i=j}^{j'-1} \frac{1}{n} \int_{\mathbb{R}^d} f\left(\tfrac{i}{n}, \tfrac{j}{n}, x, z\right) g\left(\tfrac{j'}{n}, \tfrac{i}{n}, z, y\right) dz, \end{aligned}$$

where L^n and $\tilde{L}^{n,y}$ are infinitesimal operators of chains (5) and (6), where $\tilde{p}_n^y\left(\frac{i}{n}, x, y\right)$ is the transition density of chain (6).

The transition density of the Markov chain (5) can be represented as

$$p_n\left(\tfrac{j'}{n}, \tfrac{j}{n}, x, y\right) = \textstyle\sum_{r=0}^{j'-j} \left(\tilde{p}_n \otimes_n H_n^{(r)}\right)\left(\tfrac{j'}{n}, \tfrac{j}{n}, x, y\right). \tag{7}$$

The proximity of left-hand sides of representations (4) and (7) can be established with a detailed analysis of the series. One can show that

$$\tilde{p} \approx \tilde{p}_n, \quad H^{(r)} \approx H_n^{(r)}, \quad \otimes \approx \otimes_n.$$

Applications (A. Kozhina).

1. [2] We study the sensitivity of the densities of non degenerate diffusion processes and related Markov Chains with respect to a perturbation of the coefficients. Natural applications of these results appear in models with misspecified coefficients or for the investigation of the weak error of the Euler scheme with irregular coefficients.

For a fixed given deterministic final horizon $T > 0$ let us consider the "perturbed" version of the non-degenerate SDE:

$$dX_t^{(\varepsilon)} = b_\varepsilon(t, X_t^{(\varepsilon)})dt + \sigma_\varepsilon(t, X_t^{(\varepsilon)})dW_t,\ t \in [0, T]. \tag{8}$$

Coefficients $b, b_\varepsilon : [0,T] \times \mathbb{R}^d \to \mathbb{R}^d,\ \sigma, \sigma_\varepsilon : [0,T] \times \mathbb{R}^d \to \mathbb{R}^d \otimes \mathbb{R}^d$ are in some sense meant to be close when ε is small. Under some assumptions one can prove that *for a fixed $\varepsilon > 0$, a final deterministic time horizon $T > 0$ and $q > d$ there exist $C := C(q) \geq 1, c := c(q) \in (0, 1]$ s.t. for all $0 \leq s < t \leq T, (x, y) \in (\mathbb{R}^d)^2$:*

$$p_c(t-s, y-x)^{-1}|(p - p_\varepsilon)(s,t,x,y)| \leq C\Delta_{\varepsilon,\gamma,q},$$

where $p(s,t,x,.)$, $p_\varepsilon(s,t,x,.)$ respectively stand for the transition densities at time t of equations (1), (8)*, starting from x at time s.*

$$p_c(u,z) := \frac{c^{d/2}}{(2\pi u)^{d/2}} \exp(-c\frac{|z|^2}{2u})$$

and $\Delta_{\varepsilon,b,\infty} := \sup_{(t,x)\in[0,T]\times\mathbb{R}^d}\{|b(t,x) - b_\varepsilon(t,x)|\},\ \forall q \in (1, +\infty),\ \Delta_{\varepsilon,b,q} := \sup_{t\in[0,T]} \|b(t,.) - b_\varepsilon(t,.)\|_{L^q(\mathbb{R}^d)},\ \Delta_{\varepsilon,\sigma,\gamma} := \sup_{u\in[0,T]} |\sigma(u,.) - \sigma_\varepsilon(u,.)|_\gamma$, for $\gamma \in (0, 1]$, $|.|_\gamma$ stands for a usual Hölder norm in space $C_b^\gamma(\mathbb{R}^d, \mathbb{R}^d \otimes \mathbb{R}^d)$. For $q \in (1, +\infty]$, $\Delta_{\varepsilon,\gamma,q} := \Delta_{\varepsilon,\sigma,\gamma} + \Delta_{\varepsilon,b,q}$.
The same result holds for Markov Chains.

2. [3], [4] Weak error estimation. We consider the cases of Hölder continuous coefficients for parametrix applications to SDE's solutions. Under mentioned assumptions the weak error associated with the Euler scheme of diffusion processes has been studied. Also the case of the degenerate diffusions are going to be presented.

Technique's development (A. Markova).

We consider the diffusion process and its approximation by a Markov chain with increasing trends. The usual parametrix method is not applicable because these models have unbounded trends. We describe a procedure that allows to exclude increasing trend and move to stochastic differential equation with bounded trend and diffusion coefficients.

We consider the SDE (1) in the following form:

$$\begin{aligned} dY_t &= \{F(t,Y_t) + m(t,Y_t)\}\, dt + \sigma(t,Y_t)dW_t, \\ Y_0 &= x_0 \in R^d,\ 0 \leq t \leq T, \end{aligned} \tag{9}$$

where $F(t,y)$ is nonlinear d-dimentional vector-functions, $m(t,y)$ is bounded d-dimentional vector-functions, W_t – d-dimentional Wiener process, the corresponding ODE

$$\frac{dx}{dt} = F(t,x) \tag{10}$$

and transformation $G : [0,T] \times \mathbb{R}^d \to [0,T] \times \mathbb{R}^d$ defined as

$$G(t, \mathbf{x}) = (t, \mathbf{g}(t; t_0, \mathbf{x}))\,,$$

where

$$\mathbf{g}(t; t_0, \mathbf{x}) = (\mathbf{g}_1(t; t_0, \mathbf{x}), \ldots, g_d(t; t_0, \mathbf{x}))$$

is a solution of ODE (10) satisfying the initial condition $\mathbf{g}(t_0; t_0, \mathbf{x}) = x$.

We set $t_0 = 0$ and consider a new process:

$$\tilde{Y}_t = \mathbf{g}^{-1}(0; t, Y_t), \quad \tilde{Y}_0 = Y_0 = x_0, \tag{11}$$

where Y_t is the solution of SDE (9).

Theorem 1 [5]. *Under some suitable assumptions the process* $\tilde{Y}_t$ *defined in* (11) *is a diffusion process with stochastic differential*

$$d\tilde{Y}_t = \tilde{m}\left(t, \tilde{Y}_t\right) dt + \tilde{\sigma}\left(t, \tilde{Y}_t\right) dW_t,$$

where

$$\tilde{m}(t,y) = \mathbf{g}_*^{-1}(t;0,y)\left\{ m(t,\mathbf{g}(t;0,y)) + \frac{1}{2}\sum_{i,j=1}^{d} c_{ij}(t,0,y)\sum_{p=1}^{d}\sigma_{ip}(t,\mathbf{g}(t;0,y))\sigma_{jp}(t,\mathbf{g}(t;0,y))\right\},$$

$$\tilde{\sigma}(t,y) = \mathbf{g}_*^{-1}(t;0,y)\sigma(t,\mathbf{g}(t;0,y)).$$

Moreover, all the components of the trend $\tilde{m}(t,y)$ *are bounded.*

Note that in linear case [6] we have

$$\tilde{m}\left(t,\tilde{Y}(t)\right) = \Phi^{-1}(t) m\left(t,\Phi(t)\tilde{Y}(t)\right)$$

$$\tilde{\sigma}\left(t,\tilde{Y}(t)\right) = \Phi^{-1}(t)\sigma\left(t,\Phi(t)\tilde{Y}(t)\right),$$

where $\Phi(t)$ is a fundamental matrix corresponding to (10).

We consider also a grid (non-uniform in general) $0 = t_0^n < t_1^n < ... < t_n^n = T$ and a Markov chain (5) in the following form:

$$X(t_{k+1}^n) = X(t_k^n) + h_k^n\{F(t_k^n, X(t_k^n)) + m(t_k^n, X(t_k^n))\} +$$

$$+\sqrt{h_k^n}\sigma(t_k^n, X(t_k^n))\varepsilon(t_{k+1}^n),$$

$$X(0) = x \in \mathbb{R}^d, k = 0, ..., n-1, h_k^n = t_{k+1}^n - t_k^n,$$

where $F(t_k^n, X(t_k^n))$ is nonlinear component of trend, $m(t_k^n, X(t_k^n)$ is bounded component of trend.

Consider a difference equation

$$\frac{\widehat{\mathbf{g}}(t_{k+1}^n;0,x) - \widehat{\mathbf{g}}(t_k^n;0,x)}{h_k^n} = F(t_k^n, \widehat{\mathbf{g}}(t_k^n;0,x)), \quad \widehat{\mathbf{g}}(0;0,x) = x,$$

where $\widehat{\mathbf{g}}(t_k^n;0,x)$ are Euler's broken lines on the grid $0 = t_0^n < t_1^n < \cdots < t_n^n = T$ starting from x at time 0 and constructed for solution $\mathbf{g}(t;0,x)$ of equation $\dot{\mathbf{x}} = F(t,\mathbf{x}), \mathbf{g}(0;0,x) = x$.

Theorem 2 [5]. *Under some suitable assumptions* $\widetilde{X}(t_k^n) = \widehat{\mathbf{g}}^{-1}(0;t_k^n, X(t_k^n))$ *is a Markov chain*

$$\widetilde{X}(t_{k+1}^n) = \widetilde{X}(t_k^n) + h_k^n\widetilde{m}(t_k^n, \widetilde{X}(t_k^n), \varepsilon(t_{k+1}^n)) + \sqrt{h_k^n}\widetilde{\sigma}(t_k^n, \widetilde{X}(t_k^n), \varepsilon(t_{k+1}^n))\varepsilon(t_{k+1}^n)$$

with bounded coefficients

$$\widetilde{m}(t_k^n, \widetilde{X}(t_k^n), \varepsilon(t_{k+1}^n)) = \left(\int_0^1 \widehat{\mathbf{g}}_*^{-1}(0;t_{k+1}^n, \Psi_u(\widetilde{X}(t_k^n), \varepsilon(t_{k+1}^n)))du\right) m(t_k^n, \widehat{\mathbf{g}}(t_k^n;0,\widetilde{X}(t_k^n))),$$

$$\widetilde{\sigma}(t_k^n, \widetilde{X}(t_k^n), \varepsilon(t_{k+1}^n)) = \left(\int_0^1 \widehat{\mathbf{g}}_*^{-1}(0;t_{k+1}^n, \Psi_u(\widetilde{X}(t_k^n), \varepsilon(t_{k+1}^n)))du\right) \sigma(t_k^n, \widehat{\mathbf{g}}(t_k^n;0,\widetilde{X}(t_k^n))).$$

Note that for $F(t_j^n, \cdot) = b(t_j^n)$ and $\widehat{\mathbf{g}}_*^{-1}(t_{k+1}^n;0,\Psi_u(\widetilde{X}(t_k^n))) = \prod_{j=0}^{k}[I + h_{j+1}^n b(t_j^n)]^{-1}$ we have the same results as in [6].

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Properties for generalized cumulative measures of information*

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The (differential) Shannon entropy based on the probability density function is a key information measure with applications in different areas. Some alternative information measures have been proposed: the cumulative residual entropy (based on the survival function) and the cumulative past entropy (based on the distribution function). Recently some extensions of these measures have been studied. Here we obtain some properties for the generalized cumulative past entropy and its dynamic version. We also study these measures in coherent reliability systems. Moreover, we define a new closely related generalized cumulative Kerridge inaccuracy measure.

Definition and some properties of GCPE and DGCPE

The generalized cumulative residual entropy (GCRE) was defined in Psarrakos and Navarro [4] as

$$\mathcal{E}_n(X) = \int_0^{+\infty} \overline{F}_X(x) \frac{[\Lambda_X(x)]^n}{n!} dx \tag{1}$$

for $n = 0, 1, 2, \ldots$, where $\Lambda_X(x) = -\log \overline{F}_X(x)$ and where, by convention, $0(\log 0)^n = 0$ for $n = 1, 2, \ldots$.

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In analogy with (1), Kayal [2] defined the *generalized cumulative past entropy* (GCPE) of a non-negative random variable X as

$$\mathcal{CE}_n(X) = \int_0^{+\infty} F_X(x) \frac{[T_X(x)]^n}{n!} dx, \qquad \text{for } n = 0, 1, 2, \dots \tag{2}$$

where $T_X(t) = -\log F_X(t)$. Kayal [2] also introduced the *dynamic generalized cumulative past entropy* (DGCPE), as

$$\mathcal{CE}_n(X;t) = \frac{1}{n!} \int_0^t \frac{F_X(x)}{F_X(t)} \left[-\log \frac{F_X(x)}{F_X(t)} \right]^n dx, \qquad t > 0 : \ F_X(t) > 0, \tag{3}$$

for $n = 0, 1, 2, \dots$.

We prove that, under some assumptions, $\mathcal{CE}_n(X;t)$ for a fixed n uniquely determines $F_X(t)$.

GCPE of coherent systems

Let T is the lifetime of a coherent system with m identically distributed (i.d.) components, then its distribution function F_T can be written as

$$F_T(t) = q(F_X(t)),$$

where F_X is the common distribution of the component lifetimes and where q is a *distortion function*, that is, $q : [0,1] \to [0,1]$ is a continuous increasing function such that $q(0) = 0$ and $q(1) = 1$. The function q depends on the structure of the system and on the copula of the component lifetimes. Then the GCPE of such system can be written as

$$\mathcal{CE}_n(T) = \frac{1}{n!} \int_0^1 \frac{\phi_n(q(u))}{f_X(F_X^{-1}(u))} du, \tag{4}$$

where $\phi_n(u) := u[-\log(u)]^n \geq 0$ (with $\phi(0) = \phi(1) = 0$) and F_X^{-1} is the inverse function of F_X. We analyze the relationship between $\mathcal{CE}_n(T)$ and $\mathcal{CE}_n(X_1)$, we obtain bounds for $\mathcal{CE}_n(T)$ in terms of $\mathcal{CE}_n(X_1)$ and we compare GCPE of two systems.

Generalized cumulative Kerridge inaccuracy

Given two random lifetimes X and Y having distribution functions F_X and F_Y defined on $(0, \infty)$, recall the definition of the cumulative Kerridge inaccuracy, given by Di Crescenzo and Longobardi [1]:

$$K[F_X, F_Y] = -\int_0^{+\infty} F_X(u) \log F_Y(u) du. \tag{5}$$

In analogy with (5), let us now introduce the *generalized cumulative Kerridge inaccuracy of order* n defined as

$$K_n[F_X, F_Y] = \frac{1}{n!} \int_0^{+\infty} F_X(u)[-\log F_Y(u)]^n du. \tag{6}$$

We obtain a connection between our measure of discrimination, $\mathcal{CE}_n(X)$ and some stochastic orders.

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Asymptotic results for a multivariate version of the alternative fractional Poisson process

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Fractional Poisson processes are widely studied in the literature by considering a version of some known equations (for the state probabilities) with fractional derivatives and/or fractional difference operators. Moreover they can be represented in terms of non-fractional Poisson processes with independent random time-changes.

A multivariate (space and/or time) fractional Poisson process

$$\{\underline{N}^{\eta,\nu}(t) = (N_1^{\eta,\nu}(t), \dots, N_m^{\eta,\nu}(t)) : t \geq 0\}$$

was recently defined in [2] by considering a common independent random time-change (in terms of the stable subordinator and/or its inverse) for a m-dimensional vector of independent non-fractional Poisson processes $\{(N_1(t), \dots, N_m(t)) : t \geq 0\}$, with intensities $\lambda_1, \dots, \lambda_m > 0$. In particular it was proved that, for each fixed $t \geq 0$, for all integers $k_1, \dots, k_m \geq 0$ we have

$$P\left(N_1^{\eta,\nu}(t) = k_1, \dots, N_m^{\eta,\nu}(t) = k_m \Big| \sum_{i=1}^m N_i^{\eta,\nu}(t) = \sum_{i=1}^m k_i\right) = \frac{(\sum_{j=1}^m k_j)!}{k_1! \cdots k_m!} \prod_{i=1}^m \left(\frac{\lambda_i}{\sum_{j=1}^m \lambda_j}\right)^{k_i}$$

(see a part of the proof of Proposition 4 in [2]); thus we have a conditional multinomial distribution of the components of given their sum, which does not depend on t and on the fractional parameters $\eta, \nu \in (0, 1]$. In this paper we consider another multivariate process

$$\{\underline{M}^{\nu}(t) = (M_1^{\nu}(t), \dots, M_m^{\nu}(t)) : t \geq 0\}$$

with the same conditional distributions of the components given their sums, but we change the distribution of the sums of the components. More precisely, if we consider the alternative fractional Poisson process $\{\hat{N}_{\nu,\lambda}(t) : t \geq 0\}$ in [3] (see Section 4) for some $\lambda > 0$, then we assume that, for each fixed $t \geq 0$, the random variable $\sum_{i=1}^m M_i^{\nu}(t)$ is distributed as $\hat{N}_{\nu,\sum_{i=1}^m \lambda_i}(t^{\nu})$. In this way, for $m = 1$, we recover the alternative fractional (univariate) Poisson process in [1].

The theory of large deviations gives an asymptotic computation of small probabilities on exponential scale (see e.g. [4] as a reference on this topic). In this paper we prove the two following results by applying the Gärtner Ellis Theorem (see e.g. Theorem 2.3.6 in [4]).

*Speaker

Proposition 1 (Large deviations) *The family of random variables* $\left\{\frac{\underline{M}^\nu(t)}{t} : t > 0\right\}$ *satisfies the large deviation principle with speed* $v_t = t$ *and good rate function* I_{LD} *defined by*

$$I_{\mathrm{LD}}(\underline{x}) := \begin{cases} \sum_{i=1}^m x_i \log\left(\frac{\nu^\nu}{\lambda_i}\frac{x_i}{(s(\underline{x}))^{1-\nu}}\right) - \nu s(\underline{x}) + (\sum_{i=1}^m \lambda_i)^{1/\nu} & \text{if } \underline{x} \in [0,\infty)^m \\ \infty & \text{otherwise,} \end{cases}$$

where $\underline{x} = (x_1, \ldots, x_m)$ *and* $s(\underline{x}) := \sum_{i=1}^m x_i$ *(we recall that* $0 \log 0 = 0$ *and* $0 \log 0/0 = 0$*).*

Proposition 2 (Moderate deviations) *For all families of positive numbers* $\{a_t : t > 0\}$ *such that* $a_t \to 0$ *and* $ta_t \to \infty$ *as* $t \to \infty$*, the family of random variables* $\left\{\sqrt{ta_t} \cdot \frac{\underline{M}^\nu(t) - \mathbb{E}[\underline{M}^\nu(t)]}{t} : t > 0\right\}$ *satisfies the large deviation principle with speed* $1/a_t$ *and good rate function* I_{MD} *defined by*

$$I_{\mathrm{MD}}(\underline{x}) := \sup_{\underline{\theta} \in \mathbb{R}^m} \left\{\langle \underline{\theta}, \underline{x}\rangle - \frac{1}{2}\langle \underline{\theta}, C\underline{\theta}\rangle\right\},$$

where $\underline{x} = (x_1, \ldots, x_m)$ *and the matrix* $C = (c_{jk}^{(\nu)})_{j,k \in \{1,\ldots,m\}}$ *is defined by*

$$c_{jk}^{(\nu)} := \begin{cases} \frac{1}{\nu}\left(\frac{1}{\nu} - 1\right)(\sum_{i=1}^m \lambda_i)^{1/\nu - 2}\lambda_j\lambda_k & \text{if } j \neq k \\ \frac{1}{\nu}\left(\frac{1}{\nu} - 1\right)(\sum_{i=1}^m \lambda_i)^{1/\nu - 2}\lambda_j^2 + \frac{1}{\nu}(\sum_{i=1}^m \lambda_i)^{1/\nu - 1}\lambda_j & \text{if } j = k. \end{cases}$$

Moreover, if C *is invertible, then we have* $I_{\mathrm{MD}}(\underline{x}) = \frac{1}{2}\langle \underline{x}, C^{-1}\underline{x}\rangle$*.*

We conclude with some remarks.

1) Proposition **??** is a generalization of a known result (see Proposition 4.1 in [1]) concerning the univariate case $m = 1$.
2) If $\nu = 1$, then the components of $\{\underline{M}^\nu(t) = (M_1^\nu(t), \ldots, M_m^\nu(t)) : t \geq 0\}$ are independent and C is a diagonal matrix. So, as one can expect, the rate functions are the sums of the "marginal rate functions", namely

$$I_{\mathrm{LD}}(\underline{x}) := \sum_{i=1}^m I_{\mathrm{LD}}^{(i)}(x_i), \text{ where } I_{\mathrm{LD}}^{(i)}(x_i) = \begin{cases} x_i \log\left(\frac{x_i}{\lambda_i}\right) - x_i + \lambda_i & \text{if } x_i \geq 0 \\ \infty & \text{otherwise,} \end{cases}$$

and

$$I_{\mathrm{MD}}(\underline{x}) := \sum_{i=1}^m I_{\mathrm{MD}}^{(i)}(x_i), \text{ where } I_{\mathrm{MD}}^{(i)}(x_i) = \frac{x_i^2}{2\lambda_i}.$$

3) If we take $a_t = 1$ in Proposition **??** (so only $ta_t \to \infty$ as $t \to \infty$ holds), then the family of random variables converges weakly to the centered Normal distribution with covariance matrix C (as $t \to \infty$).

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Fractional equilibrium distributions, fractional probabilistic Taylor's and mean value theorems*

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Taylor's theorem is the most important result in differential calculus since it gives a sequence of approximations of a differentiable function in the neighborhood of a given point by polynomials with coefficients depending only on the derivatives of the function at that point. Therefore, given the derivatives of a function at a single point, it is possible to describe the behavior of the function at nearby points. At a later time fractional Taylor series and mean value theorems have been introduced with the idea of approximating non-integer power law functions (cf. for example [6] and [9]). Among others, they have been used to study a fractional conservation of mass ([7] and [10]), a fractional order model for HIV infection [8] and a fractional order model for MINMOD Millennium in order to estimate insulin sensitivity in glucose–insulin dynamics [1]. Probabilistic extensions of both theorems have been developed too (cf. [2], [4] and [5]). Inspired by such improvements, in [3] we unify these two approaches by presenting a fractional probabilistic Taylor's theorem and a fractional probabilistic mean value theorem. We first introduce the nth-order fractional equilibrium distribution by means of the Weyl fractional integral and study its main properties. Specifically, we show a characterization result by which the nth-order fractional equilibrium distribution is identical to the starting distribution if and only if it is exponential, this generalizing the classical case. The nth-order fractional equilibrium density is then used to prove a fractional probabilistic Taylor's theorem based on derivatives of Riemann-Liouville type. A fractional analogue of the probabilistic mean value theorem is thus developed for pairs of nonnegative random variables ordered according to the survival bounded stochastic order. We also provide some related results, both involving the normalized moments and a fractional extension of the variance, and a formula of interest to actuarial science. In conclusion, we discuss the probabilistic Taylor's theorem based on fractional Caputo derivatives.

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Epidemic on networks with stochastic infection rates

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Networks represent the backbone of many complex systems and they appear for a large variety of real-world systems in ecology, epidemiology and neuroscience. In particular, there is a close relationship between epidemiology and network theory, indeed diffusion of infectious disease, between interacting agents, strongly depends by the intrinsic characteristics of the population contact network. The epidemic models describe well a wide range of others phenomena, like social behaviors, opinion spreading, diffusion of information and computer viruses. In this framework, understanding what will be the behavior of such processes in the long term is of fundamental importance.

With this regard, we study the diffusion of an SIS-type epidemics on a network, under the presence of a random environment that enters in the definition of the infection rates of the nodes. Accordingly, we model the infection rates in the form of independent stochastic processes. To analyze the problem, we apply a mean field approximation, which allows to get a stochastic differential equations for the probability of infection in each node, and classical tools about stability, which require to find suitable Lyapunov's functions. We prove that the unique global solution remains within $(0,1)^N$ whenever it starts from this region. Then, we concentrate on the long time behavior of the solution. We find conditions which guarantee, respectively, extinction and stochastic persistence of the epidemics. We show that there exist two regions, given in terms of the coefficients of the model, one where the system goes to extinction almost surely, and the other where it is stochastic permanent. These two regions are, unfortunately, not adjacent, as there is a gap between them, whose extension depends on the specific level of noise. In this last region, we perform numerical analysis to suggest the true behavior of the solution.

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Phase transition for the Maki-Thompson rumor model on a small-word network *

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We consider the Maki-Thompson model for the stochastic propagation of a rumour within a population. We extend the original hypothesis of homogenously mixed population by allowing for a small-world network embedding the model. This structure is realized starting from a k-regular ring and by inserting, in the average, c additional links in such a way that k and c are tuneable parameter for the population architecture. We prove that this system exhibits a transition between regimes of localization (where the final number of stiflers is at most logarithmic in the population size) and propagation (where the final number of stiflers grows algebraically with the population size) at a finite value of the network parameter c. A quantitative estimate for the critical value of c is obtained via extensive numerical simulations.

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Extensions of the Yule Model

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We propose two generalizations of the classical Yule model for macroevolution in the context of network growth. The first accounts for the possibility of detachment of links and the second introduces persistent memory in the system.

Real-world networks may exhibit detachment phenomenon determined by the cancelling of previously existing connections. We discuss a tractable extension of Yule model to account for this feature. Analytical results are derived and discussed both asymptotically and for a finite number of links. Comparison with the original model is performed in the supercritical case. The first-order asymptotic tail behavior of the two models is similar but differences arise in the second-order term. We explicitly refer to World Wide Web modeling and we show the agreement of the proposed model on very recent data.

Regarding the second generalization, nonlinearity is introduced by replacing the linear birth process governing the growth of the in-links of each specific webpage with a fractional nonlinear birth process with completely general birth rates. Among the main results we derive the explicit distribution of the number of in-links of a webpage chosen uniformly at random taking separated the contribution to the asymptotics and the finite time correction. The mean value of the latter distribution is also calculated explicitly in the most general case. Furthermore, in order to show the usefulness of our results, we particularize them in the case of specific birth rates giving rise to a saturating behaviour, a property that is often observed in nature.

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*Speaker

SPADE: Spike Pattern Detection and Evaluation in Massively Parallel Spike Trains

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Introduction

The cerebral cortex is composed by millions of neuronal cells. Such cells communicate via the emission of fast electrical impulses (spikes). The connections between neuronal cells are called synapses and they form a highly interwoven network. Cell assemblies [1], i.e. interacting groups of neurons, were suggested as the building blocks of information processing in the brain. Spatio-temporal spike patterns (STPs) are widely considered a signature of such cell assemblies in electrophysiological recordings [2,3]. Modern techniques allow to record the spiking activity of hundreds of neurons simultaneously, thereby increasing the chances to observe neurons involved in assemblies expressed by spatio-temporal spike patterns.

Method

We recently developed [4] a statistical method, named SPADE, to detect STPs in massively parallel spike data (MPST), i.e. on the order of 100 or more neurons. The method is able to deal with the huge number of patterns occurring in such high-dimensional data by employing a combination of frequent item set mining [5] and a stability analysis algorithm [6] to efficiently extract repeating precise sequences of spikes from the data. In order to evaluate the statistical significance of such pattern candidates under the null hypothesis that neurons fire independently from each other we use a Monte-Carlo approach. In order to cope with the multiple testing problem presented by testing for the statistical significance of so many patterns we then reduce the dimensionality of the space of the pattern candidate. As a result, the method extracts STPs that occur significantly more times than chance events.

Results

We evaluate our method on test MPST data generated by stochastic point process simulations. The performance of the method (in terms of false positive and false negative detections) depends on

*Speaker

a variety of parameters, such as the number of STP occurrences, the number of neurons involved in each pattern, and the firing rates of the neurons. We validate the performance of SPADE taking into account various features of experimental data, such as non-stationary spiking rate in time or inhomogeneity across neurons, and inter-spike interval regularity. To test the robustness of our methods, we use a battery of different artificial datasets which replicate such features. The results of our validation show that SPADE is suited for the analysis of STPs in massively parallel spike trains thereby offering the possibility to relate such patterns to behavior and investigate their computational relevance.

Acknowledgments

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Exponential Models by Orlicz Spaces and Applications

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Statistical exponential models built on Orlicz spaces arise in several fields, such as differential geometry, algebraic statistics and information theory. To our knowledge, their application to finance has not been investigated yet, although the use of Orlicz spaces in utility maximization and in risk measure theory is known (see, e.g., Biagini and Frittelli (2008)).

The aim of our work is to provide a first investigation in this direction, particularly concerning maximal exponential models, using some recent results contained in Santacroce, Siri and Trivellato (2016).

The theory of non-parametric maximal exponential models centered at a given positive density p starts with the work by Pistone and Sempi (1995). In that paper, and subsequently in Cena and Pistone (2007), using the Orlicz space associated to an exponentially growing Young function, the set of positive densities is endowed with a structure of exponential Banach manifold. Such a manifold setting turns out to be well-suited for applications in physics as some recent papers show (see, e.g., Pistone (2013)).

One of the main result in Cena and Pistone (2007) states that any density belonging to the maximal exponential model centered at p is connected by an *open* exponential arc to p and viceversa, (by *open*, we essentially mean that the two densities are not the extremal points of the arc). In Santacroce, Siri and Trivellato (2016), the equivalence between the equality of the maximal exponential models centered at two (connected) densities p and q and the equality of the Orlicz spaces referred to the same densities is proved.

This work is a natural continuation of the previous one and, moreover, it includes applications to finance. It is essentially composed of two parts. In the first part we give new theoretical results concerning exponential models which can be useful to understand their underlying geometrical structure. We show that the equality of Orlicz spaces referred to connected densities is equivalent to the existence of a transport mapping between the corresponding coniugate spaces. Furthermore, we deal with densities projections on sub-σ-algebras and relate them to exponential sub-models, proving that exponential connection by arc is stable with respect to projections and that projected densities belong to suitable sub-models.

The second part of the work addresses the classical problem of exponential utility maximization in incomplete markets. In the literature, the study of the optimal solution of the corresponding dual problem is often related to the so-called Reverse Hölder condition (see Delbaen et al. (2002)). Assuming this condition, we show that the minimal entropy martingale density measure belongs to a maximal exponential model. This reflects on the solution of the primal problem, which translates

[*]Speaker

into a smoothness condition on the optimal wealth process. We use the exponential connection by arcs to slightly improve some well-known duality results and we do it by exploiting the equivalent conditions proved in Santacroce, Siri and Trivellato (2016). Finally, our results are illustrated in some classical examples of financial markets taken from the literature.

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Pricing American options in Heston-type models

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We study analytical results linking the price function of an American option to the associated obstacle parabolic problem. Our approach is based on variational inequalities and extends recent results of Daskalopouos and Feehan (2011)[1].

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*Speaker

A new firing paradigm for Integrate and Fire stochastic neuronal models: an excursion approach.

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Integrate and Fire (IF) models are among the most used descriptions of the single neuron membrane potential dynamics. However, in many instances, data are not consistent with a relevant feature of such models. We refer to the absorbing assumption imposed to the membrane potential at the threshold level, i.e. the firing condition. The presence of the absorbing boundary is often disregarded, introducing important errors in the estimation procedure [1, 2].
Mainly motivated by statistical purposes, we propose here a new definition of the firing time of a neuron. The new model relaxes the absorption condition and allows crossing of the threshold without firing. We assume that a spike is generated as the membrane potential reaches a fixed threshold level and remains above it for a sufficiently long time interval. The firing time is defined as

$$H = \inf \{ t \geq 0 \mid (t - g_t) \cdot \mathbf{1}_{V_t \geq S} \geq \Delta \} , \tag{1}$$

where V_t is the neuron membrane potential, $\mathbf{1}_A$ is the indicator function of the set A, Δ is the time window that the process has to spend above the threshold S and $\forall t$

$$g_t = \sup\{s \leq t; V_s = S\} \tag{2}$$

In order to derive the Laplace transform of H for a general diffusion process V_t, we study H in the framework of Ito excursion theory [3]. In particular, we review the question of the first excursion of a diffusion process V_t above a certain level S with length strictly greater than Δ. Main references related to this problem are [4] and [5]. Finally, we specialize our results for the three diffusion processes thst appear in (Leaky) Integrate and Fire neuronal models: Wiener, Ornstein-Uhlenbeck and Feller processes.

The results just discussed are seminal to approach the estimation of the parameters for this new familiy of neural models with a MDE method [6].

The ultimate aim of this proposal is the validation of the new model, that has to be perform through the comparison of the model with intracellular recording data.

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On some fractional stochastic differential equations and applications

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In the last two decades, stochastic models essentially based on fractional differential equations have been extensively studied due to large number of experimental results revealing some delays and dependences in resultant dynamics ([8],[16],[18]). These kind of models turn out very useful to describe complex systems subject to random correlated fluctuations and jumps, and to investigate all phenomena affected by memory effects as, for instance, adaptation behaviors, multi-time scales and long-range dynamics in many fields of applications ([2], [17]).

There are different ways to model these phenomenological evidences. One is to construct fractional models based on stochastic differential equations including the fractional Brownian motion (FBM) $B^H(t)$ ([13]), indexed by the Hurst parameter $H \in (0,1)$. FBM is a zero mean Gaussian process with covariance function

$$\mathbb{E}[B^H(t)B^H(s)] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right).$$

For $H = 1/2$, it is the Brownian motion (BM). For $H \neq 1/2$, FBM has stationary increments; it is self-similar but not Markovian and nor a semimartingale. However, the stochastic differential equation (SDE) driven by FBM can be considered:

$$dX(t) = a(t,X)dt + b(t,X)dB^H(t) \tag{1}$$

specifying the integral with respect to FBM $B^H(t)$. To do this, in literature, there are some different approaches based on the pathwise Riemann-Stieltjes method ([5]), on regularization techniques and rough paths ([14]), on the stochastic calculus of variations or Malliavin calculus ([1], [6]). Here, along the lines of [7], starting from fractional Ornstein-Uhlenbeck (FOU) processes, one of them defined as solution of (1) with constant coefficients, the other one constructed by means a specialized Doob transformation, we focus the attention on processes related to SDE (1) with a more general kind of coefficients, also exploiting different integral approaches. The theoretical extensions and/or specializations of such kind of processes allow to provide fractional models able to include not only the effect of time-dependent inputs in stochastic dynamics, as, for instance, in neuronal dynamics ([3],[4]), but also to describe more complex dynamics such as the firing activity of coupled neurons and neuronal networks.

Another parallel possibility is to consider models based on SDEs having the form ([9])

$$dX(t) = a(t,X)dL(t) + b(t,X)dB(L(t)) \tag{2}$$

where $L(t)$ is a path-continuous independent random time-change. In this case the memory introduced is semi-Markovian [12], i.e., $E(X(t)|\mathcal{F}_s) = E(X(t)|X(s), \gamma(s))$ where $\gamma(s)$ is the sojourn time

*Speaker

in the current position of X. It is worth recalling that the connection between PDEs (Fokker-Planck equations) and the SDE (2) is known in this situation. The governing PDE here is, under suitable assumptions on L, the fractional counterpart of the original one, in which a first-order time derivative is replaced by the time-fractional order derivative ([9]). Results in this direction are, for instance, those related to continuous-time random walks (CTRWs) and time-changed limit processes ([11, 15])). This direction lead to different fractional models which can be useful to accomplish investigation strategies and simulation techniques.

Moreover, again in the context of correlated neuronal models, we recall that the additive white noise term in the classical stochastic Leaky-Integrate and Fire (LIF) model is often replaced by a colored noise term ([10]); this means that an Ornstein-Uhlenbeck process is involved in the stochastic differential equation in place of BM. With this kind of models, suitable comparisons can be done.

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Vine copula modeling of high-dimensional inputs in uncertainty quantification problems

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The quantification of uncertainty (UQ) in the response of a system subject to a stochastic input requires to build a map of the input-output relationship, and to study how the components/parameters of the input influence the output.

Advanced UQ methods based on spectral decomposition, such as polynomial chaos expansions (PCE, [1]), accomplish this task under the assumption that the components of the input are statistically independent, or that they can be mapped onto independent variables by means of isoprobabilistic transformations like Rosenblatt or Nataf [2]. Such transformations, however, are in general difficult to compute, both analytically and numerically, especially in large dimensions.

In this contribution we propose an effective approach to model the input's dependence structure (copula) via vine copulas. Vine copulas consist of a factorization of a joint copula into pair copulas of its components [3, 4]. The advantage of this approach is two-fold: it grants great flexibility in modeling the pairwise dependencies of the data, while at the same time providing a natural framework to transform the input model into the independent unit hypercube via the Rosenblatt transform. The latter can be used to build a map of the input onto the output by, e.g., PCE.

We further investigate how to embed this approach into surrogate-based frameworks for UQ. Specifically, once the vine model of the input data is defined, it can be used to create space-filling samples of its joint distribution (e.g. with latin hypercube sampling or pseudorandom series), via the inverse Rosenblatt transform. The resulting sample enables one to perform a large class of UQ analyses (e.g. reliability analysis or propagation by polynomial chaos expansions) at limited computational cost.

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*Speaker

Polynomial representations of Bayesian network classifiers*

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We construct polynomial representations of the decision functions associated with classifiers induced by probabilistic graphical models and in particular Bayesian network classifiers. For every fixed-structure Bayesian network classifier we link the topology of the network with a particular class of polynomials which exactly represent the decision functions compatible with the conditional independence statements depicted by the Bayesian network. We show how this polynomial representation can be useful to study some expressive problems of such classifiers (e.g. to bound the fraction of decisions representable by a given model) and we study the family of generated polynomials within the framework of algebraic statistics.

Inverse first passage time problem for a diffusion process constrained by two boundaries

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Several modeling problems describe the dynamics of the variables of interest via suitable stochastic processes constrained by boundaries. Often the focus is on the first passage time (FPT) of Markov processes through constant or time varying boundaries. Yet explicit solutions to the first-passage problem are known only in a limited number of special cases and efficient algorithms for their study were proposed in literature [1].

Generally one describes the dynamics of the involved variables via a suitable stochastic process $\{X_t, t \geq 0\}$ constrained by an assigned boundary $b : \langle 0, \infty\rangle \to \mathbb{R}$ satisfying $b(0+) \geq 0$ and investigates distribution features of the FPT)

$$T_b = \inf \{ t > 0 \mid W_t \geq b(t) \} \tag{1}$$

of X_t over b. This is the direct FPT problem. However there are also instances when the underlying stochastic process is assigned, one knows or estimates the FPT distribution F_b and wishes to determine the corresponding boundary shape. This is the inverse first passage time (IFPT) problem.

In [2] we studied this problem in the case of a Wiener process constrained by a single boundary. We proposed an algorithm to determine the unknown eventually time dependent boundary when the distribution of the FPT is assigned and we studied its convergence properties. In [3] we generalized the approach in order to apply it to an Ornstein-Uhlenbeck process and we proposed its use for a classification problem in neuroscience.

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Here we re-consider the IFPT problem assuming that the diffusion process is constrained by two boundaries: given the quantities $P(T_a < t, T_a < T_b)$ and $P(T_b < t, T_a > T_b)$, we propose an algorithm to approximate the unknown boundaries $a(t)$ and $b(t)$.

Examples and modeling motivation for this study are also discussed.

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