Proceedings of the 2007 American Control Conference
Marriott Marquis Hotel at Times Square
New York City, USA, July 11-13, 2007

Direct Virtual Sensor (DVS) design in vehicle sideslip angle estimation

Mario Milanese, Diego Regruto, Andrea Fortina

Abstract— **The vehicle sideslip angle is one of the most important variable to evaluate vehicle stability during dynamic manoeuvres. In this paper a nonlinear estimator is proposed, which use measurements of lateral accleration, steering angle, yaw rate and longitudinal velocity as input signals and provide the sideslip angle estimate as output. The design of such an estimator is based on a recently proposed direct approach to the design of virtual sensor. The obtained estimator has been experimentally tested on a huge number of different manoeuvres showing quite good results in a large range of operation covering both the linear and the nonlinear behaviour of the car.**

I. INTRODUCTION

The vehicle sideslip angle is one of the most important variable to evaluate vehicle stability during dynamic manoeuvres. The knowledge of such a variable is of paramount importance in order to improve the performance of vehicle control systems designed to guarantee stability of the vehicle motion in emergency situations. Unfortunately, direct measurement of such a variable requires the use of complex and quite expensive devices usually not available in ordinary cars. Several papers in the literature describe methods for designing observers for estimating the sideslip angle from variables that can be more easily measured (e.g. yaw rate, lateral acceleration,..) and from suitable models of lateral dynamics, [1], [2], [3], [4], [5], [6], [7]. Different kind of models (linear, nonlinear, single track, double track) and of observers (Extended Kalman Filters, Extended Luenberger Observers, Sliding Mode Observers, Adaption of Quality Function Observers,..) have been considered. Common to all these approaches, a two-step procedure is adopted. First, a model of the car is identified exploiting measurements obtained from a suitably equipped testing car. The measured variables includes quantity actually measured on production cars (lateral acceleration, yaw rate, steering angle) as well as measurements of the sideslip angle provided by a suitable device mounted on the car. Second, an observer for estimating the sideslip angle, which exploits only the variables actually measured on the production cars, is designed on the basis of the estimated model. Note that sideslip angle measurements on the testing car are required, since, though not explicitly used in the model identification step, they are necessary at least for estimation accuracy verification.

In this paper we apply the alternative Direct Virtual Sensor (DVS) design approach proposed in [8], [9]. An observer is a system using variables actually measured on the production cars as inputs and producing the sideslip angle estimate as output. The DVS design approach is based on the idea of directly identifying an observer model from the measurements of these variables performed on the testing car. The interest for this new approach stems from the fact that it has been proved in [9] that even in the most favorable situations, e.g. no modeling errors and actually computable minimum variance observers, the two-step procedure is proved to give performances not better than the direct approach. More importantly, in presence of modeling errors, the directly identified sensor, though giving deteriorated performances with respect to the absolute optimal, is anyway the minimum variance estimator, among the selected approximating observer class. A similar result is not assured by the two-step design, whose performance deterioration caused by modeling errors may be significantly larger. Another relevant point is that minimum variance filters for nonlinear systems are in general difficult to derive and/or to implement, and widely used approximate solutions, such as Extended Kalman filters, quite often exhibit poor performances. On the contrary, the progresses of last years in nonlinear identification has led to advanced methods which may allow the direct filter identification for quite complex nonlinear models.

In this paper the Set Membership DVS method presented in [8] is used in designing a nonlinear direct virtual sensor which use measurements of lateral acceleration, steering angle, yaw rate and longitudinal velocity as input signals and provide the sideslip angle estimate as output. The obtained estimator has been experimentally tested on an Alfa Romeo car on a number of different manoeuvres including steering angle steps of different amplitude from 30 to 80 degrees, lane change and double lane change. The obtained results show that the proposed nonlinear estimator is able to provide a good sideslip angle estimation in a large range of operation covering both the linear and the nonlinear behaviours of the car.

II. NONLINEAR VIRTUAL SENSOR DESIGN FROM DATA

Consider a nonlinear discrete-time system in state-space form:

$$\begin{aligned} x^{t+1} &= F\left(x^t, u^t\right) \\ y^t &= H_y\left(x^t, u^t\right) \\ z^t &= H_z\left(x^t, u^t\right) \end{aligned} \tag{1}$$

where F, H_y and H_z are continuous and differentiable functions, $x^t \in \mathcal{X} \subset R^n$, $u^t \in \mathcal{U} \subset R^{m_u}$, $y^t \in \mathcal{Y} \subset R^{m_y}$ and $z^t \in \mathcal{Z} \subset R^{m_z}$, $\mathcal{X}, \mathcal{U}, \mathcal{Y}$ and $\mathcal{Z}$ are compact sets.
Suppose that noise corrupted measurements $(\widetilde{u}^t, \widetilde{y}^t)$ of u^t

Mario Milanese and Diego Regruto are with Dipartimento di Automatica e Informatica, Politecnico di Torino, Italy mario.milanese@polito.it, diego.regruto@polito.it

Andrea Fortina is with Engineering & Design, Chassis & Vehicle Dynamics, Fiat Auto S.p.A andrea.fortina@fiat.com

and y^t are available for any time t and it is of interest to know z^t, which is measured only in the time interval $(0, T_m)$. In cases that F, H_y and H_z are known and z is observable from the couple (u, y), observer/Kalman filter theory can be applied. In cases that F, H_y and H_z are not known, the standard approach is to identify a model and then design an observer based on the identified model. Clearly, an observer is a system using $(\widetilde{u}^t, \widetilde{y}^t)$ as inputs and producing an estimate of z^t as output. The direct approach to the design of virtual sensor, proposed in [8] and exploited in this paper, is based on the idea of directly identifying an observer model from the available data $(\widetilde{u}^t, \widetilde{y}^t)$ and $\widetilde{z}^t$ in the time interval $(0, T_m)$. Such a model can be used to generate estimates of z^t for $t > T_m$, thus representing a "virtual" sensor of z^t which can be used when the "actual" sensor is no longer available. In the rest of this section, the main results of [8] are briefly summarized.

The following theorem shows that z^t may be expressed as a nonlinear function of a finite number of past values of u^t and y^t.

Theorem 1: [8] Consider the system (1). If (F, H_y) is observable, then $\exists f_o$ and integers n_y, n_u such that:

$$\begin{array}{r}z^t = f_o\left(Y^t, U^t\right) \\ Y^t = \left[y^t, y^{t-1}, \ldots, y^{t-n_y+1}\right] \\ U^t = \left[u^t, u^{t-1}, \ldots, u^{t-n_u+1}\right]\end{array} \tag{2}$$

■

This theorem shows that the variable of interest z^t could be computed as $z^t = f_o\left(Y^t, U^t\right)$, if f_o, Y^t, U^t would be known. However, the function f_o is not known, since F, H_y, H_z in (1) are not supposed to be known, and only noise corrupted measurement $(\widetilde{Y}^t, \widetilde{U}^t)$ of the input-output data (Y^t, U^t) are available. Thus, the problem is to compute an estimate $\widehat{z}^t$ of z^t from the noisy data available in the time interval $(0, T_m)$ which can be described by the following equation:

$$\widetilde{z}^t = f_o\left(\widetilde{Y}^t, \widetilde{U}^t\right) + d^t, \; t = 0, 1, .., T_m \tag{3}$$

where d^t accounts for the fact that z^t, y^t, u^t are not exactly known.

To this end the Nonlinear Set Membership method proposed in [10] is used. The motivation for this choice is that most of the other estimation methods in the literature assume a parametric functional form for f_o and reliable statistical models of noise. However, as just noted above, it is difficult to have information regarding the functional form of f_o and on the statistics of d^t, even in the case of reliable information about F, H_y, H_z and on the statistics of the noise affecting the measurements z^t, y^t, u^t. In the NSM method, weaker assumptions are taken, not requiring the choice of a functional form for f_o, but related to its rate of variation. Moreover, the noise sequence d^t is only supposed to be bounded.

Prior assumptions on f_o:

There exists a γ such that:

$$f_o \in K \doteq \left\{f \in C^1 \colon \|f'(Y, U)\| \leq \gamma, \forall (Y, U) \in W\right\}$$

where f' denotes the gradient of f, $\|\cdot\|$ is the Euclidean norm and W is a bounded subset of the regressor space.

■

Prior assumptions on noise:

There exist ε^t and δ^t such that:

$$|d^t| \leq \varepsilon^t + \gamma\delta^t, \; \forall t$$

■

A key role in this Set Membership framework is played by the Feasible Systems Set, often called "unfalsified systems set", i.e. the set of all systems consistent with prior information and measured data.

Definition 1: The Feasible Systems Set FSS^T is:

$$FSS^T \doteq \left\{f \in K : \left|\widetilde{z}^t - f\left(\widetilde{Y}^t, \widetilde{U}^t\right)\right| \leq \varepsilon^t + \gamma\delta^t, \; t = 1, 2, ..., T_m\right\}$$

■

The Feasible Systems Set FSS^T summarizes all the information (measured data and prior information on f_o and noise d) that is available up to time T_m on the mechanism generating the data. As required in any identification theory, the problem of checking the validity of prior assumptions arises. Indeed, the only thing that can be actually done is to check if prior assumptions are invalidated by data, evaluating if no unfalsified system exists, i.e. if FSS^T is empty. However, it is usual to introduce the concept of prior assumption validation as follows:

Definition 2: Prior assumptions are considered validated if $FSS^T \neq \emptyset$.

■

Necessary and sufficient conditions for checking the assumptions validity, are given below in Theorem 2, which is reported from [10].

Let us introduce the following quantities:

$$\begin{aligned}\overline{f}(Y, U) &\doteq \min_{t=1,\ldots,T_m}\left(\overline{h}^t + \gamma\left\|(Y, U) - (\widetilde{Y}^t, \widetilde{U}^t)\right\|\right) \\ \underline{f}(Y, U) &\doteq \max_{t=1,\ldots,T_m}\left(\underline{h}^t - \gamma\left\|(Y, U) - (\widetilde{Y}^t, \widetilde{U}^t)\right\|\right)\end{aligned}$$

where $\overline{h}^t \doteq \widetilde{z}^t + \varepsilon^t + \gamma\delta^t$ and $\underline{h}^t \doteq \widetilde{z}^t - \varepsilon^t - \gamma\delta^t$.

Theorem 2: [10]

i) A necessary condition for prior assumptions to be validated is:

$$\overline{f}\left(\widetilde{Y}^t, \widetilde{U}^t\right) \geq \underline{h}^t \qquad t = 1, 2, ..., T-1$$

ii) A sufficient condition for prior assumptions to be validated is:

$$\overline{f}\left(\widetilde{Y}^t, \widetilde{U}^t\right) > \underline{h}^t \qquad t = 1, 2, ..., T-1$$

■

Theorem 2 can be used for assessing the values of ε^t, δ^t and γ, in order to have a non-empty FSS^T.

In the space $(\varepsilon^t, \delta^t, \gamma)$, the function:

$$\gamma^*\left(\varepsilon^t, \delta^t\right) \doteq \inf_{FSS^T \neq \emptyset} \gamma \tag{4}$$

represents a surface that separate falsified values of ε, δ and γ from validated ones. Clearly, ε, δ and γ must be chosen in the validated parameters region (see section 7 of [10] for some more details on the selection of these constants).

A virtual sensor is a function ϕ giving the estimate of z^t as $\widehat{z}^t = \phi(\widetilde{Y}^t, \widetilde{U}^t)$. Indeed, an almost optimal virtual sensor can be derived. Here, optimality refers to minimizing, with respect to all virtual sensors ϕ, the worst case estimation error:

$$WE(\phi) = \sup_{f \in FSS^T} \sup_{(Y,U) \in B_\delta} \left|\phi(\widetilde{Y}^t, \widetilde{U}^t) - f(Y,U)\right|$$

where:

$$B_\delta\left(\widetilde{Y}^t, \widetilde{U}^t\right) \doteq \{(Y,U) : ||(Y,U) - (\widetilde{Y}^t, \widetilde{U}^t)|| \leq \delta^t\}$$

Almost optimality of virtual sensor ϕ^* means that its worst case estimation error is not greater than twice the optimal one, i.e.:

$$WE(\phi^*) \leq 2 \inf_\phi WE(\phi)$$

Consider the virtual sensor provided by the so called *central estimate* ϕ_c:

$$\phi_c(Y,U) = \frac{1}{2}\left[\underline{f}(Y,U) + \overline{f}(Y,U)\right] \tag{5}$$

Theorem 3: [8] The virtual sensor:

$$\widehat{z}^t = \phi_c\left(\widetilde{Y}^t, \widetilde{U}^t\right)$$

is almost optimal.

■

So far a global bound on $\|f_o'(w)\|$ over all W is assumed. However, a local approach can be taken in order to obtain improvements in identification accuracy by assuming a non-constant gradient bound, i.e., $\|f_o'(Y,U)\| \leq \gamma(Y,U)$. A very simple approach allowing to use variable bound on f_o' (local nonlinear Set-membership method), is based on the evaluation of a function f_a approximating f_o (using any desired method) and on the application of the Set-membership method to the residue function $f_\Delta(Y,U) \doteq f_o(Y,U) - f_a(Y,U)$ using the set of values $\Delta z^t = \widetilde{z}^t - f_a\left(\widetilde{Y}^t, \widetilde{U}^t\right)$, $t = 1, 2, ..., T_m$. Then, the estimate:

$$f_c^L(Y,U) = f_a(Y,U) + \phi_\Delta^c(Y,U) \tag{6}$$

is used, where $\phi_\Delta^c(Y,U)$ is the central estimate of $f_\Delta(Y,U)$ obtained from data $\Delta z^t,\ t = 1,2,...,T_m$.

Note that assuming a global bound $\|f_\Delta'(Y,U)\| = ||f_o'(Y,U) - f_a'(w)|| \leq \gamma_\Delta$ on the residue function f_Δ implies the local bound $\|f_a'(Y,U)\| - \gamma_\Delta \leq \|f_o'(Y,U)\| \leq ||f_a'(Y,U)|| + \gamma_\Delta$ for function f_o.

III. Sideslip angle virtual sensor design from data

The sideslip virtual sensor, designed exploiting the direct approach summarized in the previous section, has been tested on the experimental data obtained from a huge number of different manoeuvres (including steering angle steps of different amplitude from 30 to 80 degrees, lane change and double lane change) performed on dry roads with a passenger car provided by Fiat Auto S.p.A. The sideslip angle β is the output to be estimated $z^t = \beta^t$, the steering angle α_s is assumed as measured input $u^t = \alpha_S^t$ while the lateral acceleration a_y, the yaw rate $\dot{\psi}$ and the longitudinal velocity v_x have been used as measured output $y^t = [a_y^t\ \dot{\psi}^t\ v_x^t]$. Sideslip angle and vehicle velocity measurements have been collected using a DATRON® sensor. Although the DATRON® sensor is usually not available in ordinary car, in this work the longitudinal velocity measurements provided by such an instrument were used in y^t. The problem of designing a suitable virtual sensor for the estimation of the vehicle longitudinal speed v_x from measurements of quantities usually available in ordinary cars and the problem of evaluating the effects of such virtual sensor on the estimation of the sideslip angle are currently under study.

The application of the local nonlinear Set-membership method (NSM) to such a set of data, provided the following virtual sensor:

$$\widehat{\beta}^t = f_a(\widehat{\theta}, \widetilde{Y}^t, \widetilde{U}^t) + \phi_\Delta^c\left(\widetilde{Y}^t, \widetilde{U}^t\right)$$

where the function f_a is a one hidden layer neural network with $r = 5$ neurons:

$$f_a(\widehat{\theta}, \widetilde{Y}^t, \widetilde{U}^t) = \sum_{i=1}^{r} \alpha_i \sigma\left\{\mu_i \cdot \left[\widetilde{Y}^t, \widetilde{U}^t\right] - \lambda_i\right\} + \zeta \tag{7}$$

where $\widehat{\theta} = \{\alpha_i, \lambda_i, \zeta \in \Re,\ \mu_i \in \Re^n,\ i = 1, ..., r\}$ is a set of parameters, and $\sigma(x) = 2/(1 + e^{-2x}) - 1$ is a sigmoidal function. The dimension of Y^t has been set to $n_u = 10$ for a_y and $\dot{\psi}$ and to $n_u = 0$ for v_x, while the dimension of U^t has been set to $n_y = 10$. The neural network has been trained using standard algorithms of the MATLAB® Neural Networks toolbox®. The central estimate $\phi_\Delta^c\left(\widetilde{Y}^t, \widetilde{U}^t\right)$ of the residue function $f_\Delta\left(\widetilde{Y}^t, \widetilde{U}^t\right)$ has been estimated exploiting equation (5). The values $\varepsilon^t = 0.3$ degrees $\forall t$ and $\delta^t = 0\ \forall t$ have been assumed according to the sensor characteristics. The value $\gamma_\Delta = 2.56$ has been computed solving problem (4). In this section a subset of the obtained results is presented. More specifically the following maneuvers are considered:

- Steering angle step of 30 degrees performed at the velocity of 100 km/h (Figures 1,2 and 3).

- Steering angle step of 80 degrees performed at the velocity of 100 km/h (Figures 4,5 and 6).
- Double lane change performed at the velocity of 80 km/h (Figures 7,8 and 9).

The obtained results show that the proposed nonlinear estimator is able to provide a good sideslip angle estimation in a large range of operation covering both the linear ($a_y < 0.6$g) and the nonlinear ($a_y > 0.6$g) behaviour of the car.

IV. Conclusion

In this paper we have presented a sideslip angle estimator based on a recently proposed direct approach for the design of virtual sensors. The obtained sideslip virtual sensor has been tested on number of different manoeuvres including steering angle steps of different amplitude from 30 to 80 degrees and double lane change. The obtained results showed that the proposed nonlinear estimator is able to provide a good sideslip angle estimation in a large range of operation covering both the linear and the nonlinear behaviour of the car.

V. Acknowledgements

This research was supported by FIAT S.p.A., under the contract "Metodologie di impostazione e sviluppo algoritmi e strategie di controllo dell'autotelaio". The authors are grateful to Ing. G. Caviasso and Ing. S. Campo of FIAT for the fruitful discussions during the development of the project. The authors also wish to thank Ing. L. Zaffalon which helped in the development of the Matlab® routines for the design of the sideslip virtual sensor.

References

[1] J. Stèphant, A. Charara, and D. Meizel, "Experimental validation of vehicle sideslip angle observers," in *Proc. of 2004 IEEE Intelligent Vehicle Symposium*, pp. 150 – 155, 2004.

[2] J. Stèphant, A. Charara, and D. Meizel, "Evaluation of sliding mode observer for vehicle sideslip angle," in *Proc. of the 16th IFAC World Congress*, 2005.

[3] A. von Vietinghoff, M. Hiemer, and U. Kiencke, "Nonlinear observer design for lateral vehicle dynamics," in *Proc. of the 16th IFAC World Congress*, 2005.

[4] J. Stèphant, A. Charara, and D. Meizel, "Virtual sensor: Application to vehicle sideslip angle and transversal forces," *IEEE Transactions on Industrial Electronics*, pp. 278 – 289, 2004.

[5] U. Kiencke and A. Daiß, "Observation of lateral vehicle dynamics," *Control Eng. Practice*, pp. 1145 – 1150, 1997.

[6] A. Hac and M. D. Simpson, "Estimation of vehicle side slip angle and yaw rate," in *Proc. of SAE 2000 World Congress*, 2000.

[7] A. Krantz, J. Neubeck, and J. Wiedemann, "Estimation of side slip angle using measured tire forces," in *Proc. of SAE 2002 World Congress*, 2002.

[8] K. Hsu, M. Milanese, C. Novara, and K. Poolla, "Nonlinear virtual sensors design from data," in *14th IFAC Symposium on System Identification, SYSID 2006*, (Newcastle, Australia), 2006.

[9] M. Milanese, C. Novara, K. Hsu, and K. Poolla, "Filter design from data: direct vs. two-step approaches," in *American Control Conference 2006*, (Minneapolis, Minnesota, USA), 2006.

[10] M. Milanese and C. Novara, "Set membership identification of nonlinear systems," *Automatica*, vol. 40/6, pp. 957–975, 2004.

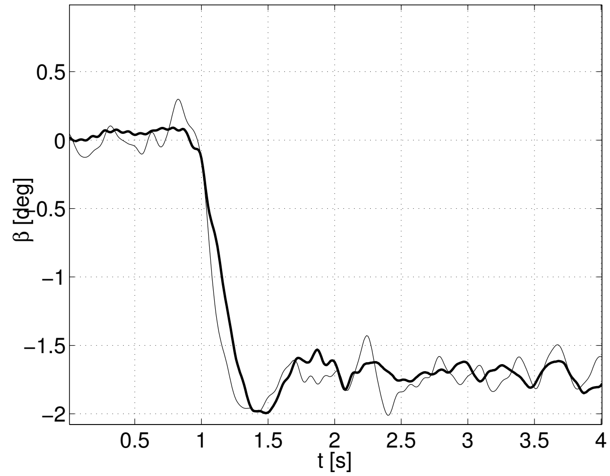

Fig. 1. Steering angle step of 30 degrees: measured β (thin) and estimated $\hat{\beta}$ (thick)

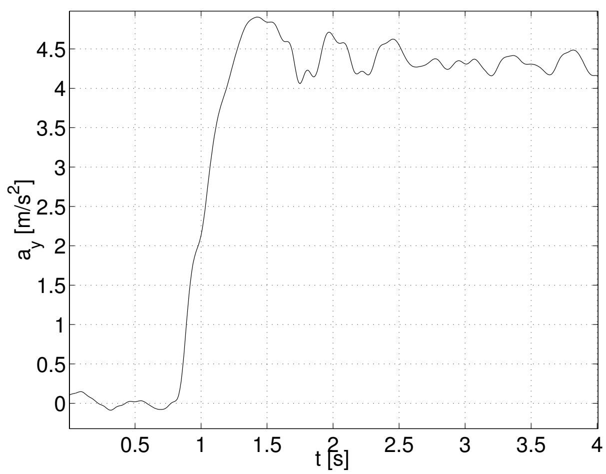

Fig. 2. Steering angle step of 30 degrees: lateral acceleration a_y

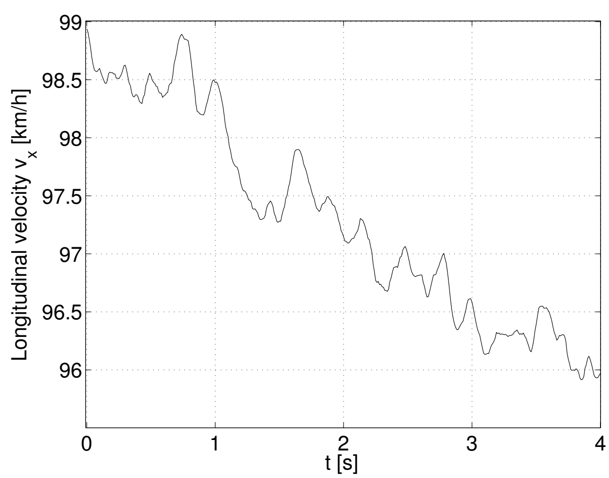

Fig. 3. Steering angle step of 30 degrees: longitudinal velocity v_x

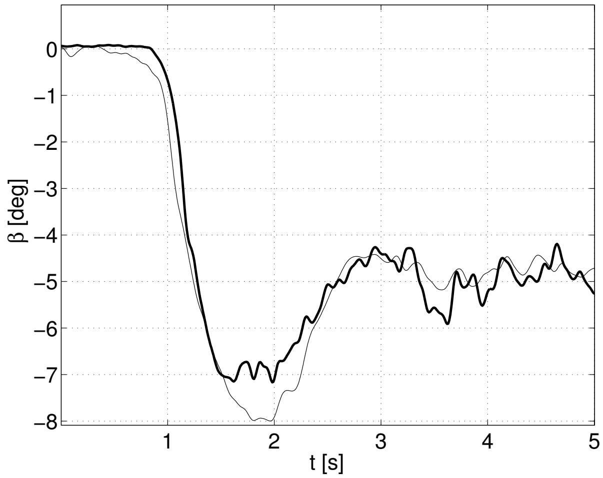

Fig. 4. Steering angle step of 80 degrees: measured β (thin) and estimated $\hat{\beta}$ (thick)

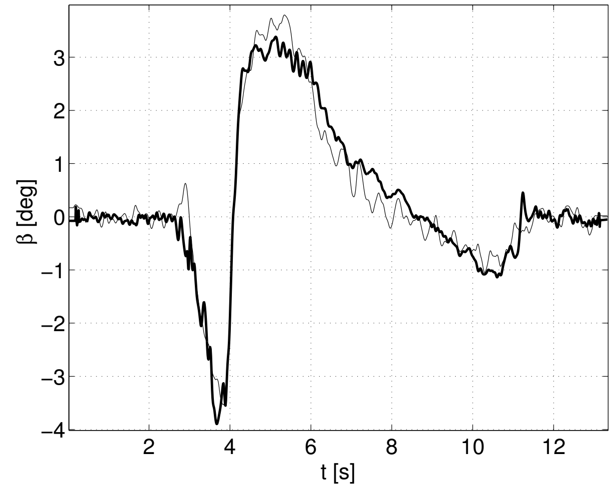

Fig. 7. Double lane change: measured β (thin) and estimated $\hat{\beta}$ (thick)

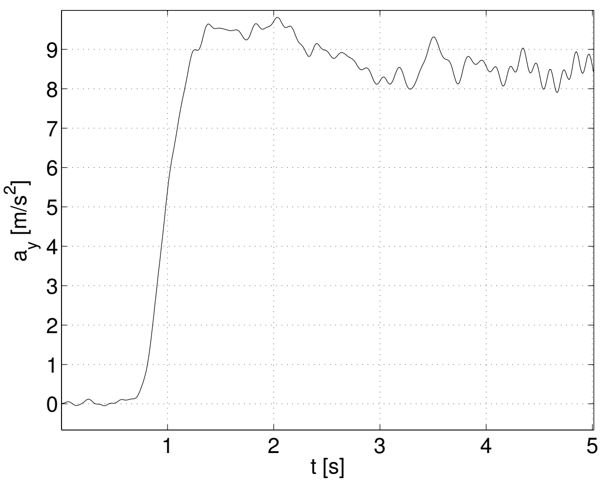

Fig. 5. Steering angle step of 80 degrees: lateral acceleration a_y

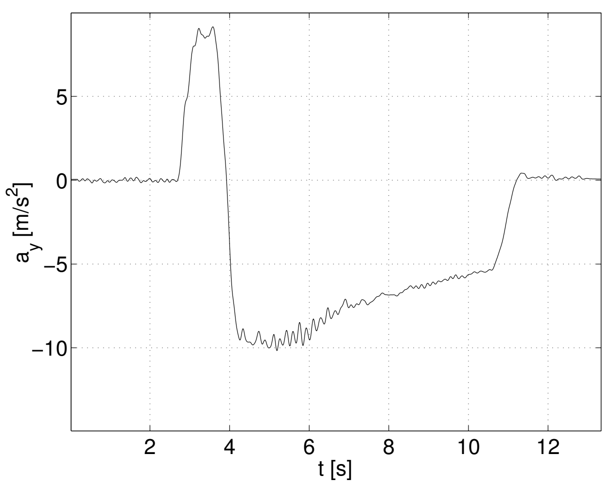

Fig. 8. Double lane change: lateral acceleration a_y

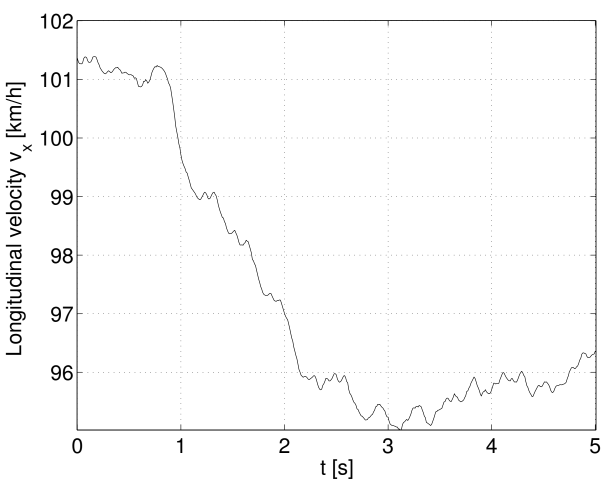

Fig. 6. Steering angle step of 80 degrees: longitudinal velocity v_x

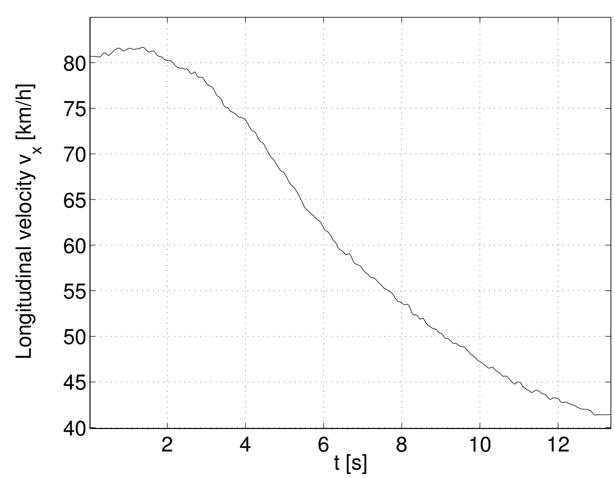

Fig. 9. Double lane change: longitudinal velocity v_x