An upwind numerical method for predicting ideal MHD high speed flows is presented. It is based on a flux-difference splitting procedure with an approximate solver for the solution of the Riemann problem. The method is used for investigating numerically the admissibility of discontinuities that satisfy the jump conditions of the 1D planar problem. Fast and slow magnetic shock waves are numerically predicted and also particular shocks, which result from the merging of a slow and a fast magnetic compression waves. Moreover, compound waves are computed where a compression shock is attached to a rarefaction fan of the same family. Both slow and fast compound waves are predicted numerically, as well as transitional compound waves which represent the limiting case of a fast and a slow rarefaction fans attached each other through a compression shock.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{F}$</td>
<td>fluxes vector</td>
</tr>
<tr>
<td>$\mathbf{W}$</td>
<td>conservative variables vector</td>
</tr>
<tr>
<td>$D_N F$</td>
<td>flux-difference for left travelling waves</td>
</tr>
<tr>
<td>$D_R F$</td>
<td>flux-difference for right travelling waves</td>
</tr>
<tr>
<td>$\hat{e}, \hat{j}, \hat{k}$</td>
<td>unit vectors in the x,y,z-directions</td>
</tr>
<tr>
<td>$\hat{\mathbf{B}}$</td>
<td>magnetic field vector</td>
</tr>
<tr>
<td>$\hat{\mathbf{q}}$</td>
<td>velocity vector</td>
</tr>
<tr>
<td>$a$</td>
<td>acoustic speed of sound</td>
</tr>
<tr>
<td>$B_t$</td>
<td>tangential component of the magnetic field</td>
</tr>
<tr>
<td>$B_x$</td>
<td>magnetic field component in the x-direction</td>
</tr>
<tr>
<td>$B_y$</td>
<td>magnetic field component in the y-direction</td>
</tr>
<tr>
<td>$B_z$</td>
<td>magnetic field component in the z-direction</td>
</tr>
<tr>
<td>$c_A$</td>
<td>Alfvén speed</td>
</tr>
<tr>
<td>$c_f$</td>
<td>fast magnetoacoustic speed</td>
</tr>
<tr>
<td>$c_s$</td>
<td>slow magnetoacoustic speed</td>
</tr>
<tr>
<td>$D_N F$</td>
<td>flux-difference</td>
</tr>
<tr>
<td>$e$</td>
<td>total energy per unit volume</td>
</tr>
<tr>
<td>$N$</td>
<td>computational cell index</td>
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<tr>
<td>$p$</td>
<td>pressure</td>
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<tr>
<td>$R_i$</td>
<td>$i^{th}$ Riemann invariant</td>
</tr>
<tr>
<td>$R_{ij}$</td>
<td>$i^{th}$ Riemann invariant in region $j$</td>
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<tr>
<td>$t$</td>
<td>time</td>
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<tr>
<td>$U$</td>
<td>normalized pressure</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity component in the x-direction</td>
</tr>
<tr>
<td>$V$</td>
<td>normalized $B_y$</td>
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<tr>
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<td>velocity component in the y-direction</td>
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<tr>
<td>$w$</td>
<td>velocity component in the z-direction</td>
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<tr>
<td>$x, y, z$</td>
<td>coordinate directions</td>
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</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
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<tbody>
<tr>
<td>$i$</td>
<td>characteristic wave index</td>
</tr>
<tr>
<td>$j$</td>
<td>Riemann problem region index</td>
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<tr>
<td>$L$</td>
<td>left state of a discontinuity</td>
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<tr>
<td>$l$</td>
<td>region to the left of a characteristic wave</td>
</tr>
<tr>
<td>$R$</td>
<td>right state of a discontinuity</td>
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<tr>
<td>$r$</td>
<td>region to the right of a characteristic wave</td>
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**Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta_f$</td>
<td>fast magnetoacoustic coefficient</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>slow magnetoacoustic coefficient</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>specific heats ratio</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>characteristic wave speed</td>
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<tr>
<td>$\rho$</td>
<td>density</td>
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**Superscripts**

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<th>Description</th>
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<tr>
<td>-</td>
<td>average value</td>
</tr>
<tr>
<td>*</td>
<td>sonic conditions</td>
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I. Introduction

Upwind numerical methods are considered appropriate and reliable tools for the prediction of compressible flows in the high speed regime. They have shown excellent performances as far as both robustness and accuracy are concerned, over a broad spectrum of applications. Today they are embedded in the core of several codes widely used for scientific and technical purposes.

The upwind label given to these methods refers to the treatment of the convective terms in the governing equations. Specifically, it applies to the procedure followed to evaluate the relevant fluxes of the conservation laws (CL) on the surfaces of each cell defined in the finite volume approximation. In particular, the wave propagation, that is described by the partial differential equations (PDE) derived from the CL, plays a key role in such an evaluation and allows for the achievement of excellent predictions.

A large variety of upwind methods has been developed worldwide, since the end of the 70’s, in the field of classical gasdynamics (CGD), that is in the absence of any magnetic interaction. Despite the upwind label common to all these methods, they are conceived according to different approaches and present rather different numerical performances in dealing with discontinuities, such as shock waves and contact/shear surfaces. The reader can refer to Refs.1 and 2, where critical reviews are reported on several upwind methods for CGD, currently used today through the scientific community.

Many of the upwind methods successfully experimented in CGD have been, more recently, extended to the prediction of compressible flows described by the ideal magneto-hydrodynamics (MHD) equations. Such extensions are not trivial, because the 1D wave propagation mechanism at the base of any upwind approach has to cope with the structure of seven characteristic waves instead of the three ones typical of CGD: \((u \pm a)\) and \(u\). More precisely, the role of the thermodynamic speed of sound \((a)\) is replaced in MHD by those of the fast magnetoacoustic speed \((c_f)\), of the Alfven speed \((c_A)\) and of the slow magnetoacoustic speed \((c_s)\). It follows that the MHD wave structure comprises seven waves: \((u \pm c_f), (u \pm c_A), (u \pm c_s)\) and \(u\).

A further basic difference between CGD and MHD shows up in defining discontinuities. In CGD, the jump conditions lead to the identification of the leftwards and the rightwards oriented compression shocks (expansion shocks are assumed to collapse into expansion fans) and the contact/shear surface. Therefore, beside shocks, contact/shear surfaces and expansion fans, no other waves are expected to be generated in the solution of Riemann problems (RP) that characterize upwind methods in CGD. On the contrary, more and different kinds of discontinuities can be identified from the jump conditions resulting from the larger set of algebraic equations in MHD. An open debate appears in the literature on the admissibility of these discontinuities. In fact, some of them are expected to collapse by generating not only expansion fans, as in CGD, but also compound waves (CW), where an expansion remains attached to a compression shock of the same family, without any mutual interaction.

In the present paper we would like to propose an approximate solver for the RP, specifically conceived for a flux-difference splitting (FDS) upwind method in MHD. We have used this method for investigating numerically the admissibility of discontinuities that satisfy the jump conditions dictated by the planar 1D MHD conservation laws. Here we discuss numerical results that confirm the admissibility of some of the above discontinuities and show, on the contrary, the collapse of others and, in particular cases, the generation of so called compound waves.

The paper is organized as follows. In Section II, the governing CLs for MHD are recalled for the 1D problem. Then, the relevant PDEs are recalled in Section III, where the related wave speeds, signals and compatibility equations are defined. In Section IV, the proposed approximate solver for the RP in MHD is presented and in the following Section V the structure of a FDS code, which includes this approximate solver and that will be used for following 1D MHD numerical experiments, is briefly described. The discontinuities that algebraically result from the jump conditions of the planar 1D MHD problem are identified and classified in Section VI and, finally, in Section VII, the admissibility of some of these discontinuities is investigated numerically with the above mentioned code.

II. Conservation laws in the 1D MHD problem

The flow velocity \(\vec{q}\) and the magnetic field intensity \(\vec{B}\) are defined as:

\[
\vec{q} = u \hat{i} + v \hat{j} + w \hat{k} \\
\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}
\] (1) (2)

2 of 18

American Institute of Aeronautics and Astronautics Paper 2004-2164
where \( \vec{i} \) (along the \( x \)-coordinate) refers to the significant direction for the 1D problem and \( \vec{j} \) and \( \vec{k} \) (along the coordinates \( y \) and \( z \)) lay on the transversal plane normal to \( \vec{i} \).

The CLs are:

\[
\frac{d}{dt} \int_{x_1}^{x_2} W \, dx = -(F_2 - F_1) \tag{3}
\]

where the conservative variables \( (W) \) and the fluxes \( (F) \) are defined by:

\[
W = \begin{pmatrix}
\rho \\
\rho u \\
\rho v \\
\rho w \\
B_y \\
B_z \\
\frac{e + B_t^2}{2}
\end{pmatrix}; \quad F = \begin{pmatrix}
\rho u \\
p + \rho u^2 + \frac{B_t^2}{2} \\
\rho uv - B_x B_y \\
\rho uw - B_x B_z \\
uB_y - vB_z \\
uB_z - wB_y \\
\frac{e}{\gamma - 1} + \frac{\rho}{2} (u^2 + v^2 + w^2)
\end{pmatrix} \tag{4}
\]

with:

\[ e = \frac{p}{\gamma - 1} + \frac{\rho}{2} (u^2 + v^2 + w^2) \quad ; \quad B_t^2 = B_x^2 + B_y^2 \tag{5} \]

### III. Wave propagation in the 1D MHD problem

From Eqs. (3) and (4) we obtain, in the general case of \( B_x \neq 0 \) and \( B_t \neq 0 \), the governing PDE:

\[
\rho_t + u \rho_x + \rho u_x = 0 \tag{6a}
\]

\[
u_t + u \nu_x + \frac{B_y}{\rho} (B_y)_x + \frac{B_z}{\rho} (B_z)_x = 0 \tag{6b}
\]

\[
w_t + u w_x - \frac{B_z}{\rho} (B_z)_x = 0 \tag{6c}
\]

\[
p_t + u p_x + \gamma \rho u x = 0 \tag{6d}
\]

\[
(B_y)_t + u (B_y)_x + B_y u_x - B_x v_x = 0 \tag{6e}
\]

\[
(B_z)_t + u (B_z)_x + B_z u_x - B_x w_x = 0 \tag{6f}
\]

Since it must be \( \nabla \cdot \vec{B} = 0 \), then \( B_x \) is a prescribed constant.

By rearranging Eqs. (6), we define seven distinct characteristic directions:

\[
\lambda_1 = u - c_f \quad ; \quad \lambda_2 = u - c_A \quad ; \quad \lambda_3 = u - c_s \\
\lambda_4 = u \\
\lambda_5 = u + c_s \quad ; \quad \lambda_6 = u + c_A \quad ; \quad \lambda_7 = u + c_f \tag{7}
\]

The characteristics \( \lambda_1 \) and \( \lambda_7 \) are related to the fast magnetoaoustic waves, \( \lambda_2 \) and \( \lambda_6 \) refer to the Alfvén waves and \( \lambda_3 \) and \( \lambda_5 \) are related to the slow magnetoaoustic waves. The contact/shear surface (entropy/tangential momentum wave) propagates along \( \lambda_4 \).

The fast magnetoaoustic speed \( (c_f) \), the Alfven speed \( (c_A) \) and the slow magnetoaoustic speed \( (c_s) \) are
defined as:

\[ c_f = \sqrt{\frac{1}{2} \left( a^2 + c_A^2 + \frac{B_t^2}{\rho} + \sqrt{(a^2 + c_A^2 + \frac{B_t^2}{\rho})^2 - 4a^2c_A^2} \right)} \]  
(8a)

\[ c_A = \frac{B_x}{\sqrt{\rho}} \]  
(8b)

\[ c_s = \sqrt{\frac{1}{2} \left( a^2 + c_A^2 + \frac{B_t^2}{\rho} - \sqrt{(a^2 + c_A^2 + \frac{B_t^2}{\rho})^2 - 4a^2c_A^2} \right)} \]  
(8c)

where \( a = \sqrt{\frac{\gamma \rho}{\rho}} \) represents the thermodynamic speed of sound.

From Eqs. (6), seven compatibility equations are obtained:

\[ R_{it} + \lambda_i R_{ix} = 0 \quad \text{with} \quad i = 1, 2, 3, 4, 5, 6, 7 \]  
(9)

The signals \( R_i \), also known as Riemann invariants, do not change along the relevant characteristic \( \lambda_i \). They are be expressed as inexact differentials (in fact, the fourth one could be written as an exact differential introducing the entropy and writing \( dR_4 = dS \)):

\[ dR_1 = dp - \rho f \, du + \beta_f B_x B_y \, dv + \beta_f B_x B_z \, dw + c_f \beta_f B_y \, dB_y + c_f \beta_f B_z \, dB_z \]  
(10a)

\[ dR_2 = -B_z \, dv + B_y \, dw - \frac{1}{\sqrt{\rho}} B_z \, dB_y + \frac{1}{\sqrt{\rho}} B_y \, dB_z \]  
(10b)

\[ dR_3 = dp - \rho c_s \, du + \beta_s B_x B_y \, dv + \beta_s B_x B_z \, dw + c_s \beta_s B_y \, dB_y + c_s \beta_s B_z \, dB_z \]  
(10c)

\[ dR_4 = dp - \frac{dp}{a^2} \]  
(10d)

\[ dR_5 = dp + \rho c_s \, du - \beta_s B_x B_y \, dv - \beta_s B_x B_z \, dw + c_s \beta_s B_y \, dB_y + c_s \beta_s B_z \, dB_z \]  
(10e)

\[ dR_6 = -B_x \, dv + B_y \, dw + \frac{1}{\sqrt{\rho}} B_x \, dB_y - \frac{1}{\sqrt{\rho}} B_y \, dB_z \]  
(10f)

\[ dR_7 = dp + \rho f \, du - \beta_f B_x B_y \, dv - \beta_f B_x B_z \, dw + c_f \beta_f B_y \, dB_y + c_f \beta_f B_z \, dB_z \]  
(10g)

where coefficients \( \beta_f \) and \( \beta_s \) are respectively:

\[ \beta_f = \frac{c_f}{(c_f^2 - c_A^2)} \quad ; \quad \beta_s = \frac{c_s}{(c_s^2 - c_A^2)} \]  
(11)

IV. Approximate solution of the Riemann problem in MHD

The upwind method we are going to present belongs to the group of the FDS methods. Consequently, it requires the solution of a RP, that is the prediction of the evolution of a discontinuity prescribed by initial data.

In the CGD, the exact solution of a RP can be achieved easily, at the price, however, of a large computing time. To reduce this shortcoming, approximate solvers of the RP have been conceived.\(^3\),\(^4\),\(^5\)

Dealing with MHD, the exact solution of a RP appears prohibitive as computational times are concerned, but also in view of the admissibility of discontinuous solutions. Therefore, the use of simple approximate solvers appears appealing. We remind contributions to this subject from Refs.6,7,8 and 9, which have been fruitfully put in operation. The flux-vector splitting procedure described and experimented for MHD in Refs.10 and 11 belongs to a different group of upwind methods.

The approximate solver we show here is the extension to MHD of the solver proposed for CGD in Ref. 5 and refers to the general case with \( B_x \neq 0 \) and \( B_t \neq 0 \). When \( B_x = 0 \) or \( B_t = 0 \), multiplicity appears in the characteristic waves structure, with different definitions of signals in the compatibility equations:\(^12\) in these cases, the approximate solver looks much simpler and more similar to the one seen for CGD.\(^5\)
The basic approximation of the present solver is represented by the assumption that any signal remains constant while it propagates along its characteristic ($\lambda_i$), even if the signal itself travels across a discontinuity. As it occurs in CGD, this approximation implies that shock waves are simulated by isentropic converging fans. Moreover, with a further approximation, the signals defined in Eqs. (10) as inexact differentials will be here approximated as exact differentials by assuming the coefficients that appear in their definition as constant. Therefore, the approximated finite form of the Riemann invariants for MHD results as:

$$R_i = p - \frac{pc}{\rho} + \beta_f B_x B_y w + \beta_f B_z B_y B_z + \frac{\rho}{\sqrt{\rho}} \beta_y B_z B_y + \frac{\rho}{\sqrt{\rho}} B_z B_z$$

(12a)

$$R_2 = - \frac{\rho}{\sqrt{\rho}} B_z w + \frac{\rho}{\sqrt{\rho}} B_y B_z$$

(12b)

$$R_3 = p - \frac{pc}{\rho} + \beta_f B_x B_y v + \beta_f B_x B_z B_x w + \beta_f B_z B_z B_y + \frac{\rho}{\sqrt{\rho}} B_z B_z$$

(12c)

$$R_4 = \rho - \frac{pc}{\rho}$$

(12d)

$$R_5 = p + \frac{pc}{\rho} - \beta_x B_x B_y v - \beta_x B_x B_z B_x w + \beta_x B_z B_z B_y + \frac{\rho}{\sqrt{\rho}} B_z B_z$$

(12e)

$$R_6 = - \frac{\rho}{\sqrt{\rho}} B_z w + \frac{\rho}{\sqrt{\rho}} B_y B_z$$

(12f)

$$R_7 = p - \frac{pc}{\rho} + \beta_f B_x B_y v + \beta_f B_x B_z B_x w + \beta_f B_z B_z B_y + \frac{\rho}{\sqrt{\rho}} B_z B_z$$

(12g)

The constant coefficients denoted by the bar in Eqs. (12) are estimated as the arithmetic average of the initial values of the RP (regions $a$ and $b$ defined in the next lines).

Within the framework of these two approximations, we expect a pattern of seven waves to be generated by the collapse of the initial discontinuity of the RP at time $t_0$ and on the coordinate $x_{N+1/2}$. Eight regions ($j = a, b, c, d, e, f, g, h$) appear in Fig. 1, separated each other by the seven waves ($i = 1, 2, 3, 4, 5, 6, 7$). Regions $a$ and $b$ represent the prescribed initial values of the RP; in the first order integration scheme, they coincide with the flow properties in the cells $N$ and $N + 1$ at time $t_0$. The six regions $c, d, e, f, g, h$ have to be evaluated in order to proceed with the FDS method. Because of the invariance of the signals defined in Eqs. (12), the value of any Riemann invariant $R_i$ in a $j$ region ($j = c, d, e, f, g, h$) is given by

$$R_{ij} = R_{ia} \quad \text{or} \quad R_{ij} = R_{ib}$$

(13)

depending on which region, $a$ or $b$, the characteristic $\lambda_i$ arriving in region $j$ has been originated from.

The values of $R_{ij}$ are defined in Table 1. Once the set of signals $R_{ij}$ is known at any region $j$, the

<table>
<thead>
<tr>
<th>Region</th>
<th>$R_{1j}$</th>
<th>$R_{2j}$</th>
<th>$R_{3j}$</th>
<th>$R_{4j}$</th>
<th>$R_{5j}$</th>
<th>$R_{6j}$</th>
<th>$R_{7j}$</th>
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<tbody>
<tr>
<td>$j = c$</td>
<td>$R_{1b}$</td>
<td>$R_{2a}$</td>
<td>$R_{3a}$</td>
<td>$R_{4a}$</td>
<td>$R_{5a}$</td>
<td>$R_{6a}$</td>
<td>$R_{7a}$</td>
</tr>
<tr>
<td>$j = d$</td>
<td>$R_{1b}$</td>
<td>$R_{2b}$</td>
<td>$R_{3a}$</td>
<td>$R_{4a}$</td>
<td>$R_{5a}$</td>
<td>$R_{6a}$</td>
<td>$R_{7a}$</td>
</tr>
<tr>
<td>$j = e$</td>
<td>$R_{1b}$</td>
<td>$R_{2b}$</td>
<td>$R_{3b}$</td>
<td>$R_{4a}$</td>
<td>$R_{5a}$</td>
<td>$R_{6a}$</td>
<td>$R_{7a}$</td>
</tr>
<tr>
<td>$j = f$</td>
<td>$R_{1b}$</td>
<td>$R_{2b}$</td>
<td>$R_{3b}$</td>
<td>$R_{4b}$</td>
<td>$R_{5a}$</td>
<td>$R_{6a}$</td>
<td>$R_{7a}$</td>
</tr>
<tr>
<td>$j = g$</td>
<td>$R_{1b}$</td>
<td>$R_{2b}$</td>
<td>$R_{3b}$</td>
<td>$R_{4b}$</td>
<td>$R_{5b}$</td>
<td>$R_{6a}$</td>
<td>$R_{7a}$</td>
</tr>
<tr>
<td>$j = h$</td>
<td>$R_{1b}$</td>
<td>$R_{2b}$</td>
<td>$R_{3b}$</td>
<td>$R_{4b}$</td>
<td>$R_{5b}$</td>
<td>$R_{6b}$</td>
<td>$R_{7a}$</td>
</tr>
</tbody>
</table>
corresponding primitive variables are easily decoded as:

$$p_j = \frac{\overline{c_x} \beta_x (R_{\tau j} + R_{4 j}) - \overline{c_f} \beta_f (R_{6 j} + R_{3 j})}{2(\overline{c_x} \beta_x - \overline{c_f} \beta_f)}$$  \hspace{1cm} (14a)

$$u_j = \frac{\beta_x (R_{\tau j} - R_{1 j}) - \beta_f (R_{5 j} - R_{3 j})}{2 \rho (\overline{c_x} \beta_f - \overline{c_f} \beta_f)}$$  \hspace{1cm} (14b)

$$v_j = \frac{1}{2 B_t^2} \left( \frac{B_y \overline{c_x} (R_{\tau j} - R_{1 j}) - \overline{c_t} (R_{5 j} - R_{3 j})}{(\overline{c_x} \beta_f - \overline{c_f} \beta_f)} - \overline{c_z} (R_6 j + R_{2 j}) \right)$$  \hspace{1cm} (14c)

$$w_j = \frac{1}{2 B_t^2} \left( \frac{B_z \overline{c_x} (R_{\tau j} - R_{1 j}) - \overline{c_t} (R_{5 j} - R_{3 j})}{(\overline{c_x} \beta_f - \overline{c_f} \beta_f)} - \overline{c_y} (R_6 j + R_{2 j}) \right)$$  \hspace{1cm} (14d)

$$\rho_j = \frac{p_j}{\rho}$$  \hspace{1cm} (14e)

$$\begin{align*}
(B_y)_j &= \frac{1}{2 B_t^2} \left( B_y \frac{(R_{\tau j} + R_{3 j}) + (R_{5 j} + R_{3 j})}{\overline{c_x} \beta_x - \overline{c_f} \beta_f} + \sqrt{\rho} B_z (R_6 j - R_{2 j}) \right) & (14f) \\
(B_z)_j &= \frac{1}{2 B_t^2} \left( B_z \frac{(R_{\tau j} + R_{1 j}) + (R_{5 j} + R_{3 j})}{\overline{c_x} \beta_x - \overline{c_f} \beta_f} - \sqrt{\rho} B_y (R_6 j - R_{2 j}) \right) & (14g)
\end{align*}$$

We can now estimate the slopes of the characteristic $\lambda_i$ on the two sides of the $i$-wave, $\lambda_{i l}$ on the left side and $\lambda_{i r}$ on the right one. Also, we can estimate the flux-difference, $(D_N F)_i$, across this $i$-wave. The specific values of $\lambda_{i l}$, $\lambda_{i r}$ and $(D_N F)_i$, are listed in Table 2.

<table>
<thead>
<tr>
<th>wave</th>
<th>$\lambda_{i l}$</th>
<th>$\lambda_{i r}$</th>
<th>$(D_N F)_i$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$\lambda_{1 a}$</td>
<td>$\lambda_{1 c}$</td>
<td>$F_c - F_a$</td>
</tr>
<tr>
<td>i = 2</td>
<td>$\lambda_{2 c}$</td>
<td>$\lambda_{2 d}$</td>
<td>$F_d - F_c$</td>
</tr>
<tr>
<td>i = 3</td>
<td>$\lambda_{3 d}$</td>
<td>$\lambda_{3 e}$</td>
<td>$F_e - F_d$</td>
</tr>
<tr>
<td>i = 4</td>
<td>$\lambda_{4 e}$</td>
<td>$\lambda_{4 f}$</td>
<td>$F_f - F_e$</td>
</tr>
<tr>
<td>i = 5</td>
<td>$\lambda_{5 f}$</td>
<td>$\lambda_{5 g}$</td>
<td>$F_g - F_f$</td>
</tr>
<tr>
<td>i = 6</td>
<td>$\lambda_{6 g}$</td>
<td>$\lambda_{6 h}$</td>
<td>$F_h - F_g$</td>
</tr>
<tr>
<td>i = 7</td>
<td>$\lambda_{7 h}$</td>
<td>$\lambda_{7 b}$</td>
<td>$F_b - F_h$</td>
</tr>
</tbody>
</table>

A. Regular splitting

We define as regular the splitting that results from the approximate solution of the RP if, for each $i$-wave, both $\lambda_{i l}$ and $\lambda_{i r}$ travel on the same direction, leftwards or rightwards. In this case, we construct the term $\overline{D_N F}$ as the sum of contributions coming from the waves with negative $\lambda_{i l}$ and $\lambda_{i r}$ and the term $\widetilde{D_N F}$ as the sum of contributions coming from the waves with positive $\lambda_{i l}$ and $\lambda_{i r}$:

$$\overline{D_N F} = \sum_{i=1}^{7} \frac{\lambda_{i l} - |\lambda_{i l}|}{2 \lambda_{i l}} (D_N F)_i$$  \hspace{1cm} (15a)

$$\widetilde{D_N F} = \sum_{i=1}^{7} \frac{\lambda_{i l} + |\lambda_{i l}|}{2 \lambda_{i l}} (D_N F)_i$$  \hspace{1cm} (15b)

Of course, the splitting must respect the condition that:

$$D_N F = \overline{D_N F} + \widetilde{D_N F}$$  \hspace{1cm} (16)
B. Splitting within a magnetoacoustic wave

It may occur that, just for one wave, \( \lambda_i \ell < 0 \) and \( \lambda_i r > 0 \), or that \( \lambda_i \ell > 0 \) and \( \lambda_i r < 0 \). This happens only for the slow magnetoacoustic waves \( (i = 3, 5) \) or for the fast \( (i = 1, 7) \) ones. In this case, the splitting is not anymore regular, but requires a particular treatment, since the wave bounded by characteristics with opposite signs travels partly to the left and partly to the right.

There can be different situations, depending on which is the wave that presents this configuration. For example, let us assume that from the approximate solver we find that this change in sign of the characteristics occurs for the magnetoacoustic fast wave with \( i = 7 \) (i.e. for the characteristic \((u + c_f)\)). If \( \lambda_7 \ell < 0 \) and \( \lambda_7 r > 0 \), the wave 7 is an expansion fan, while if \( \lambda_7 \ell > 0 \) and \( \lambda_7 r < 0 \) this wave is a compression fan that approximates a shock. In both cases we have a characteristic embedded in the fan which is vertical \((\lambda_7^* = u^* + c_f^* = 0)\). The values of the seven variables \( \rho^*, u^*, v^*, w^*, p^*, (B_y)^*, (B_z)^* \) corresponding to \( \lambda_7^* \) and therefore the fluxes \( F^* \) can be found by solving the system:

\[
\begin{align*}
(R_i)^* &= (R_i)_b \quad (i = 1, 2, 3, 4, 5, 6) \\
u^* + c_f^* &= 0 
\end{align*}
\] (17a)

The flux-difference through wave 7 is now evaluated as

\[
(D_N F)_7 = F_b - F_h = (F_b - F^*) + (F^* - F_h) 
\] (18)

Its contribution is not completely given to \( \overline{D_N F} \) or to \( \overline{D_N F} \), as in the regular splitting (Eqs. (15)), but it is split in two parts, one contributing to \( \overline{(D_N F)_7} \) and to other to \( \overline{(D_N F)_7} \).

If wave 7 is an expansion fan, then:

\[
\begin{align*}
(D_N F)_7 &= F^* - F_h \quad \text{contributes to } \overline{D_N F} \\
(D_N F)_7 &= F_b - F^* \quad \text{contributes to } \overline{D_N F}
\end{align*}
\] (19a)

while if the wave 7 is a compression fan, we have:

\[
\begin{align*}
(D_N F)_7 &= F_b - F^* \quad \text{contributes to } \overline{D_N F} \\
(D_N F)_7 &= F^* - F_h \quad \text{contributes to } \overline{D_N F}
\end{align*}
\] (20a)

A similar procedure is followed when the situation described in the present Subsection occurs with the other fast magnetoacoustic wave (that is, when \( \lambda_1^* = u^* - c_f^* = 0 \)) or with any of the two slow magnetoacoustic waves (that is, when \( \lambda_5^* = u^* - c_s^* = 0 \) or \( \lambda_5^* = u^* + c_s^* = 0 \)). This particular kind of splitting is just the plain extension of the procedure proposed for the CGD in Ref. 5.

C. Overlapping of magnetoacoustic waves

A particular situation shows up when two adjacent fast and slow magnetoacoustic waves present, both of them, a vertical characteristic. With reference, for example, to waves 7 and 5, we can have expansion processes such that

\[
\begin{align*}
\lambda_{7, h} &< 0 & \lambda_{7, b} &> 0 \\
\lambda_{5, f} &< 0 & \lambda_{5, g} &> 0
\end{align*}
\] (21a)

or compression processes where

\[
\begin{align*}
\lambda_{7, h} &> 0 & \lambda_{7, b} &< 0 \\
\lambda_{5, f} &> 0 & \lambda_{5, g} &< 0
\end{align*}
\] (22a)

We can interpret this situation as the overlapping generated by the merging of waves 7 and 5. On this basis, the fast and the slow magnetoacoustic fans present their vertical characteristics \( \lambda_7^* = u^* + c_f^* \) and
\[ \lambda_5^* = u^* + c_s^* \text{ coincident and null. Now the seven primitive variables } \rho^*, u^*, v^*, w^*, p^*, (B_y)^*, (B_z)^* \text{ on these coincident characteristics } (\lambda_7^* = \lambda_6^* = \lambda_5^* = 0) \text{ are found by solving the system:} \]

\[
\begin{align*}
(R_i)^* &= (R_i)_b \quad (i = 1, 2, 3, 4) \\
(u^* + c_f^*) &= 0 \\
(u^* + c_A^*) &= 0 \\
(u^* + c_s^*) &= 0
\end{align*}
\] (23a) (23b) (23c) (23d)

The flux-difference through waves 5, 6 and 7 is now evaluated as

\[ (D_N F)_{567} = F_b - F_f = (F_b - F^*) + (F^* - F_f) \] (24)

Instead of the separate contributions \((D_N F)_{567}\), \((D_N F)_{67}\) and \((D_N F)_{7}\) in Eqs. (15), the term \((D_N F)_{567}\) contributes with its \((D_N F)_{567}\) part to \(D_N F\) and with its \((D_N F)_{567}\) part to \(D_N F\). If waves 5 and 7 are expansion fans, then:

\[
\begin{align*}
(D_{N F})_{567} &= F^* - F_f & \text{contributes to} & \quad D_N F \\
(D_{N F})_{567} &= F_b - F^* & \text{contributes to} & \quad D_N F
\end{align*}
\] (25a) (25b)

while, in case these waves are compression fans, we have:

\[
\begin{align*}
(D_{N F})_{567} &= F_b - F^* & \text{contributes to} & \quad D_N F \\
(D_{N F})_{567} &= F^* - F_f & \text{contributes to} & \quad D_N F
\end{align*}
\] (26a) (26b)

This particular splitting plays a fundamental role in some of the numerical experiments discussed later.

V. Structure of a 1D MHD code

Here we report briefly on the structure of the 1D MHD code used in performing the numerical predictions shown in the following.

The governing CLs of Eq.3 are discretized according to the cell centered finite volume approximation. The integration in time is given by:

\[ W_N^{K+1} = W_N^K - \frac{\Delta t}{\Delta x} (F_{N+1/2} - F_{N-1/2}) \] (27)

where \(N\) is the index of the cell (finite volume), \(\Delta x\) its length, \(K\) the integration step, \(\Delta t\) the integration time interval, and \(W_N^{K+1}\) and \(W_N^K\) are average values of the conservative variable over the cell \(N\), at the integration steps \(K + 1\) and \(K\). In the first order integration scheme, the flux \(F_{N+1/2}\) (or \(F_{N-1/2}\)) at the interface \(N + 1/2\) (or \(N - 1/2\)) will be evaluated according to the solution of a RP where the initial values in regions \(a\) and \(b\) correspond to the values at the step \(K\) in the cells \(N\) and \(N + 1\) (or \(N - 1\) and \(N\)). An ENO reconstruction\(^\text{13}\) based also on variables in cells \(N - 1\) and \(N + 2\) (or \(N - 2\) and \(N + 1\)), always at step \(K\), and the use of the MINMOD limiter provide a reconstruction of the initial data \(a\) and \(b\) for the RP that leads to a second order of accuracy in space. Second order of accuracy in time is achieved by adding a further correction to these values to account for the time dependency of the solution. The computational domain is assumed to be limited by the boundaries at \(x = 0\) and \(x = 1\), where we have imposed simple-wave non-reflecting conditions.

VI. Discontinuities in the planar 1D MFD

In this section we focus the attention on stationary discontinuities that satisfy the jump conditions dictated by the CLs in the \textbf{planar} 1D MHD problem. To this purpose, we consider the fluxes defined in Eq. (4) in the case of \(w = 0\) and \(B_z = 0\). By denoting with \(L\) and \(R\) the left and right sides of a stationary discontinuity, the following condition must hold:

\[ F_R = F_L \] (28)
Here, we follow the analysis presented in Refs. 14, 15, 16 and we consider stationary discontinuities, permeable to gas flow that moves from right to left. These discontinuities are related to the characteristics of the families \( \lambda_7 = u + c_f \) (fast magnetoacoustic wave) and \( \lambda_5 = u + c_s \) (slow magnetoacoustic wave). Note that, in the planar problem, the Alfvén waves vanish.

By following the above mentioned analysis, if we prescribe the pressure \( p_L \) and the \( y \)-component of the magnetic field \( B_{yL} \) on the left side \( L \) of a discontinuity, the corresponding values of \( p_R \) and \( B_{yR} \) on the right side \( R \) can be easily obtained solving a simple second degree equation, once the constants \( \gamma \) and \( B_z \) are given.\(^{12}\) The loci of the possible solutions can be represented on a curve as the ones shown in Fig. 2. Here the abscissa refers to the normalized pressure, \( U \), and the ordinate to the normalized \( y \)-component of the magnetic field, \( V \), where

\[
U = \frac{\gamma p}{B_z^2} \quad \quad V = \frac{B_y}{B_z} \quad (29)
\]

The curve is shaped like a curl and it is characterized by the prescribed values at the left side, represented by the coordinates \((U_L, V_L)\) of the crossing point \( L \) on the curl. The coordinates of any point on the curl denote the values at the right side, \((U_R, V_R)\), that satisfy the jump conditions for a discontinuity at rest (Eq. (28)). The curls shown in Fig. 2 correspond to the ones reported in Fig.1b and Fig.1c of Ref. 16. The complete description of the flow on the two sides \( L \) and \( R \) requires also one prescribed value of density \( \rho_L \) (or \( \rho_R \)) and of the \( y \)-component of velocity \( v_L \) (or \( v_R \)). Other flow properties, such as \( \rho_R \) (or \( \rho_L \)), \( u_L, u_R \), and \( v_R \) (or \( v_L \)) can be reconstructed from the prescribed or computed values described above.

We can classify each point on a curl curve by looking at the sign of the characteristics, \( \lambda_7 \) and \( \lambda_5 \), on both sides of the discontinuity. In principle, we should expect 16 possible configurations for the combinations of sign of the four values of \( (\lambda_7)_L,R \) and \( (\lambda_5)_L,R \). However, because \( c_f > c_s \) (and therefore \( \lambda_7 > \lambda_5 \)), only 9 of them will be possible. These configurations are shown in Fig. 3, where the upper solid arrows and the lower dotted ones refer respectively to \( \lambda_7 \) and to \( \lambda_5 \), and are identified there as it follows:

- fast shock (FS) discontinuity, when \( \lambda_7 > 0 \) and \( \lambda_7 < 0 \) while \( \lambda_5 > 0 \) and \( \lambda_5 < 0 \);
- slow shock (SS) discontinuity, when \( \lambda_7 > 0 \) and \( \lambda_5 < 0 \) while \( \lambda_7 > 0 \) and \( \lambda_7 > 0 \);
- fast expansion (FE) discontinuity, when \( \lambda_7 < 0 \) and \( \lambda_7 > 0 \) while \( \lambda_5 < 0 \) and \( \lambda_5 < 0 \);
- slow expansion (SE) discontinuity, when \( \lambda_5 < 0 \) and \( \lambda_5 > 0 \) while \( \lambda_7 > 0 \) and \( \lambda_7 > 0 \);
- overcompression (OC) discontinuity, where all the characteristics converge on the discontinuity: \( \lambda_7 > 0 \), \( \lambda_5 > 0 \) and \( \lambda_7 < 0 \), \( \lambda_5 < 0 \);
- overexpansion (OE) discontinuity, where all the characteristics diverge from the discontinuity: \( \lambda_7 < 0 \), \( \lambda_5 < 0 \) and \( \lambda_7 > 0 \), \( \lambda_5 > 0 \);
- undercompression (UC) discontinuity, where the fast characteristics move rightwards (\( \lambda_7 > 0 \), \( \lambda_7 > 0 \)) and the slow ones move leftwards (\( \lambda_5 < 0 \), \( \lambda_5 < 0 \));
- left transport (LT) discontinuity, where all the characteristics move leftwards (\( \lambda_7 < 0 \), \( \lambda_7 < 0 \), \( \lambda_5 < 0 \), \( \lambda_5 < 0 \));
- right transport (RT) discontinuity, where all the characteristics move rightwards (\( \lambda_7 > 0 \), \( \lambda_7 > 0 \), \( \lambda_5 > 0 \), \( \lambda_5 > 0 \)).

Note that the nomenclature used to identify possible discontinuities is not unique in the literature. Any definition given above refers to one of the configurations of characteristics displayed in Fig. 3.

The wave configurations described above can be identified along curl curves, as shown in Fig. 2. In particular, we find that, going along the upper branch of the curve moving leftwards (or rightwards) from point \( L \), SS (or FE) discontinuities are met. On the contrary, the lower branches, below the point \( L \), represent, along a limited extension, FS discontinuities (on the left) and SE discontinuities (on the right). The remaining portion of the curl loop, between the SE and FS portions, can refer to one or more different configurations. However, the sequence from SE to FS should be ordered in such a way that the transition from one configuration to the neighboring one is characterized by the change in sign of only one characteristic. In these cases, as also reported in Ref. 16, possible configurations LT and RT have never appeared.
VII. Admissibility of planar discontinuities

A wide debate appears in the literature on the admissibility of the different discontinuities that satisfy the jump conditions expressed by Eq. (28). Here, we would like to contribute to this debate by discussing the admissibility of discontinuities on the basis of numerical experiments performed with the code mentioned in Section V.

First, we select a set of L and R conditions, which define a curl curve and a point on it, respectively, and also values of $\rho_L$ (or $\rho_R$) and $v_L$ (or $v_R$). We denote each point on the curl curves with a label that indicates the branch where it is located and a subscript number that identifies the point itself in the catalog of several numerical experiments we have carried out. Then, we prescribe these L and R conditions as initial data on the left and the right side of a discontinuity located at $x = 0.5$ in the 1D computational domain $0 < x < 1.0$. Finally, we predict the evolution in time of such a discontinuity with the computational MHD code described above. We consider as admissible any planar discontinuity that remains stationary and preserves the initial data save the small numerical disturbances arising from the transition from the initial jump conditions to the captured numerical discontinuity. On the contrary, we consider as inadmissible a discontinuity that collapses generating different waves.

We performed numerical experiments by selecting sets of L-R initial conditions on the curl curves shown in Fig. 2. The corresponding left side conditions are given in Table 3 being $\gamma = 1.667$ and $B_x = 1.0$. We have assumed $\rho_L = 1.0$ and $v_L = 0$. The computations have been performed with a second order of accuracy, a CFL number equal to 0.8 and 800 cells over the computational domain.

First, we present results for admissible discontinuities; then, we discuss results where the discontinuities collapse and are therefore inadmissible.

A. Admissible discontinuities

We report on three cases where the numerical prediction of the evolution of the initial discontinuity has only generated the negligible and inevitable transition to a sharp numerically captured discontinuity at rest.

1. A fast shock (FS) discontinuity

Here we use the initial data relevant to the point $FS_1$ located on the FS branch of Fig. 2(b) and characterized by $U_R = 0.847$ and $V_R = 0.100$. The pressure distribution along the x-coordinate shown in Fig. 4(a) has been obtained after few steps of integration and remains perfectly stable for any arbitrary large number of steps. The $FS_1$ initial data represent an admissible discontinuity, specifically a fast magnetoacoustic shock wave.

2. A slow shock (SS) discontinuity

We use the initial data related to point $SS_1$ on the upper SS branch in Fig. 2(b). Here the right conditions are $U_R = 0.855$ and $V_R = 0.700$. The pressure distribution is shown in Fig. 4(b) and presents the same features as the previous $FS_1$ case. The $SS_1$ initial data refer to an admissible discontinuity, specifically a slow magnetoacoustic shock wave. Note that the numerical capturing of fast and slow shocks, performed with the splitting presented in Subsection IV.B, is neat and sharp as it is usually found in CGD.

3. An overcompressive (OC) discontinuity

We assume now the initial data of the point $OC_6$ located in Fig. 2(b), on the branch OC that connects the branch FS and the branch SS on the lower part of the curl. The right conditions here are given by $U_R = 0.380$ and $V_R = -0.270$. Note that in this case both the fast and the slow characteristics on the sides

| Table 3. Left side conditions for the curves of Fig. 2 |
|-------------|-------------|
| Fig. 2(a)   | 0.352       | 0.300       |
| Fig. 2(b)   | 1.490       | 0.380       |

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L and R converge on the discontinuity (see also Fig. 3). We interpret this configuration as the overlapping of a fast and a slow shock; in fact, the splitting in the numerical experiment follows the rules indicated in Subsection IV.C. The pressure distribution as it is obtained after few steps of integration is reported in Fig. 4(c) and, as in the previous cases, is very sharp and perfectly stable without any variation in time. We consider admissible the discontinuity of the point $OC_6$, which can be interpreted as a magnetoacoustic shock wave formed by fast and slow waves. We didn’t find in the literature any previous numerical prediction of this particular shock wave.

B. Inadmissible discontinuities

When an initial discontinuity is not preserved by the computation, we consider such a discontinuity as inadmissible. During its collapse, the discontinuity generates different waves. One of them is predominant and other waves are needed to match the prescribed initial conditions to the conditions that bound the predominant wave. The typical inadmissible discontinuity in CGD is the so called expansion shock, a discontinuity that satisfies the jump conditions, with a subsonic incoming flow. The divergence of the characteristics of the same family on the two sides of this discontinuity promotes its collapse and allows also for the respect of the second principle of thermodynamics. However, not only the expansion fan is generated as the predominant wave, but also a small shock of the opposite characteristic family and a small contact surface appear in order to match the predominant isentropic rarefaction fan to the initial expansion shock. Similar situations are expected in MHD also. In the following, we will show the evolution in time of some cases of inadmissible discontinuities.

1. A fast expansion (FE) discontinuity

We use as initial data the values pertinent to point $FE_4$ on the upper FE branch in Fig. 2(a), characterized by $U_R = 0.740$ and $V_R = 0.920$. The prescribed discontinuity collapses, as shown in Fig. 5(a) by the pressure distribution at time $t = 0.45$. The predominant wave is the fast rarefaction fan centered at about $x = 0.50$. The slow rarefaction fan on its left and other minor waves (a contact surface and magnetoacoustic waves of the opposite family that already escaped through the left boundary of the computational domain in Fig. 5(a)) are the matching waves. Definitely, the $FE_4$ discontinuity is inadmissible.

2. A slow expansion (SE) discontinuity

A very similar result is obtained by using as initial data those of point $SE_2$ on the SE branch in Fig. 2(a), characterized by $U_R = 1.033$ and $V_R = 0.100$. The pressure distribution at time $t = 1.0$ is shown in Fig. 5(b). The predominant wave is now the slow rarefaction fan, always centered at about $x = 0.50$. The weak fast rarefaction fan on its right and other minor waves play the role of matching waves. Also this discontinuity is inadmissible. In this case, as in the previous one ($FE_4$), the splitting presented in Subsection IV.B has prompted the collapse of the initial discontinuity.

3. An undercompressive (UC) discontinuity

The undercompressive discontinuity $UC_1$ located at the border of the UC branch of Fig. 2(b) and characterized by $U_R = 1.698$ and $V_R = -0.210$ has been also found to be inadmissible. Its collapse generated a predominant slow CW, centered at about $x = 0.5$, and, as matching wave, a fast rarefaction fan that propagates rightwards, as shown by the pressure distribution at time $t = 1.0$ in Fig. 6(a). A slow CW, predicted numerically, was shown years ago in Ref. 6 and it was reproduced later in many other computations by different authors. Recently, it has been shown in Ref. 17 that this kind of CW is only acceptable in the planar configuration. If non-planar configurations occur, a CW structure can appear in numerical simulations carried out on “relatively” rough meshes, but a grid convergence study shows that with a very fine mesh such a CW is transformed into a slow shock and an Alfvén wave. In our present context, we deal with a planar problem and we should consider as acceptable the compound wave. Nonetheless, we have performed a grid convergence study. The results with different numbers of points (1250, 2500, 5000, 10000, 20000) are reported in Fig. 6(b), where the enlarged view of the right portion of the compound wave is shown. No doubt that the grid convergence study confirms the description of the CW as the expansion fan on the left attached to the shock on the right. However, while grid convergence is well satisfied as far as the expansion is concerned, the shock converges to a captured discontinuity with an overshoot in pressure. We think that such a numerical
prediction is due to the way the approximate solver and the following splitting are conceived. Attention should be paid in the next future to improve this aspect of the numerical prediction.

4. An overexpansion (OE) discontinuity

The last cases of inadmissible discontinuities we discuss refer to points OE3, OE4, and OE7, located on the OE branch of Fig. 2(a). The pressure distribution at the time \( t = 1.0 \) is shown in Fig. 7(a) (OE3), Fig. 7(b) (OE4) and Fig. 7(c) (OE7).

In the case OE3, where \( U_R = 1.528 \) and \( V_R = -0.280 \), the collapse of the initial discontinuity generates a slow CW on the left and a fast rarefaction fan on the right. It is not possible here to distinguish between predominant and matching waves, since the two have similar intensities.

By moving downwards along the OE branch of Fig. 2(a), the point OE4 can be selected, with \( U_R = 1.522 \) and \( V_R = -0.330 \). The relative pressure distribution is shown in Fig. 7(b). Now, the slow wave is a plain rarefaction fan and the small shock that was present in the slow CW of case OE3 has moved to merge with the fast rarefaction wave to form a fast CW. Note that the shock follows the fan in the fast CW and precedes the fan in the slow compound wave.

The last case we consider refers to point OE7, always on Fig. 2(a), which presents \( U_R = 1.526 \) and \( V_R = -0.310 \). This case is half way between the two previous ones and has been selected to show the transition from a slow CW to a fast one. In fact, the pressure distribution reported in Fig. 7(c) shows a shock wave attached to both the slow and the fast fans so that the resulting wave can be labelled as a transitional CW. As far as we know, fast compound and transitional waves have been never predicted numerically, even though they have been anticipated on the basis of a different analysis and sketched in Fig.5 of Ref. 15.

VIII. Conclusions

An approximate solver for the solution of the Riemann problem in ideal MHD has been proposed and implemented in a flux-difference splitting method. A 1D code based on such a method has been used for investigating the admissibility of stationary discontinuities obtained from the jump conditions of the 1D planar problem. Admissible discontinuities have been investigated numerically with a very sharp and neat capturing of both fast and slow magnetic shocks. Moreover, particular shocks which can be interpreted as the overlapping of fast and slow magnetic compression waves have been predicted. Inadmissible discontinuities have been found to break down in fast or slow rarefaction fans, but also in some cases in compound waves. These are formed by a shock wave attached to a rarefaction fan of the same family and can belong to the slow or to the fast wave families. The shock precedes the fan in the slow compound wave and follows the fan in the fast compound wave. The existence of fast compound waves have been anticipated in the literature, but never predicted numerically, as well as the magnetic shocks resulting from fast and slow overlapping compression waves. Even though the capturing of magnetic shocks shows a remarkable sharpness, in the case of compound wave a pressure overshoot comes out from a grid convergence study that requires some further attention in the next future.

References

Figure 1. The ideal MHD Riemann problem.
(a) Locus of the possible discontinuities corresponding to left-side conditions $U_L = 0.352$, $V_L = 0.300$

(b) Locus of the possible discontinuities corresponding to left-side conditions $U_L = 1.490$, $V_L = 0.380$

Figure 2. Curl curves for two different left-side conditions.
Figure 3. Possible characteristic waves configurations in ideal MHD.
Figure 4. Collapse of initial discontinuities indicated as $FS_1$, $SS_1$ and $OC_6$ in Fig. 2(b).
Figure 5. Collapse of initial discontinuities indicated as \( FE_4 \) and \( SE_2 \) in Fig. 2(a).

Figure 6. Compound wave generated by the collapse of the initial discontinuity indicated as \( UC_1 \) in Fig. 2(b).
Figure 7. Compound waves generated by the collapse of the overexpansion discontinuities indicated as $OE_3$, $OE_4$ and $OE_7$ in Fig. 2(a).