Size-scale effects on interaction diagrams for reinforced concrete columns

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A B S T R A C T

The use of $N$–$M$ interaction diagrams is well established in the design of reinforced concrete columns, when the second order effects can be neglected. According to the stress–strain constitutive laws usually adopted to compute the resistant domains, complex phenomena such as size effects and concrete confinement cannot be considered in practical applications. On the other hand, several experimental evidences, and some analytical models available in the literature, emphasize the influence of such effects. In the present paper, a numerical approach based on the integrated Cohesive/Overlapping Crack Model is applied to compute the interaction diagrams. Compared to classical approaches, different constitutive laws are assumed for concrete in compression and tension, based on Nonlinear Fracture Mechanics models, and a step-by-step analysis is performed instead of limit state analysis. The proposed model permits the size and the confinement effects to be predicted, according to the experimental results. Moreover, the obtained results completely agree with previous extensive applications of the model to plain concrete specimens subjected to uniaxial compression and reinforced concrete beams in bending.

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1. Introduction

Columns are structural members subjected to combinations of axial compression, $N$, due to vertical loads, and bending moments, $M_x$ and $M_y$, induced by unbalanced moments at connecting beams, vertical misalignments, or lateral forces resulting from wind or seismic action. From a practical point of view, the design of reinforced concrete (RC) columns is performed by means of seismic action. From a practical point of view, the design of reinforced concrete (RC) columns is performed by means of seismic action. From a practical point of view, the design of reinforced concrete (RC) columns is performed by means of seismic action.

Typically, the interaction diagrams are drawn in dimensionless form, by normalizing the axial force and the bending moment with geometrical and mechanical parameters:

\[ v = \frac{N_{rd}}{A_{fc}} \]

\[ \mu = \frac{M_{rd}}{A_{fc}h_{fc}} \]

where $A_{c}$ is the gross cross-sectional area of the column, $h$ is the dimension of the cross-section side parallel to the action plane of the bending moment, and $f_{c}$ is the concrete compressive strength. As a result, the obtained curves are only functions of the position of the reinforcing bars within the cross-section and of the mechanical percentage of the longitudinal reinforcement, defined as follows:

\[ \omega = \frac{A_{ls}f_{y}}{A_{fc}} \]

where $A_{ls}$ is the total area of the longitudinal reinforcement, and $f_{y}$ is the steel yielding strength.

The interaction diagrams obtained on the basis of the Eurocode 2 [1] prescriptions for mechanical reinforcement percentages equal to 0.00, 0.25 and 0.50 are shown in Fig. 1 (note that positive axial forces represent compression loads). In particular, they have been computed by assuming a stress–strain curve composed by an initial parabola followed by a horizontal straight line (parabola-rectangular law) for concrete in compression and an elasto-plastic constitutive relation for steel. All the safety coefficients are set equal to unity, since we are interested in the actual resistant

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domains, and not to those used for design purposes. The concrete compressive strength has been set equal to 40 MPa, and the steel yield strength to 400 MPa. Only two layers of reinforcement have been considered, 0.10 \( h \) apart from the cross-section sides. In order to emphasize the effect of the constitutive laws, the interaction diagrams computed according to the prescriptions of the ACI 318 Building Code [2] are also shown in Fig. 1 (dashed lines). The main differences are related to the constitutive law for concrete in compression (equivalent rectangular stress distribution instead of parabola-rectangular) and to the limit strain for concrete (0.003 instead of 0.0035 as prescribed by Eurocode 2). Furthermore, the value of \( \mu \) relative to the case of pure compression is lower than that of Eurocode, because a minimum eccentricity is assumed to account for accidental eccentricities not considered in the Eurocode 2 approach.

Due to the large number of parameters involved, the present topic has been deeply studied from an experimental point of view since 1950s. Many experimental campaigns have been performed to investigate the effects of biaxial loadings, sustained loadings and slenderness on columns made of conventional concrete. More recently, similar investigations have been carried out on high strength concrete columns. In this case, the slenderness effect becomes predominant, since the increase in the concrete strength gives rise to the design of slender frames, more sensitive to the second order effects. A comprehensive collection of experimental tests carried out in the second half of the last century is reported in Pallarés et al. [3]. Finally, in the last few years, the interest is mainly turned to the behaviour of reinforced concrete columns confined by fibre reinforced polymers (FRP) jackets, due to the frequent use of this technique to increase load carrying capacity and/or ductility. Many experimental tests on columns of circular or rectangular cross-section investigated the increase in the load carrying capacity as a function of the mechanical parameters and the amount of FRP [see Rocca et al. [4] for a summary of the results]. In this context, some analytical models have also been proposed to generate interaction diagrams of concrete columns confined by stirrups and/or FRP jackets [4–6]. As far as size effects are concerned, only few experimental investigations are available in the literature: the eccentric compression tests on RC columns by Bažant and Kwon [7], the compact compression tests on plain concrete specimens by Barr et al. [8], the eccentric compression tests on plain concrete elements by Kim et al. [9], and the axial compression tests on plain and reinforced concrete columns by Sener et al. [10,11]. In general, a reduction in the dimensionless load-carrying capacity by increasing the specimen size has been highlighted. Such a phenomenon is particularly evident in the results from Bažant and Kwon, shown in Fig. 2, although they are also affected by the slenderness. In the light of these experimental evidences, it is worth noting that the usual interaction diagrams for design purposes, Fig. 1, do not account the effect of the size, and they are not influenced by concrete confinement.

In the present paper, a new approach based on nonlinear fracture mechanics is proposed to compute interaction diagrams for
RC elements subjected to a combination of axial force and uniaxial bending moment. The main novelty with respect to classical approaches based on stress–strain constitutive relations is the use of the Cohesive Crack Model and the Overlapping Crack Model. Such constitutive laws relate the stresses to the relative normal displacements evaluated along the region where strain localization takes place. They are independent of the specimen size. First, the main features of these models will be introduced, and their effectiveness will be proved in comparison with experimental data. Then, a numerical algorithm will be proposed for a step-by-step analysis of the mechanical response of a RC column up to the ultimate load. In this case, in fact, due to the complexity of the assumed constitutive laws, the load carrying capacity cannot be obtained by means of simple equilibrium equations written with reference to the ultimate limit state deformed configurations. Finally, size-scale and confinement effects on the interaction curves will be elucidated with detailed numerical examples.

### 2. Constitutive models

#### 2.1. Overlapping Crack Model for concrete in compression

The stress–strain constitutive laws proposed by Standards for concrete are suitable for practical purposes, although they represent simplifications of the actual material behaviour. The softening branch as well as the localization of strain and the energy dissipation in the post-peak regime are neglected. More precisely, when a nonlinear analysis is carried out, constitutive laws taking into account the softening branch are also provided, as for example the Sargin’s parabola. However, also in this case, as remarked in the comments to Section 2.1.4.4 of the Model Code 90 [12], such a relationship is reasonably accurate only for a very limited range of member sizes, since it is not a material property. According to experimental results and analytical models, scale-independent constitutive laws may be obtained only if stress–displacement (fictitious interpenetration) relationships are adopted for the post-peak behaviour, instead of the classical stress–strain laws. In this case, good predictions of several compression tests carried out on concrete-like specimens with different sizes and slendernesses have been obtained on the basis of the Overlapping Crack Model, proposed by Carpinteri et al. in 2007 [13,14]. The fundamental hypotheses of such a model are detailed below.

1. The constitutive law used for the undamaged material is a linear-elastic stress–strain relationship characterized by the values of the elastic modulus, $E_c$, the compressive strength, $f_c$, and the ultimate elastic strain, $\varepsilon_c$ (see Fig. 3a).
2. The crushing zone develops when the maximum compressive stress reaches the concrete compressive strength. The crushing zone is then represented by a fictitious interpenetration, analogous to the fictitious crack in tension.
3. The damaged material in the process zone is assumed to be able to transfer compression stresses between the overlapping surfaces. Concerning such stresses, they are assumed to be decreasing functions of the interpenetration displacement, $w^c$ (see Fig. 3b). In compression, experimental results have shown that the stress does not vanish, but it tends to an asymptotic residual value for overlapping displacements larger than the critical interpenetration $w^c_c \approx 1\, \text{mm}$ (see Fig. 3b). As a result, quite complicated relationships should be assumed for pure compression tests, as that proposed by Carpinteri et al. [14] for plain and fibre reinforced concrete. However, the following simplified linear softening expression is considered for modelling purposes:

\[
\sigma = f_c \left(1 - \frac{w^c}{w^c_c}\right).
\]

This is a reasonable approximation because only the range $0.0 < w^c/w^c_c < 0.5$ is usually utilized in the numerical simulations of plain concrete specimen subjected to eccentric compression, as well as of RC elements in bending (see Carpinteri et al. [15,16] for a wide comparison with experimental results).

The crushing energy, $G_c$, defined as the area below the post-peak softening curve of Fig. 3b, is considered as a material property, since it is not affected by the structural size. The following empirical equation for calculating the crushing energy has recently been proposed by Suzuki et al. [17] for unconfined concrete:

\[
G_{c,0} = 80 - 50k_0,
\]

where $k_0$ depends on the concrete compressive strength:

\[
k_0 = \frac{40}{f_c} \leq 1.0.
\]

By varying the concrete compressive strength from 20 to 90 MPa, Eq. (5) gives a crushing energy ranging from 30 to 58 N/mm. In the case of confinement exerted by stirrups, the crushing energy...
turns to be a function of the stirrups yield strength and the stirrups volumetric content (see again Suzuki et al. [17]):

$$G_C = \frac{G_{c0}}{1 + 10,000 \cdot k_f / k_c},$$  \hspace{1cm} (7)

where $f_s$ is the average concrete compressive strength, $k_s$ is a parameter depending on the stirrups strength and volumetric content:

$$k_s = 1 + k_f \cdot (f_y - f_s) / f_y,$$  \hspace{1cm} (8)

$p_c$ is the effective lateral pressure:

$$p_c = k_c \cdot p_{f_s} / f_y.$$  \hspace{1cm} (9)

$k_c$ is the effective confinement coefficient:

$$k_c = \left( 1 - \frac{w_s^2}{b_s d_s} \right) \left( 1 - \frac{s^2}{2b_c} \right) \left( 1 - \frac{s^2}{2d_c} \right) / (1 - \rho_{c0}).$$  \hspace{1cm} (10)

2.2. Cohesive Crack Model for concrete in tension

In the present formulation, the Cohesive Crack Model is adopted in order to accurately describe the tensile contribution of concrete. Proposed by Hillerborg et al. in 1976 [18], it has been largely used, in the past, to study the ductile-to-brittle transition in plain concrete beams in bending [19]. According to this model, the constitutive law for the undamaged zone is a $\sigma$–$e$ linear-elastic relationship up to the achievement of the tensile strength, $f_t$ (Fig. 3c). In the process zone, the damaged material is still able to transfer a tensile stress across the crack surfaces. The cohesive stresses are considered to be linear decreasing functions of the crack opening displacement, $w'$ (see Fig. 3d):

$$\sigma = f_t \left( 1 - \frac{w'}{w_{cr}^t} \right).$$  \hspace{1cm} (12)

where $w_{cr}$ is the critical value of the crack opening corresponding to $\sigma = 0$. The area beneath the stress vs. displacement curve in Fig. 3d represents the fracture energy, $G_f$.

2.3. Steel–concrete interaction

According to the constitutive laws adopted for concrete in tension and compression, also the behaviour of steel is described by a couple of relationships. As long as the reinforcing bars lie in the elastic region, perfect bond between concrete and steel is assumed, and the stress–strain linear-elastic relationship shown in Fig. 3e is used for calculation. When they fall into the fictitious crack or fictitious overlapping zones, a more consistent stress versus displacement law is introduced (Fig. 3f). Such a relation is derived from considering the slip between reinforcing bars and surrounding concrete. More precisely, from considering the relationship between the tangential stress along the steel–concrete interface and the relative tangential displacement, as proposed in the Model Code 90 [12]. By imposing equilibrium and compatibility conditions, it is possible to correlate the reinforcement reaction, given by the integration of the bond stresses acting along the bar, to the relative slip at the crack edge, which corresponds to half the crack opening displacement. Typically, the obtained relationship is characterized by an ascending branch up to steel yielding, after which the reaction is nearly constant. In this way, the actual bond-slip shear stress distribution is replaced by a pair of concentrated forces acting along the crack faces, functions of the opening displacement at the reinforcement level (see Ruiz et al. [20] for a parametric study on the bond-slip properties). The same constitutive law has been adopted in tension as well as in compression.

3. Numerical algorithm for eccentric compression tests on RC members

In this section, an improved version of the numerical algorithm developed by Carpinteri et al. [16] describing the mechanical behaviour of RC beams in bending is proposed as an alternative way for evaluating the $N$–$M$ interaction diagrams. To this aim, a RC element having cross-section height $h$, width $b$, and length $l$ equal to twice $h$, as shown in Fig. 4, is analysed to obtain the bending moment capacity for a given axial load. By increasing the applied moment, the considered element shows the development and growth of a crushing zone in the compression side and of a tensile crack in the opposite one. In close analogy with the behaviour of concrete specimens subjected to uniaxial compression [14], all the nonlinear contributions in the post-peak regime are localized along the middle cross-section where interpenetration and crack propagation take place, while the two half-specimens exhibit an elastic behaviour, as shown in Fig. 4b and c.

It is assumed that the stress distribution in the middle cross-section is linear-elastic until the maximum compression stress reaches the concrete compressive strength. When this threshold is reached, concrete crushing is assumed to take place and a fictitious overlapping zone propagates towards the opposite side of the specimen. Outside the overlapping zone, the material is assumed to behave linear-elastically. The stresses in the overlapping zone are assumed to be a function of the interpenetration displacement, according to the overlapping cohesive law in Fig. 3b.

The middle cross-section of the specimen is subdivided into finite elements by $n$ nodes (Fig. 4b). In this scheme, overlapping or cohesive stresses are replaced by equivalent nodal forces by integrating the corresponding pressures or tractions over each element size. Such nodal forces depend on the nodal closing or opening displacements according to the overlapping or cohesive softening laws. The vertical forces, $F$, acting along such a cross-section can be computed as follows:

$$F = [K_w] \cdot \{w\} + [K_M] \cdot M + [K_N] \cdot N$$  \hspace{1cm} (13)

where $\{F\}$ is the vector of nodal forces, $[K_w]$ is the matrix of the coefficients of influence for the nodal displacements, $\{w\}$ is the vector of nodal displacements, $[K_M]$ is the vector of the coefficients of influence for the bending moment, $M$, and $[K_N]$ is the vector of the coefficients of influence for the axial force, $N$. In the generic situation shown in Fig. 4c, the following equations can be considered, taking...
into account, respectively, the stress-free crack (Eq. (14a)), the linear cohesive softening law (Eq. (14b)), the undamaged zone (Eq. (14c)), the linear overlapping softening law (Eq. (14d)), and the steel constitutive law (Eq. (14e)):  

\[
\begin{align*}
F_i &= 0 \quad \text{for } i = 1, \ldots, (j - 1) \\
F_i &= F_c \left(1 - \frac{w_i}{W_0}\right) \quad \text{for } i = j, \ldots, (m - 1) \\
w_i &= 0 \quad \text{for } i = m, \ldots, p \\
F_i &= F_c \left(1 - \frac{w_i}{W_0}\right) \quad \text{for } i = (p + 1), \ldots, n \\
F_i &= f(w_i) \quad \text{for } i = r, s.
\end{align*}
\]  

Eqs. (13) and (14) constitute a linear algebraic system of \((2n + 1)\) equations in \((2n + 1)\) unknowns, namely \(\{F\}, \{w\}\) and \(M\), the axial load \(N\) being fixed. A possible additional equation can be chosen: we can set either the force in the cohesive crack tip, \(m\), equal to the ultimate tensile force, or the force in the overlapping crack tip, \(p\), equal to the ultimate compression force. In the numerical scheme, we select the situation which is closer the critical condition. This criterion will ensure the uniqueness of the solution on the basis of physical arguments. The driving parameter of the process is the position of the tip that in the considered step has reached the related resistance. Only this tip is moved when passing to the next step.

Finally, at each step of the algorithm, it is possible to calculate the beam rotation, \(\vartheta\), as follows:  

\[
\vartheta = \{D_w\}^T \{w\} + D_\theta M .
\]  

where \(D_w\) is the vector of the coefficients of influence for the nodal displacements, and \(D_\theta\) is the coefficient of influence for the applied bending moment.

### 3.1. Computation of the elastic coefficients

Eq. (13) permits to analyse the fracturing and crushing processes in the mid-span cross-section, taking into account the elastic behaviour of the RC member. To this aim, all the coefficients are computed \(a \ priori\) using a finite element analysis. Due to the symmetry of the problem, a homogeneous concrete rectangular region, corresponding to half the tested specimen shown in Fig. 4, is discretized by means of plane stress quadrilateral elements with uniform nodal spacing, as shown in Fig. 5. In particular, 101 nodes have been considered along the horizontal symmetry edge. Then, vertical constraints are applied to such nodes. The coefficients \(K_i\) entering Eq. (13), which relate the nodal force \(F_i\) to the nodal displacement \(w_i\), have the physical dimensions of a stiffness and are computed by imposing a unitary vertical displacement to each of the constrained nodes. On the other hand, by applying a unitary external bending moment, it is possible to compute the coefficients \(K_i\). They have the physical dimensions of \([L]^{-1}\). Analogously, the coefficients \(K_i\) represent the vertical constraint reactions due to a unitary axial force applied to the free edge.

The contribution of the reinforcing bars is taken into account by introducing a homogenized elastic modulus for the finite elements within the strips containing the rebars (shaded areas in Fig. 5). Such a modulus is determined on the basis of the relative amount of concrete and steel and their elastic moduli, \(E_c\) and \(E_s\);  

\[
E_{\text{hom}} = E_c (1 - \rho^*) + E_s \rho^* .
\]

where \(\rho^*\) is the ratio between the reinforcement area and the area of the strip containing the rebars:  

\[
\rho^* = \frac{A_{\text{steel,tot}}}{2cb} .
\]

\(A_{\text{steel,tot}}\) is the total amount of steel reinforcement, subdivided into two layers close to the cross-section sides, and \(b\) is the cross-section...
width. The thickness of the strips, \( c \), and the number of finite elements affected by the presence of steel is proportional to the global amount of steel reinforcement. In this study, three different values for the mechanical steel percentage have been considered: \( \omega = 0.00 \), which means zero reinforcement, \( \omega = 0.25 \), to which corresponds \( c = 0.03h \) (two finite elements), and \( \omega = 0.50 \), to which corresponds \( c = 0.05h \) (four finite elements).

As regards the analysis of size-scale effects, the coefficients entering Eqs. (13) and (15) are connected by a simple relation of proportionality to the structural dimension. This means that it is not necessary to repeat the finite element analysis for any different considered beam size. More precisely, if all the three specimen sizes (height \( h \), span \( L \), width \( b \)) are multiplied by a factor \( k \), then the coefficients are transformed as follows:

\[
K_w^{(h)}(kh) = k K_w^{(h)},
\]

\[
K_m^{(h)}(kh) = \frac{1}{k} K_m^{(h)},
\]

\[
D_w^{(h)}(kh) = \frac{1}{k} D_w^{(h)},
\]

\[
D_h^{(h)}(kh) = \frac{1}{k} D_h^{(h)}.
\]

4. Numerical results

4.1. Size-scale effects

In this section, the interaction diagrams for \( \omega = 0.00 \), 0.25 and 0.50 are computed by applying the proposed integrated Cohesive/Overlapping Crack Model on columns with four different values of the cross-section side in the range 1:8 (\( h = 100, 200, 400 \) and 800 mm). As regards the concrete compressive strength and the crushing energy, they are referred to unconfined concrete \( (f_c = 40 \text{ MPa}, \sigma_C = 30 \text{N/mm}) \). The diagrams are composed of a series of straight lines connecting a few points, each of them representing a pair of values \( N_{ho} \) and \( M_{ho} \). More in detail, the step-by-step analysis is performed to evaluate the maximum bending moment corresponding to a given value for the applied axial force. The obtained curves are shown in Figs. 6–8, in dimensionless form.

As far as the case \( \omega = 0.00 \) is concerned, Fig. 6, the left portion of the diagrams (corresponding to tensile axial loads or low values of compression) obtained from numerical simulations is above that obtained on the basis of Eurocode 2 prescriptions. This is due to the concrete tensile contribution neglected by the Eurocode 2. In this zone, only slight scale effects are evidenced. On the contrary, the right portion, just after the apex of the curves, is highly affected by size effects. In this zone, where concrete crushing collapse takes place, shrunk resistant domains are obtained by increasing the structural dimension. As regards the case \( \omega = 0.25 \) (Fig. 7), in the range of \( v \) from –0.25 to 0.15, where the collapse is due to the steel yielding on the tensile side, a good overlapping is obtained with respect to the Eurocode 2 curve. On the contrary, size effects characterize the behaviour for values of \( v \) higher than 0.4. Furthermore, it has also to be noted that, after the peak, the obtained curves are below the Eurocode 2 diagram. This is due to the softening behaviour of concrete in compression and to the fact that the compressive strengths of concrete and steel are reached in correspondence with different values of the relative normal displacement. More in detail, when the concrete stress is equal to \( f_c \), the steel is not yet yielded, and when the steel is yielded, the concrete lies in the softening branch (see Fig. 3). Similar comments hold for \( \omega = 0.50 \), as can be seen in Fig. 8. A comparison between the diagrams referred to the three considered steel percentages reveals that size effects slightly decrease by increasing the steel amount. The moment versus rotation curves for \( h = 400 \text{ mm} \), \( \omega = 0.25 \) and different dimensionless axial forces are shown in Fig. 9. The overall behaviour is very ductile for low values of \( v \), due to tensile yielding of steel. Then, the ductility decreases and the maximum resistant moment increases for \( v \) values up to approximately 0.375. Finally, from 0.375 to 1.000, the load carrying capacity decreases again and the ductility slightly increases due to concrete crushing. Beyond the ultimate load, a softening branch occurs according to the assumed Overlapping Crack Model.

4.2. Effect of concrete confinement

According to the proposed model, concrete confinement influences the crushing energy and the compressive strength characterizing the Overlapping Crack Model. The higher the confinement, the higher the strength and the dissipated energy. When the confinement is exerted by stirrups, the relations expressed in Eqs. (5)–(11) can be used to define the correct constitutive law. As an example, the interaction diagrams referred to columns with cross-section \( 400 \times 400 \text{ mm} \), geometrical reinforcement ratio \( \omega = 0.25 \), and different amounts of stirrups are shown in Fig. 10.
Fig. 7. Interaction diagrams according to the proposed model for $\omega = 0.25$ and different column dimensions, $h$.

Fig. 8. Interaction diagrams according to the proposed model for $\omega = 0.50$ and different column dimensions, $h$.

Fig. 9. Moment–rotation curves obtained with the proposed model for $\omega = 0.25$, $h = 400$ mm, and different dimensionless axial loads, $v$. 
More in detail, the spacing between the stirrups is kept constant and equal to 0.15 m, whereas different diameters have been considered: 4, 6 and 8 mm, to which correspond geometrical ratios, \( q_w \), equal to 0.10%, 0.27% and 0.38%. The stirrup yielding strength has been assumed equal to 400 MPa. By assuming \( f_c = 40 \) MPa for the unconfined concrete, the values of the compressive strength obtained on the basis of Eq. (11) for the three considered stirrups amounts are: \( f_{cc} = 42.8, 45.7 \) and 47.2 MPa, respectively. The increase in the crushing energy results to be more relevant: starting from 30 N/mm for unconfined concrete, the values 61, 140 and 195 N/mm are obtained for the three different stirrups amounts according to Eq. (7). The curves in Fig. 10, compared to that prescribed by Eurocode 2, evidence a high influence of the confinement on the load carrying capacity. The right portion of the interaction diagrams, where crushing collapse prevails over steel yielding, expands as \( q_w \) increases. The curve provided by the European Standards is close to that referred to \( q_w = 0.10\% \).

The same approach can be used when the confinement is exerted by FRP jackets. In this context, some models are available in the literature to define the stress–strain response for FRP-confined concrete [4,21]. In particular, they permit to evaluate the confined compressive strength and the ultimate strain of concrete, whereas no specific study has been performed to compute the crushing energy. A comparison between numerical and experimental results is shown in Fig. 11. The experimental data are taken from the tests carried out by Iacobucci et al. [22]. Three different volumetric ratios of FRP reinforcement were considered, \( \rho_f = 1.31\%, 2.62\% \) and 3.93\%, where \( \rho_f \) is equal to \( 2n t_f (b + h) / (b h) \) for non-circular columns (\( n \) is the number of FRP layers, \( t_f \) is the layer thickness, \( b \) and \( h \) are the width and the height of the cross-section). The unconfined concrete compressive strength is equal to 36.5 MPa, the crushing energy to 30 N/mm and the steel yield strength to 465 MPa. The values of the compressive strength obtained on the basis of the model provided by Lam and Teng [21] for the three considered FRP amounts are: \( f_{cc} = 40.8, 45.6 \) and 50.4 MPa, respectively. The values of the crushing energy obtained by applying Eq. (7), entering the confining pressure obtained from the model by Lam and Teng [21] are: 803, 1598, and 2382 N/mm. The confinement effects predicted by the proposed model are slightly larger than those evidenced by the experiments, as shown in Fig. 11. In this context, a specific Overlapping Crack Model should be calibrated on the basis of the experimental results. However, it is worth noting that the load carrying capacity is highly affected by the concrete compressive strength, whereas
the crushing energy prevalently improves the ductility of the element, i.e., the rotational capacity at the ultimate condition (see also the numerical analysis carried out by Carpinteri et al. [16] on the rotational capacity of RC beams in bending).

5. Conclusions

In the present paper, the load carrying capacity of RC elements eccentrically loaded has been investigated on the basis of the integrated Cohesive/Overlapping Crack Model. In particular, interaction diagrams for columns without second order effects have been computed taking into account the main parameters involved in the problem. The use of cohesive constitutive laws for concrete in tension and compression, where the softening branch, the localization of strain and the energy dissipation in the post-peak regime are explicitly considered, permits the size effects to be described. At the same time, also the confinement effect can be effectively considered, by modifying both concrete compressive strength and crushing energy.

The obtained numerical results evidence that the load carrying capacity significantly decreases by increasing the structural dimensions, when the global collapse is due to concrete crushing, i.e., for intermediate and high values of the applied axial force. Such results represent an extension of the size-effects analysis, previously performed by the authors for RC beams in bending [23], to more generic loading conditions. In this case, further validations of the proposed approach should be accomplished by means of specific experimental campaigns, since only few experimental data are available in the literature. As far as the concrete confinement effect is concerned, the load carrying capacity turns to be an increasing function of this parameter, according to the experimental results. Also in this case, the most relevant effects are obtained when the global collapse is due to concrete crushing.

As evidenced by the comparison shown in Fig. 11, a specific Overlapping Crack Model should be calibrated on the basis of the experimental results when the confinement is exerted by FRP reinforcement. The models available in the literature, in fact, do not consider stress–displacement relationships for the post-peak behaviour and do not define the crushing energy.

Future developments may regard the extension of the proposed approach to analyze columns with more than two layers of reinforcement within the cross-sections, as well as to study the interaction between mechanical and geometrical nonlinearities, in order to consider the slenderness effects on the interaction diagrams.

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