

ARTICLE

Size-scale effects in high-performance reinforced and prestressed concrete T-beams

Renato Cafarelli¹  | Federico Accornero²  | Alberto Carpinteri^{1,3}

¹DISEG, Politecnico di Torino, Turin, Italy

²College of Engineering, Shantou University, Shantou, China

³Zhujiang (Pearl River) Professor of Guangdong Province, College of Engineering, Shantou University, Shantou, China

Correspondence

Renato Cafarelli, DISEG, Politecnico di Torino, Turin, Italy.

Email: renato.cafarelli@polito.it

Abstract

Reinforced concrete (RC) and prestressed concrete (PC) structural elements need to be designed in order to guarantee large plastic deformations, avoiding any loss in their load bearing capacity. In this framework, the rotation capacity of RC and PC beams has been demonstrated to be a function of concrete mechanical properties, reinforcement characteristics, and of the structural size. On the other hand, Theory of Plasticity as well as the International Standards completely disregard size-scale effects and ductile-to-brittle transitions, leading to an overlook of the strain-softening behavior of the concrete matrix and of the rotation capacity of RC beams. On the other hand, the Cohesive/Overlapping Crack Model is able to evaluate concrete cracking in tension and concrete crushing in compression, as well as snap-back and snap-through unstable phenomena, steel yielding and/or slippage. This Nonlinear Fracture Mechanics model predicts a reduction in the moment versus rotation plastic plateau by increasing the beam depth and/or the reinforcement percentage. The numerical investigations carried out on reinforced and prestressed high-performance concrete beams having rectangular or T-shaped cross-sections highlight the size-scale effects on plastic rotation capacity that allow to formulate new scale-dependent upper and lower limits of reinforcement percentage to guarantee a stable and ductile post-peak behavior of reinforced concrete structures.

KEYWORDS

high-performance prestressed concrete, maximum reinforcement, minimum reinforcement, reinforced concrete, scale effects, T-beams

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1 | INTRODUCTION

A comprehensive description of the flexural behavior of reinforced concrete (RC) structural elements must take into account the strain-softening and strain-localization of the concrete matrix, its different performances in tension and compression, and the mechanical instability

phenomena occurring in the loading process, such as concrete cracking in tension, reinforcement yielding, and concrete crushing in compression. Typical load-deflection diagrams can be characterized in their early stages by an elastic regime up to the first cracking load, P_{cr} , followed by a virtual unstable branch along which there is a decrease in both external load, P , and deflection, δ . This branch is commonly neglected during laboratory tests, since a snap-through behavior, with a sudden increase in deflection, or a snap-back instability, with a catastrophic loss in bearing capacity, is registered. More generally, evidences of snap-back and snap-through instabilities may appear in the response of thin cylindrical shells under axial compression as outlined by von Kármán and Tsien,¹ and in the response of complete spherical shells and spherical caps subjected to external pressure, as highlighted by Carlson et al.² and Kaplan.³ Snap-through local instabilities are often observed in the case of composite materials such as reinforced concrete and fiber-reinforced mortar, due to the fact that the reinforcements act as crack arresters, producing a global ductile response as observed by Zhu and Bartos.⁴

In this framework, effective interpretations of such instability phenomena have been proposed by means of the Bridged Crack Model⁵⁻⁷ highlighting the crucial role of a nondimensional parameter called *reinforcement brittleness number*, N_p , governing the ductile versus brittle response of the structural element. On the other hand, analytical and numerical investigations carried out by means of the Cohesive Crack Model on plain concrete specimens⁸⁻¹⁰ evidenced another nondimensional parameter, the *matrix brittleness number*, s_E , which is able to capture the transition from stable to unstable behavior either by increasing the specimen size, and/or the material tensile strength, σ_t , and/or by decreasing the fracture energy, G_F . As a matter of fact, modern design approaches require that RC structures present a sufficient deformation capacity¹¹ for warning before failure, the possibility of moment redistribution in the case of statically indeterminate structures, the ability to react to accidental loadings (robustness), together with energy dissipation in case of earthquake and fire resistance. In this framework, the nonlinear behavior of RC beams has been demonstrated¹² to be dependent on mechanical, geometrical, and loading characteristics, suggesting that simplified models¹³ are inadequate to predict plastic resources of real structures. Different numerical and experimental investigations¹⁴ have demonstrated that a decrease in plastic rotation capacity becomes manifest by increasing the reinforcement ratio, ρ , and/or by increasing the beam depth.¹⁵⁻¹⁸

Several models have been proposed to predict RC plastic rotation capacity, among which we can mention the Hillerborg Model,¹⁹ the Naples Model,^{20,21} and the Delft Model,^{22,23} among others. In the present paper, the

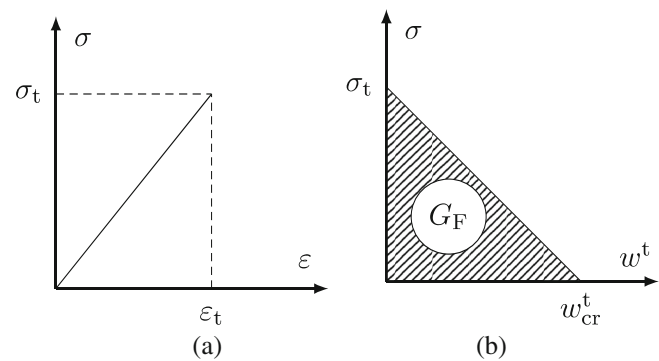


FIGURE 1 Cohesive Crack Model for concrete: (a) Linear elastic stress-strain law in tension; (b) Postpeak σ - w^t cohesive relationship.

Cohesive/Overlapping Crack Model²⁴ together with a Dimensional Analysis^{25,26} approach are used to investigate the effects of beam depth and cross-section shape, concrete grade, as well as reinforcement percentage on the plastic rotation capacity of RC beams.

The Cohesive Crack Model^{9,10} is able to describe the damage evolution in the tension zone of a concrete cross-section subjected to bending, assuming a linear elastic constitutive law (Figure 1a) up to the first peak load, and a constitutive law in the form σ - w^t , being σ the applied stress and w^t the crack opening displacement, in the zone where strain localization in tension occurs. More precisely, within the Cohesive Crack Model, a fictitious crack,^{27,28} longer than the real one, is introduced, providing a damage process zone where the material is still able to transfer tensile closing forces, albeit partially damaged. Thus, the residual bearing capacity of the structural element is simulated by means of the application of closing forces along the crack faces according to the constitutive law of Figure 1b, where w_{cr}^t is the threshold value of crack opening beyond which the closing forces vanish. The area subtended by the σ - w^t diagram represents the fracture energy of the concrete matrix, G_F , that can be determined according to Model Code 2010²⁹ as:

$$G_F = 0.073(\sigma_c + 8)^{0.18} \quad (1)$$

σ_c being the concrete compression strength. The Cohesive Crack Model has been effectively applied in the study of plain concrete or lightly reinforced/high-strength concrete beams, not considering the potential crushing failure occurring in the compression zone of RC beam cross-sections.^{27,28,30,31}

The constitutive law currently adopted in Model Code 2010²⁹ for concrete in compression is based on the experimental and numerical investigations carried out by Hognestad,³² Popovic³³ and Meyer³⁴:

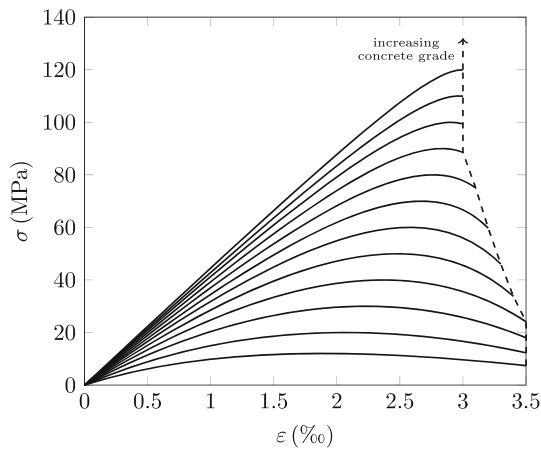


FIGURE 2 Stress–strain curves for different concrete grades (adapted from *fib* Model Code 2010).²⁹

$$\sigma = -\sigma_c \frac{k\eta - \eta^2}{1 + (k-2)\eta} \quad (2)$$

k and η being parameters that are functions of the concrete strength. Equation (2) is represented in Figure 2 for concrete grades between C12 and C120, suggesting that this constitutive law is not able to thoroughly predict the ductile-to-brittle transition occurring for high-performance concrete (HPC) since in the postpeak regime the variation from a negative (softening) to a positive (snap-back) slope by increasing the concrete strength is not captured.^{35,36} As a matter of fact, recent experimental investigations pointed out that postpeak stress–strain relationships are highly influenced by testing conditions, specimen shape, and specimen slenderness.^{37–39} In this framework, the Round Robin⁴⁰ organized by RILEM in 1997 outlined that, in order to effectively take into account the strain localization of concrete in compression, an effective constitutive law in the postpeak regime must be identified in a σ - w^c diagram, being w^c an inelastic displacement, rather than in a classic σ - ϵ diagram. The experimental investigations carried out by RILEM leads to the definition of the crushing energy, G_c , and several experimental campaigns^{41–43} have been carried out to assess this parameter for both normal-strength concrete (NSC) and HPC. Moreover, constitutive models based on a Fracture Mechanics approach have proved to be capable of thoroughly predict the damaging process of concrete in compression.^{44,45}

In this framework, Carpinteri and co-workers⁴⁵ introduced the Overlapping Crack Model, which is able to describe the crushing damage of concrete through a fictitious interpenetration zone developing in the region where compressive strain localization takes place. The parameters involved in the Overlapping Crack Model are formally similar to those of the Cohesive Crack Model. The main

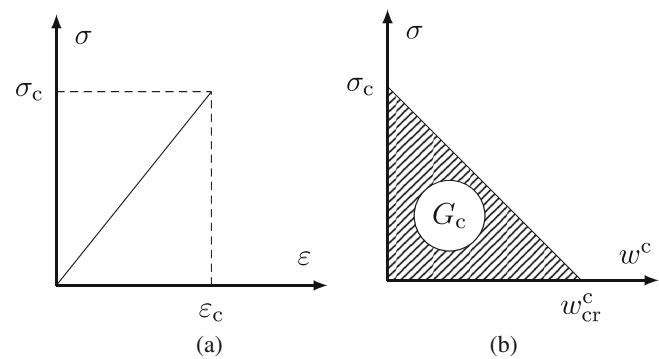


FIGURE 3 Overlapping Crack Model for concrete: (a) Linear elastic stress–strain law in compression; (b) Post-peak stress versus fictitious interpenetration relationship.

variables entailed within this model are: the concrete compression strength, σ_c , the threshold value of fictitious interpenetration beyond which the virtual opening compressive forces vanish, w^c_{cr} , and the crushing energy of concrete, G_c , which can be calculated as the area subtended by the σ - w^c diagram of Figure 3b. In this framework, Suzuki et al.⁴⁶ carried out an experimental campaign on RC columns and proposed the following relationship for G_c :

$$G_c = 80 - 50k, \quad (3)$$

being $k = 40/\sigma_c \leq 1$. According to the Overlapping Crack Model, the material behaves elastically until the concrete compression strength, σ_c , is reached (Figure 3a); then a constitutive law in the form σ - w^c (Figure 3b) is adopted.

Therefore, the Cohesive/Overlapping Crack Model (COCM)^{24,47} is able to describe by means of the two above-mentioned constitutive laws the whole structural behavior of RC elements subjected to bending, considering local and global discontinuous phenomena within the damage process: in particular, cracking and crushing instabilities are proved to set the minimum and maximum reinforcement percentages, ρ_{min} and ρ_{max} , for RC beams.^{24,48,49}

In this sense, despite several studies have proposed an equivalent stress block for non-rectangular^{50–52} and high-strength⁵³ concrete beams, a comprehensive and effective study of size-scale effects on the rotation capacity of RC T-beams is still lacking.

In the present paper, COCM is adopted to analyze the ductile-to-brittle size-scale transition in high-performance reinforced (HPRC) or prestressed concrete (HPPC) T-beams. For RC beams, this model is able to point out how the scale-dependent reinforcement limits, ρ_{min} and ρ_{max} , between which RC structures present a stable behavior without catastrophic losses in their bearing capacity due to concrete cracking (hyperstrength) or crushing, are functions of the beam cross-section shape. On the other hand, the application of the Cohesive/Overlapping Crack Model to HPPC T-beams

clearly highlights their brittleness at the Ultimate Limit State (ULS) due to the presence of the prestressing force and of the high concrete strength, demonstrating how an effective and rational maximum reinforcement ratio can represent a strict requirement for these structural elements.

2 | THE COHESIVE/OVERLAPPING CRACK MODEL (COCM)

COCM considers a beam cross-section divided into n nodes, simulating the propagation of the cohesive and the overlapping process zones by means of a step-by-step procedure. More in detail, by means of the superposition principle, it is possible to write:

$$\{w\} = [K_F]\{F\} + [K_F]\{F_p\} + \{K_M\}M, \quad (4)$$

where $\{w\}$ is the vector containing the opening/overlapping displacements; $[K_F]$ is the matrix containing the nodal displacements generated by unit nodal forces; $\{F\}$ is the vector containing the nodal forces; $\{F_p\}$ is the vector containing the nodal forces generated by a possible prestressing force; $\{K_M\}$ is the vector containing the nodal displacements generated by a unit bending moment, and M is the value of the applied bending moment (Figure 4). The unknowns involved in Equation (4) are $(2n + 1)$, namely, $\{w\}$, $\{F\}$, and M . At each step, the cohesive or the overlapping process zone advances, and, in the general case of Figure 4, the following conditions are applied:

$$F_i = 0 \quad \text{for } i = 1, \dots, (j-1), i \neq r \quad (5a)$$

$$F_i = F_t \left(1 - \frac{w_i}{w_{cr}^t}\right) \quad \text{for } i = j, \dots, (m-1) \quad (5b)$$

$$w_i = 0 \quad \text{for } i = m, \dots, p \quad (5c)$$

$$F_i = F_c \left(1 - \frac{w_i}{w_{cr}^c}\right) \quad \text{for } i = (p+1), \dots, q \quad (5d)$$

$$F_i = 0 \quad \text{for } i = (q+1), \dots, n \quad (5e)$$

$$F_i = f(w_i) \quad \text{for } i = r \quad (5f)$$

where j is the real cohesive crack tip; m is the fictitious cohesive crack tip; p is the fictitious overlapping zone tip; q is the real overlapping zone tip.

Equation (5f) expresses the bridging force,^{5,6} F_b , exerted on crack faces due to the presence of a reinforcement layer. The bridging action includes the cohesive forces of the concrete matrix, the tensile stiffness of the bar, and the hardening effect due to the interaction between concrete and the reinforcement layer (the so-called “tension stiffening” effect).⁵⁴ In the present paper, the first contribution is acknowledged automatically by means of the application of Equation (4). On the other hand, the contributions of the bar and the hardening effect are assessed by means of the equilibrium procedure reported in Ruiz et al.³⁰ and Accornero et al.,⁵⁵ leading to the definition of the following constitutive law:

$$F_b = \sqrt{\pi \phi \tau_m E_s A_s w^t} \quad \text{for } w^t < w_y^t \quad (6a)$$

$$F_b = F_y - F_p \quad \text{for } w^t \geq w_y^t \quad (6b)$$

ϕ being the bar diameter, τ_m the average shear stress at steel-concrete interface, E_s the steel Young's modulus, A_s the steel-bar area, F_y the steel yielding force, and w_y^t the crack opening displacement corresponding to steel-bar yielding. In Figure 5, the nonlinear bridging condition of Equation⁶ is reported. In this framework, the development length, ℓ_d , which is required to generate steel yielding is:

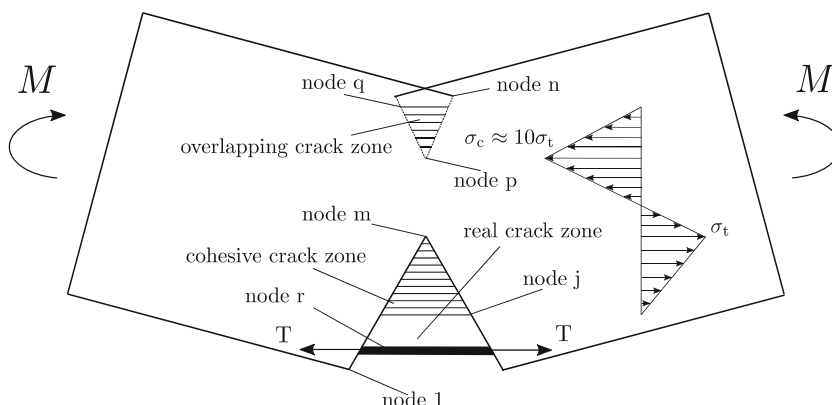


FIGURE 4 Cohesive/Overlapping Crack Model (COCM).

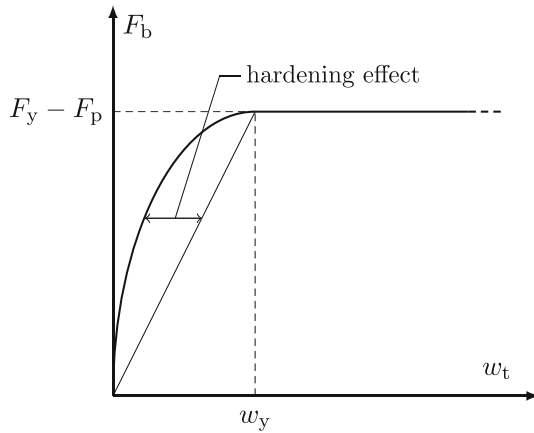


FIGURE 5 Bridging force exerted by reinforcement on crack faces.

$$\ell_d = \frac{F_y - F_p}{\pi \phi \tau_m} \quad (7)$$

When the real development length depending on beam geometry, $\ell_{d,max}$, is smaller than ℓ_d , the reinforcement slip occurs at a constant bridging force, $F_{b,s}$:

$$F_{b,s} = \frac{(F_y - F_p) \ell_{d,max}}{\ell_d} \quad (8)$$

Equations (5) reduce the unknowns of Equation (4) but do not permit the calculation of the bending moment, M . In this framework, a Crack Length Control Scheme solution technique coupled with a modified Newton–Raphson method is adopted and for each calculation step, M is fixed according to the minimum value of the external load that generates the ultimate tensile force, F_t , at the fictitious opening tip or the ultimate compressive force, F_c , at the fictitious overlapping tip. Only the tip that reaches the ultimate condition for the considered step is moved, allowing the cohesive or the overlapping process zone to extend. At the end of each calculation step, the rotation angle is computed exploiting Betti's Theorem:

$$\vartheta = \{K_M\}^T \{F\} + D_M M \quad (9)$$

D_M being the rotation generated by a unit bending moment.

The numerical procedure presented above permits the calculation of the nonlinear response of the beam section in the $M - \vartheta$ diagram. On the other hand, the structural response can be obtained in the $P - \delta$ diagram following the procedure proposed by Mattock.¹⁵ In this framework, the mid-span deflection, δ , can be regarded as the sum of two contributions (Figure 6): a displacement due to local rotation, δ_f , and a second contribution

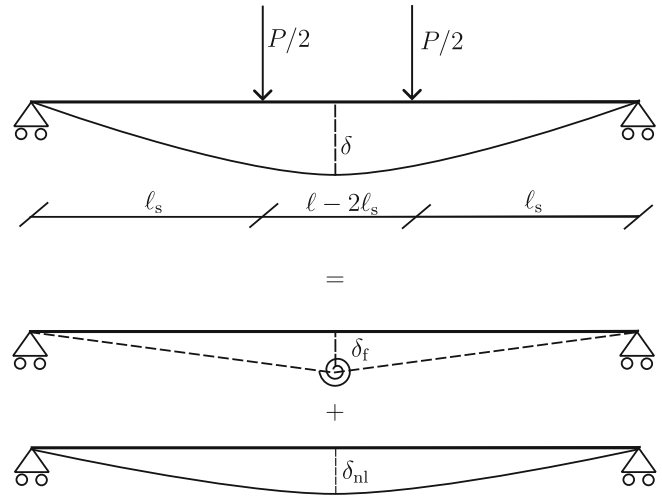


FIGURE 6 Superposition principle for the structural response of the damaged beam.

due to the nonlinear behavior of the concrete matrix along the beam span, δ_{nl} :

$$\delta = \delta_f + \delta_{nl} \quad (10)$$

Referring to Figure 6, δ_f can be computed as:

$$\delta_f = \frac{\vartheta \ell}{4} \quad (11)$$

On the other hand, we have:

$$\delta_{nl} = \frac{1}{8} \frac{M \ell^2}{EI_e} - \frac{1}{6} \frac{M \ell_s^2}{EI_e} \quad (12)$$

I_e being the equivalent inertia of the beam⁵⁶ and ℓ_s the beam shear span. I_e takes into account multiple crack formation and propagation along the beam span and it can be assessed following the procedure proposed by ACI 318–19⁵⁷ and ACI RC Design Handbook.⁵⁸ Moreover, the load, P , acting on the beam can be computed as (Figure 6):

$$P = \frac{2M}{\ell_s} \quad (13)$$

Then, when the $M - \vartheta$ curve is obtained, the $P - \delta$ diagram can be calculated by means of Equations (10–13).

3 | PARAMETRIC ANALYSES

A numerical study on the flexural behavior of RC beams is reported in Figure 7. The parametric analysis has been performed on six beam depths, h , assuming for the

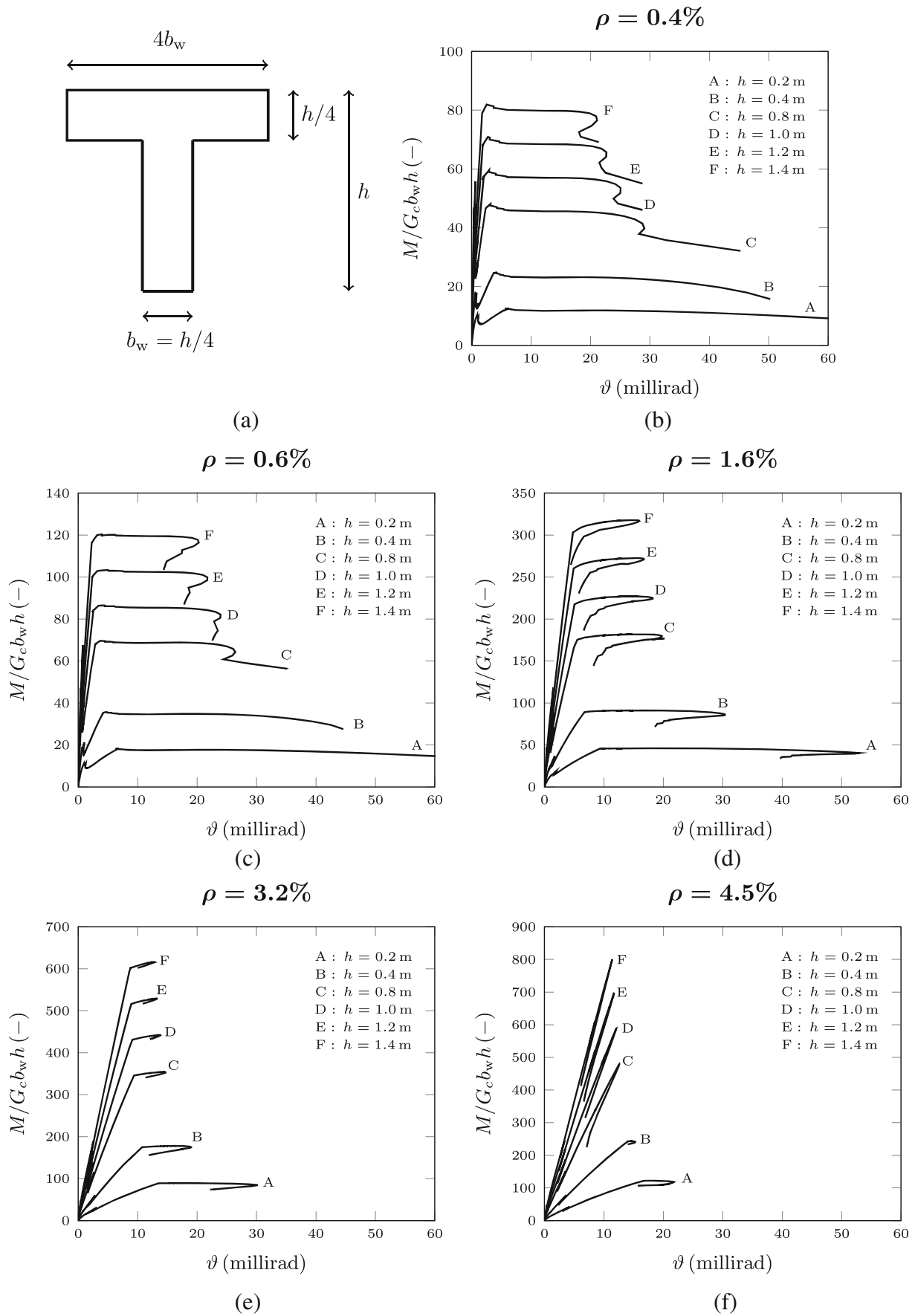


FIGURE 7 Nondimensional load versus rotation curves for RC T-beams: (a) beam cross-section; (b) $\rho = 0.4\%$; (c) $\rho = 0.6\%$; (d) $\rho = 1.6\%$; (e) $\rho = 3.2\%$; (f) $\rho = 4.5\%$.

concrete matrix a tensile strength $\sigma_t = 4$ MPa, a compression strength $\sigma_c = 40$ MPa, a fracture energy, G_F , and a crushing energy, G_c , equal to 0.15 N/mm and 30 N/mm according to Equation (1) and Equation (3), respectively. Steel reinforcement has been assumed to have a yield strength, σ_y , equal to 500 MPa, and its position is fixed in order to keep the ratio between the effective depth of the beam and the overall depth, d/h , equal to 0.9. All the investigated beams have a web thickness, b_w , and a flange thickness equal to $h/4$ with a flange width of $4b_w$ as reported in Figure 7a. The beam depth, h , varies between 0.2 m and 1.4 m in order to highlight size-scale effects.

In Figure 7b, different structural behaviors are obtained assuming $\rho = 0.4\%$. Here, the beam having $h = 0.2$ m exhibits a large rotational capacity with a wide

plastic plateau after steel yielding. Increasing the beam depth, plastic resources decrease, and early brittle crushing failure is registered. In Figure 7c, with $\rho = 0.6\%$, it is possible to observe a large reduction in plastic rotation capacity, and an even more important role played by the ultimate compressive behavior of the concrete matrix. The ductile-to-brittle transition is evident comparing Figure 7d,e, where $\rho = 1.6\%$ and $\rho = 3.2\%$ are considered. Due to the higher reinforcement ratio, the extension of the plastic plateau is reduced, and a snap-back due to crushing is present at the end of each horizontal branch for the whole beam depth range. In Figure 7f the curves C to F ($0.8 \text{ m} < h < 1.4 \text{ m}$) exhibit the most brittle behavior, since, once the peak load has been reached, a sudden loading drop is registered due to the unstable growth of the overlapping process zone without steel yielding. For

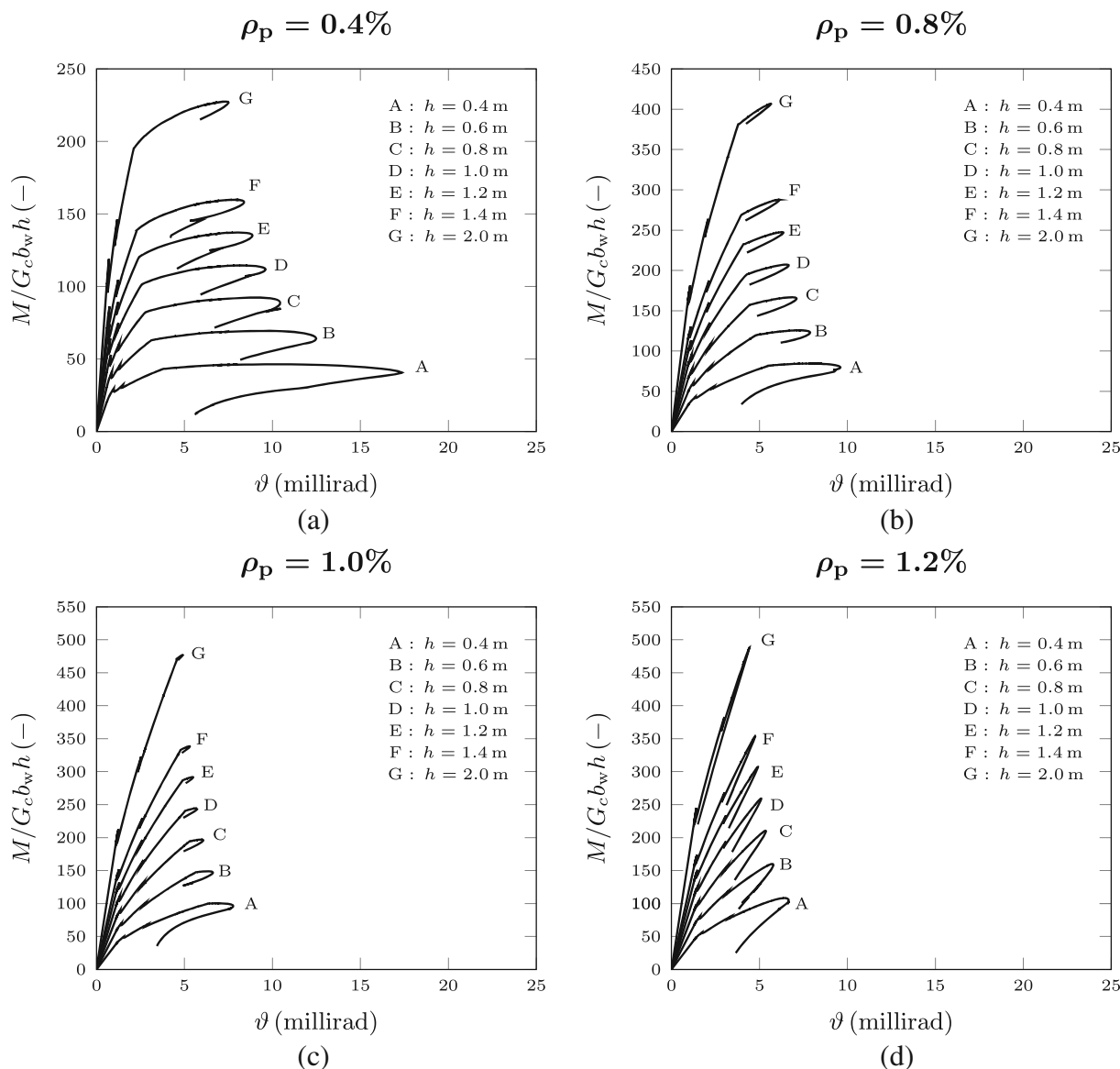


FIGURE 8 Nondimensional load versus rotation curves for PC T-beams: (a) $\rho_p = 0.4\%$; (b) $\rho_p = 0.8\%$; (c) $\rho_p = 1.0\%$; (d) $\rho_p = 1.2\%$.

this high-steel percentage, only beams with $h = 0.2$ m and $h = 0.4$ m exhibit steel yielding.

In Figure 8 a parametric analysis of the structural behavior of PC T-beams subjected to bending is reported. Within the Cohesive/Overlapping Crack Model, PC beams casted with prestressing technique or, more generally, having a straight steel strand layout can be taken into consideration. In these simulations, the concrete matrix presents the same mechanical properties as in the previous study on RC beams, the prestressing force is equal to $2/3A_{sp}\sigma_y$, being A_{sp} the prestressing steel area, and $\sigma_y = 1700$ MPa the steel yield strength. The analysis is performed on beams having a flange width of $2b_w$ with a thickness of $h/4$. In Figure 8a, where $\rho_p = 0.4\%$, it is possible to observe a first elastic branch up to the cracking load, a first descending branch due to cracking (snap-back) instability, a second ascending branch due to the steel strand activation, and a plastic plateau involving steel yielding.

At the end of each plastic plateau, a catastrophic snap-back due to concrete crushing is observed. As for RC beams, the extension of the plastic plateau decreases as reinforcement percentage and/or beam depth increases, albeit lower plastic rotations are evidenced due to the presence of the prestressing force (Figure 8a,b). The increase in the reinforcement ratio, ρ_p , beyond 0.8%, reduces the global ductility as may be observed in Figure 8c, where the beam having $h = 2.0$ m presents an almost balanced reinforcement condition with a small plastic plateau. In Figure 8d, all the curves present an unstable behavior governed by brittle concrete crushing without steel yielding.

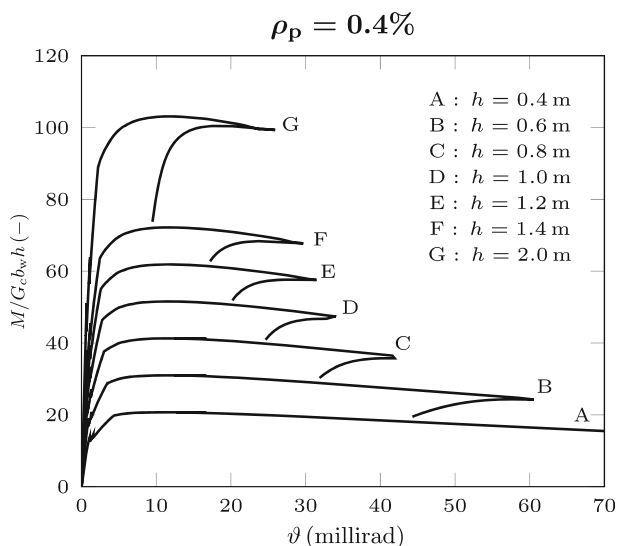


FIGURE 9 Nondimensional load versus rotation curves for HPPC T-beams ($\rho_p = 0.4\%$).

In Figure 9, a parametric analysis of the structural behavior of HPPC T-beams is reported. In this study the concrete matrix presents a compression strength $\sigma_c = 150$ MPa, a tension strength $\sigma_t = 6$ MPa, a fracture energy, G_F , and a crushing energy, G_c , equal to 0.18 N/mm (Equation (1)) and 70 N/mm (Equation (3)), respectively. The beams present the same geometry and the same prestressing force as the previous analysis and a steel percentage of $\rho_p = 0.4\%$ is considered. In this case, due to the high strength of the concrete matrix, wider plastic plateaus as well as more brittle behaviors of the post-crushing regime are detected.

4 | NUMERICAL VERSUS EXPERIMENTAL COMPARISON

In the following, a numerical versus experimental comparison is proposed to investigate the scale effects on the structural behavior of HPRC beams. The experimental campaign carried out by Ashour⁵⁹ on beams cast with three different concrete grades (see Table 1) is discussed. The concrete tension strength, σ_t , and the fracture energy, G_F , are estimated according to Model Code²⁹ and Equation (1), respectively. On the other hand, the crushing energy, G_c , is calculated by means of Equation (3). The beams have a cross section of 200×250 mm and are tested up to failure by means of a four point bending loading scheme as reported in Figure 10.

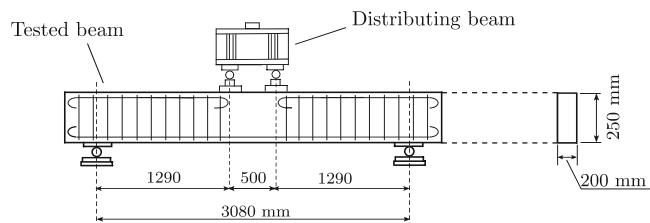
In Figure 11, the numerical (thick curve) versus experimental (thin curve) comparison is reported. It is possible to observe that COCM is able to thoroughly describe the post-cracking non-linear behavior of the beams and their ductile-to-brittle transition by increasing the reinforcement ratio, ρ . In the case of HPRC beams (H series), COCM is able to predict the increase in the plastic rotational capacity due to the increase in the concrete matrix strength. Finally, it is worth noting that snap-back phenomena can be detected during experimental testing only if an increasing function of time, such as the crack mouth opening displacement (CMOD), is used as controlling parameter. In this case, the experimental campaign carried out by Ashour was unable to follow snap-back branches in a stable manner since a load controlling technique was adopted.

5 | MINIMUM AND MAXIMUM REINFORCEMENT PERCENTAGES FOR RC BEAMS

The numerical simulations reported in Figures 7, 8 and 9 outline the existence of a scale-dependent maximum

TABLE 1 Mechanical properties of the beams tested by Ashour⁵⁹

Beam No.	ρ (%)	σ_c (MPa)	σ_t (MPa)	G_F (N/mm)	G_c (N/mm)
B-N2	1.18	48.6	4.0	0.151	38.856
B-N3	1.77	48.6	4.0	0.151	38.856
B-N4	2.37	48.6	4.0	0.151	38.856
B-M2	1.18	78.5	4.8	0.163	54.522
B-M3	1.77	78.5	4.8	0.163	54.522
B-M4	2.37	78.5	4.8	0.163	54.522
B-H2	1.18	102.4	5.3	0.170	60.469
B-H3	1.77	102.4	5.3	0.170	60.469
B-H4	2.37	102.4	5.3	0.170	60.469


FIGURE 10 Testing set-up and beam geometry adopted by Ashour.⁵⁹

reinforcement ratio, ρ_{\max} , beyond which the concrete crushing failure occurs prior to steel yielding. Currently, an effective scale-dependent minimum reinforcement ratio, ρ_{\min} , proposed by Carpinteri and co-workers²⁴ has been acknowledged by the American Association of State Highway Transportation Officials (AASHTO)⁶⁰ in order to avoid the scale-dependent hyperstrength behavior due to cracking instability. On the other hand, all the international provisions about maximum reinforcement percentage may lead to unsafe and costly structural design, since the traditional constitutive laws adopted to obtain those Standards are not able to predict the brittleness of the concrete matrix. In this framework, only an effective dimensional analysis²⁶ approach is able to clearly outline the main variables involved in the problem of determining minimum and maximum reinforcement percentages for RC T-beams.

More specifically, the variables involved in the flexural behavior of RC beams may be written as²⁴:

$$M = F(\sigma_t, G_F, \sigma_c, G_c, E, \sigma_y, \rho, h; b_w/h, b/h, \ell/h; \vartheta) \quad (14)$$

being ℓ the beam span and E the concrete Young's modulus. Note that, if we were interested in lightly reinforced concrete beams, the two variables σ_c and G_c could be neglected since beams having a low reinforcement ratio exhibit at the ULS the steel yielding rather than the concrete crushing. Thus, assuming the beam depth, h , and

the fracture toughness, $K_{IC} = (G_F E)^{0.5}$ as independent variables, Equation (14) leads to:

$$\frac{M}{K_{IC} h^{2.5}} = \Pi \left(\frac{K_{IC}}{\sigma_t h^{0.5}}, \rho \frac{\sigma_y h^{0.5}}{K_{IC}}, \vartheta \frac{K_{IC}}{E h^{0.5}} \right) \quad (15)$$

where it is possible to recognize the matrix brittleness number $s_t = K_{IC}/(\sigma_t h^{0.5})$, and the reinforcement brittleness number $N_P^L = \rho \sigma_y h^{0.5}/K_{IC}$ introduced by Carpinteri⁵ as governing parameters. On the other hand, if we are interested in the investigation of heavily reinforced beams, we may neglect σ_t and G_F in Equation (14), since for large reinforcement percentages the concrete crushing precedes steel yielding.³¹ Thus, assuming h and $(G_c E)^{0.5}$ as new fundamental variables, Equation (14) may be rewritten as:

$$\frac{M}{\sqrt{G_c E} h^{2.5}} = \Pi \left(\frac{\sqrt{G_c E}}{\sigma_c h^{0.5}}, \rho \frac{\sigma_y h^{0.5}}{\sqrt{G_c E}}, \vartheta \frac{\sqrt{G_c E}}{E h^{0.5}} \right) \quad (16)$$

where it is possible to recognize the matrix brittleness number, $s_c = (G_c E)^{0.5}/\sigma_c h^{0.5}$, and the reinforcement brittleness number, $N_P^U = \rho \sigma_y h^{0.5}/(G_c E)^{0.5}$.

A parametric analysis of RC T-beams subjected to bending can be realized by means of COCM in order to outline ρ_{\min} and ρ_{\max} trends in the planes $s_t - N_P^L$ and $s_c - N_P^U$, respectively. This analysis considers the concrete matrix characterized by a compression strength, σ_c , varying between 20 and 80 MPa, the tension strength, σ_t , and the fracture energy, G_F , being estimated according to Model Code²⁹ and Equation (1), respectively. The crushing energy, G_c , is calculated by means of Equation (3). Concerning the steel-bar reinforcement, the yield strength $\sigma_y = 500$ MPa has been considered, and the reinforcement layer has been positioned in order to keep $d/h = 0.9$. All the investigated beams have a web thickness, b_w , and a flange thickness equal to $h/4$ with a flange

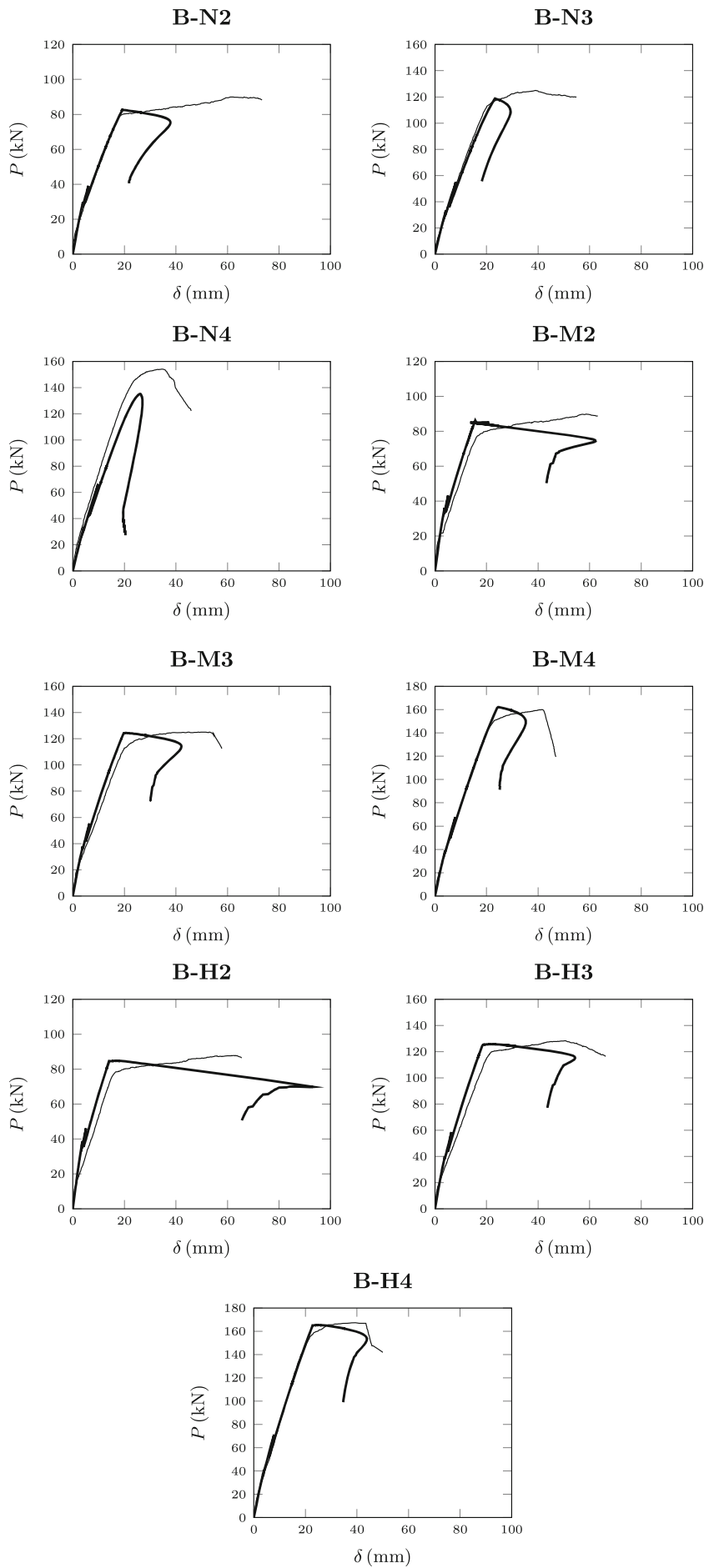


FIGURE 11 Numerical (thick) versus experimental (thin) curves.

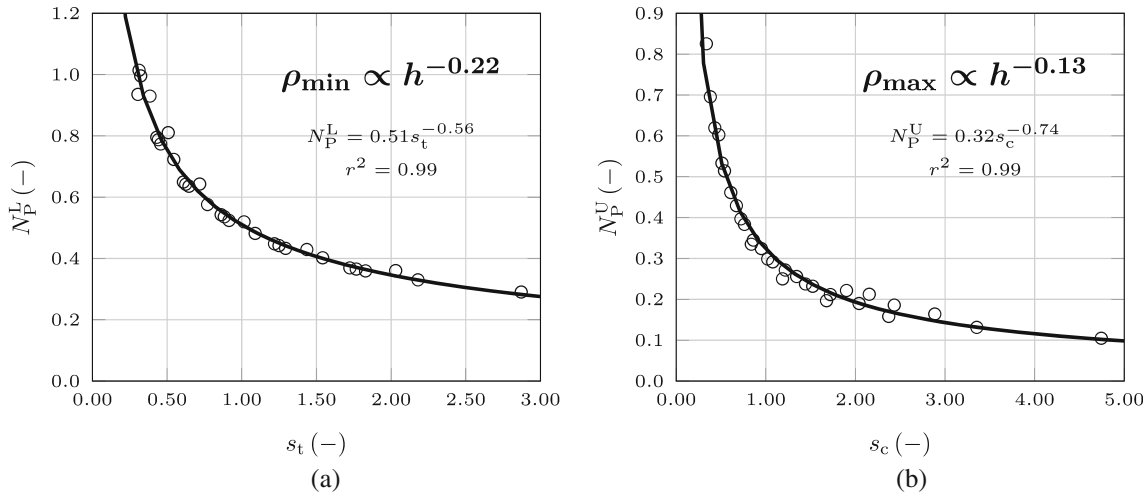


FIGURE 12 Ductile-to-brittle transitions in RC T-beams: (a) $s_t-N_P^L$, and (b) $s_c-N_P^U$ best fit.

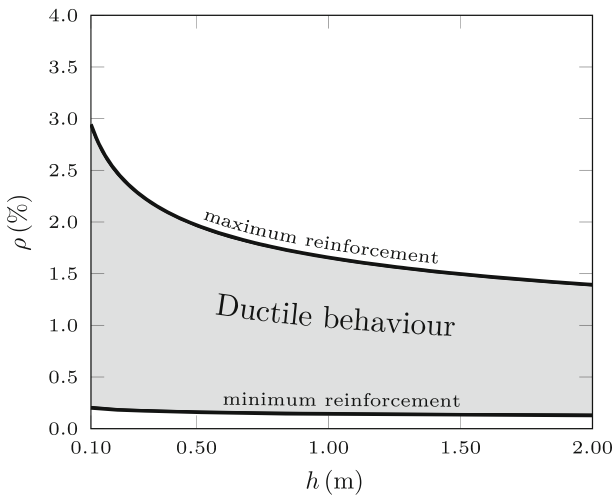


FIGURE 13 Upper and lower bounds for reinforcement ratio.

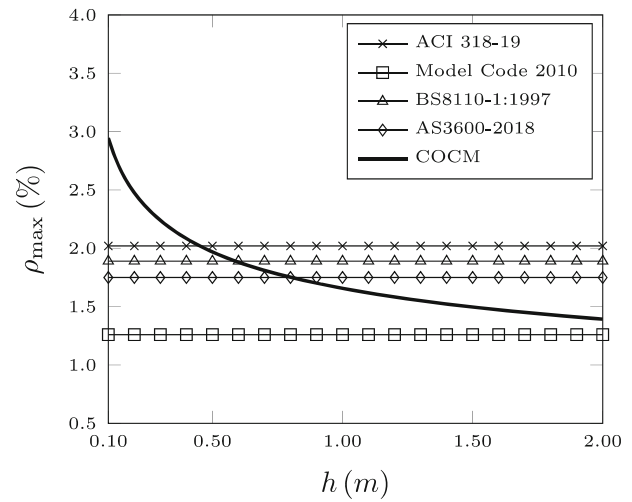


FIGURE 14 Comparison between different maximum reinforcement provisions.

width of $4b_w$ (Figure 7a), whereas the beam depth, h , varies between 0.1 m and 3.2 m. For each beam depth, the reinforcement ratio has been changed iteratively in order to obtain an ultimate bending moment equal to the cracking bending moment for the estimation of the minimum reinforcement ratio, ρ_{\min} . Similarly, the maximum reinforcement ratio, ρ_{\max} , has been estimated as the reinforcement ratio beyond which concrete crushing precedes steel yielding. The results of the parametric study and the best-fitting relations in the planes $s_t-N_P^L$ and $s_c-N_P^U$ are reported in Figure 12.

From Figure 12a, we can find:

$$N_P^L = 0.51s_t^{-0.56} \quad (17)$$

Considering the definition of N_P^L and s_t , we have:

$$\rho_{\min} = 0.51 \frac{\sigma_t^{0.56} K_{IC}^{0.44}}{\sigma_y h^{0.22}} \quad (18)$$

where a scale effect on the minimum reinforcement percentage is found to be $h^{-0.22}$.

On the other hand, in Figure 12b the results of the parametric analysis regarding the maximum reinforcement ratio, ρ_{\max} , leads to:

$$N_P^U = 0.32s_c^{-0.74} \quad (19)$$

Considering the definitions of N_P^U and s_c , it is possible to find:

$$\rho_{\max} = 0.32 \frac{\sigma_c^{0.74} (\sqrt{G_c E})^{0.26}}{\sigma_y h^{0.13}} \quad (20)$$

where the scale effect on the maximum reinforcement percentage is found to be $h^{-0.13}$.

In this context, it is interesting to recall that previous analyses of RC rectangular cross-sections^{24,48,49,61} provided a scale effect on the minimum reinforcement percentage proportional to $h^{-0.15}$, and a scale effect on the maximum reinforcement percentage proportional to $h^{-0.25}$, thus defining an effective range in which structures can develop a safe ductile behavior at ULS (Figure 13). Current Standards, such as Model Code,²⁹ BS 81110,⁶² AS 3600,⁶³ and ACI,⁵⁷ completely disregard the size-scale effects on these two reinforcement percentages leading to a wrong assessment of the rotation capacity of real structures.⁶⁴ The provisions imposed by Standards together with the scale-dependent maximum reinforcement percentage provided by the Cohesive/Overlapping Crack Model are reported in Figure 14, suggesting that current Standards are not able to fully ensure RC safety.

In the case of T-beams having flange/web ratio equal to 4, a consistent variation in the power-law exponents is found, suggesting that the beam cross-section shape can have significant effects on the ductile-to-brittle transitions occurring in RC.

6 | CONCLUSIONS

In the present paper, by means of the Cohesive/Overlapping Crack Model (COCM), an effective estimation of the minimum reinforcement percentage for RC T-beams is performed, emphasizing its $h^{-0.22}$ power-law scale dependence, where h is the beam depth. In addition, numerical studies are carried out in order to confirm how the extension of the plastic plateau developing after steel yielding is inversely proportional to beam depth, h , and reinforcement ratio, ρ . This evidence is accompanied by an analogous existence of a scale-dependent maximum reinforcement percentage for RC T-beams with a $h^{-0.13}$ power-law beyond which the structure is unable to develop any ductility due to concrete crushing occurring prior to steel yielding. In the case of the upper bound, not only economical but also safety reasons justify the present study.

For PC beams and high-performance concrete, the application of the COCM demonstrates how these structures exhibit a more brittle behavior due to the presence of the prestressing force and/or the high strength of the concrete matrix.

NOTATION

ℓ	beam span
ϑ	cross-section local rotation
$[K_F]$	matrix of nodal displacements generated by unit forces


$\{F\}$	vector of nodal forces
$\{F_p\}$	vector of nodal forces generated by the prestressing force
$\{K_M\}$	vector of nodal displacements generated by unit bending moment
$\{w\}$	vector of nodal displacements
A_s	steel area
A_{sp}	prestressed steel area
b	beam thickness
b_w	beam web thickness
d	beam effective depth
F_p	prestressing force
F_y	steel yield force
G_c	crushing energy
G_F	fracture energy
h	beam depth
M	bending moment
N_P^L	lower bound reinforcement brittleness number
N_P^U	upper bound reinforcement brittleness number
P	applied load
s_c	compression matrix brittleness number
s_t	tension matrix brittleness number
w^c	fictitious interpenetration
w_{cr}^c	critical value of the fictitious interpenetration
w^t	crack opening displacement
w_{cr}^t	critical value of the crack opening displacement
w_y^t	steel-bar yielding crack opening displacement
δ	beam mid-span deflection
ε_c	concrete ultimate compression strain
ε_t	concrete ultimate tension strain
ρ	reinforcement percentage ($A_s/b_w h$)
ρ_{max}	maximum reinforcement percentage
ρ_{min}	minimum reinforcement percentage
ρ_p	prestressing reinforcement percentage ($A_{sp}/b_w h$)
σ_c	concrete compression strength
σ_t	concrete tension strength
σ_y	steel-bar yield strength

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ORCID

Renato Cafarelli  <https://orcid.org/0000-0003-2076-1053>

Federico Accornero  <https://orcid.org/0000-0002-9638-8411>

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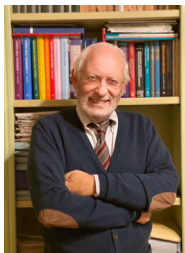
AUTHOR BIOGRAPHIES



Renato Cafarelli, DISEG, Politecnico di Torino, Turin, Italy.



Federico Accornero, Associate Professor of Structural Mechanics, College of Engineering, Shantou University, Shantou, China.



Alberto Carpinteri, Chair Professor of Structural Mechanics, DISEG, Politecnico di Torino, Turin, Italy; Zhujiang (Pearl River) Professor of Guangdong Province, College of Engineering, Shantou University, Shantou, China.

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