A theoretical model for a piezoelectric energy harvester with a tapered shape

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\textbf{Abstract}

A piezoelectric energy harvester made of a tapered cantilever surface bonded with piezoelectric patches is developed to harness energy from ambient vibrations. Compared with the available cantilever harvester of a uniform shape in the length direction, this harvester has a higher energy harvesting efficiency since a maximum collected power at each piezoelectric patch on the cantilever can be achieved. The current available models for cantilever harvesters are not applicable for the new developed tapered harvester due to the difficulties in dealing with the tapered shape. A corresponding finite differential model is hence developed to model the tapered harvester for estimating its efficiency by examining a governing differential equation with variable coefficients. The influences of some practical considerations, such as the geometry of the tapered cantilever and the width of piezoelectric patches on the root mean square of the generated electric power, are discussed. The results from the developed model show that an electric power up to 70 times higher than the available uniform cantilever harvesters can be achieved by the tapered harvester. This research provides an effective model for evaluating the high efficiency of the piezoelectric coupled tapered cantilevers in energy harvesting.

\section{Introduction}

Piezoelectric energy harvesting devices converting ambient energy into electrical energy have attracted much interest in both the academia and industry. Their applications include wearable electronics, where energy harvesting devices can power or recharge cellphones, mobile computers, radio communication equipment, high power output devices (or arrays of such devices) deployed at remote locations to serve as reliable power stations for large systems. The most common type of piezoelectric devices is the piezoelectric/metal sandwich beam mounted as a cantilever due to three major considerations. First, large mechanical strains can be produced directly owing to the piezoelectric effect of the materials during vibrations. Second, the construction of piezoelectric cantilevers is simpler compared to other harvesters. Third, the resonance frequency of the fundamental flexural modes of a cantilever is much lower than the other vibration modes of the piezoelectric element. So far, studies on piezoelectric energy harvesting technologies reported to date have involves a unimorph or bimorph cantilever design [1–5].

The exact expressions for the voltage, current, power, and tip deflection of a Timoshenko piezoelectric cantilever were derived [6]. Subsequently, several case studies were presented to examine the frequency response of vibration-based energy harvesters using this model. An analytical approach based on the Euler–Bernoulli beam theory and Timoshenko beam equations for the voltage and power generation was provided and compared with the electrical equivalent circuit and energy method [7]. The technique to adjust the performance of a piezoelectric bimorph vibrating in the flexural mode through axial preloads was studied to effectively scavenge energy from ambient mechanical vibrations/noise with varying-frequency spectra [8]. Based on a design of a bimorph cantilever located at an arbitrary position on a simply supported slender bridge, a piezoelectric power generation from moving loads was formulated [9]. An optimal design of a piezoelectric coupled cantilever structure attached by a proof mass subjected to seismic motion was introduced to achieve a higher efficient energy harvesting from high-rise buildings [10]. Two types of sea wave piezoelectric cantilever energy harvesters with proof masses were introduced and optimized to harvest electric energy from longitudinal or transverse wave motions of water particles [11,12]. Their results show that the harvesters can generate a power up to 55 W and 30 W for a practical longitudinal and transverse wave motions, respectively. A novel piezoelectric energy harvester...
device driven by a cantilever system to harvest energy from high-rise buildings was demonstrated [13,14]. This harvester can provide an efficient and practical energy harvesting in the process of dissipating the vibration energy of high-rise buildings. Dhakar et al. [15] proposed a 21-mm-long polymer extension beam with a proof mass of 0.72 g at its free end. The other end of the beam is clamped firmly to the free end of a 32-mm-long PZT-5A bimorph cantilever. Results showed that this design can tune the natural frequencies of the combinatorial cantilever notably. Azizi et al. [16] investigated the mechanical behavior of a bimorph piezoelectric micro cantilever exposed to harmonic base excitation, and the effect of load resistance and the effective piezoelectric stress constant on the equivalent damping ratio of output circuits including parallel and series connections. The effect of the impact of water droplets releasing from different various heights on the piezoelectric energy harvesting was studied [17] and the results show that a power output for one unit at around 2.5 µW. An auxiliary cantilever with a piezoelectric patch glued at its fixed end subjected to a resonance was conducted to improve power output [18].

Another innovative S-shaped meandering cantilever was designed to achieve a low resonance frequency below 30 Hz by reducing the stiffness of the beam [19]. A two-stage design was proposed in which a spring-mass system was designed to respond at the input frequency of the host structure, and then excite an array of piezoelectric structures to tune the natural frequency of a piezoelectric cantilever. Results showed that this design can tune the natural frequency device and proof mass may also increase the tuned frequency device and proof mass may also increase the frequency may not be an effective approach to improve efficiency of the tapered harvester can be studied comprehensively. The research findings are significant and helpful in designing the most efficient and economic piezoelectric coupled cantilever harvesters mentioned above to improve their power output.

Three major principles used in the available piezoelectric cantilever harvesters mentioned above to improve their power output are: (1) decreasing the natural frequency of the cantilever device by using a proof mass attached to its free end or a combination beam; (2) enhancing the excitation frequency to the vicinity of the natural frequency of the cantilever by a tuned resonance device; and (3) broadening the band response with respect to the excitation from the host structures. In the aforementioned energy harvester designs, the common intention is to make the piezoelectric coupled cantilever working at the resonance to achieve a maximum power output. However, many ambient vibration sources possess a spectrum of random frequencies. In these situations, tuning a piezoelectric energy harvester to a specific resonance frequency may not be an effective approach to improve efficiency because even a small amount of fluctuation in the input frequency will result in a large drop in the power output. Meanwhile, the tuned frequency device and proof mass may also increase the space and weight of the energy harvester device, and hence decrease the efficiency of the harvesters. And more importantly, the above-mentioned harvester devices only fully use the maximum strain at the fixed end which leaves the piezoelectric material under-utilized, resulting in sub-optimal efficiency [24].

The electrical energy from a piezoelectric material is proportional to the induced mechanical strain. Hence it is desirable to maximize the strain at each point in a beam to fully utilize the potential energy in piezoelectric materials. Under an optimal condition, the strain distribution in the beam would be completely uniform within the strain limit. The consideration suggests a triangular or trapezoidal shape cantilever which has a linearly increased wider cross-section, while reducing the size and weight of a piezoelectric cantilever. Baker et al. [25] introduced a piezoelectric harvester prototype made of a trapezoidal geometry cantilever by finite element method to examine the effect of geometry of the cantilevered piezoelectric beam on the power density. The results showed that the output power is 50% higher than that of a comparable rectangular cantilever with an even strain distribution throughout the beam. Glynne-Jones et al. [26] built a prototype of a thick-film PZT coupling trapezoidal cantilever, and then studied the effect of the load resistance and beam amplitude on the power output of the harvester. Mateu and Moll [27] provided a comprehensive study on a triangular piezoelectric bending beam structure for energy harvesting using shoe inserts. The results showed that by using a triangular cantilever, the strain along axial direction is made to be a constant and the harvested energy increases with an increase in the thickness of the triangular cantilever.

These researches aforementioned on cantilever energy harvester with varying cross-sectional area use either simulation or model testing, and focus on the cantilever tapering in the width because there is no analytical solution to the dynamic governing equations of the piezoelectric coupled cantilever tapering in both width and thickness directions. In fact, a taper in thickness of a cantilever has a greater contribution on the surface strain of the cantilever than that from the taper in width.

It is hence expected to develop a mathematical model for a tapered cantilever energy harvester in both width and thickness directions to harvest energy from ambient vibration. With this novel model, the effect of the taper in width and/or thickness of the cantilever, the width of the pzt patches, the width and the thickness at the fixed end of the cantilever on the energy harvesting efficiency of the tapered harvester can be studied comprehensively. The research findings are significant and helpful in design considerations on the most efficient and economic piezoelectric coupled tapered cantilever energy harvesters which can maximize the surface strain at each point in the beam to harvest the strain energy at a fullest extent.

2. Introduction of the analytical model

The introduction of a piezoelectric coupled cantilever harvester with a rectangular cross section tapered in both thickness and width is depicted in Fig. 1(a and b). The cantilever is subjected to a sinusoidal force $F = Y_{sin} \omega t$ at its free end. Fig. 1a schematically illustrates the normal section view of the harvester with a length of $l$. It is tapered in the thickness direction from $h_0$ at the fixed end to $h_1$ at the free end, and attached by $pzt4$ patches with the same thickness of $l_0$ one by one on the upper and lower surfaces of the cantilever. Fig. 1b is the top view of the harvester tapered in the width direction from $b_0$ at the fixed end to $b_1$ at the free end. The attached individual $pzt4$ patches fully on the surfaces of the cantilever are at a varying length $l/\pi$. Based on the model
in Fig. 1, a mathematical model for the piezoelectric tapered cantilever energy harvester is developed and solved by a finite differ-
ential method to evaluate the efficiency of the developed harvester.

3. Development of the theoretical model

The electric power of the piezoelectric coupled cantilever can be studied by setting an excitation of a harmonic sinusoidal wave as a simple representative of the ambient vibration. Therefore, the harmonic motion on the cantilever is set to be, \( f(x,t) = F(t)\delta(x) Y \sin\omega t \), where \( Y \) is the amplitude of the ambient excitation; \( \omega \) is the angular frequency of the excitation; \( \delta(x) \) is the Kronecker Delta function to represent a point force in beam models. Such a modeling for a point force should also be adopted in another work by the authors [30] although the research findings remain similar.

The motion equation for the forced lateral vibration of the piezoelectric coupled tapered cantilever can be expressed as:

\[
\frac{\partial^2}{\partial t^2} \left( EI(x) \frac{\partial^2 W(x,t)}{\partial x^2} \right) + m(x) \frac{\partial^2 W(x,t)}{\partial t^2} = F(t)\delta(x) \quad 0 \leq x \leq l
\]

It is assumed that the piezoelectric patches are bonded on the surfaces of the tapered cantilever. When bending motion takes place on the cantilever subjected to the excitation force, the generated charge \( Q_g \) and voltage \( V_g \) as well as the electric power of the piezoelectric patches are hence provided as below [31]:

\[
Q_g(t) = -e_{31} \sum_{i=1}^{m} \left( \int_{(i-1)p}^{ip} b(x) (h(x) + 2hp) \frac{\partial W(x,t)}{\partial x} dx \right)
\]

\[
V_g(t) = \sum_{i=1}^{m} \frac{Q_i(t)}{C_V} = -e_{31} \sum_{i=1}^{m} \left( \int_{(i-1)p}^{ip} h(x) + 2hp \right) \frac{\partial^2 W(x,t)}{\partial x^2} dx.
\]

\[
P_g(t) = \sum_{i=1}^{m} \frac{dQ_i(t)}{dt} V_g(t),
\]

where \( e_{31} \) is the piezoelectric constant; \( w(x, t) \) is the transverse displacement of the tapered cantilever; \( C_V \) is the electrical capacity of the piezoelectric patch; and \( C_V \) is the electric capacity per unit width of the piezoelectric patch \( (C_V = \frac{C}{W_0(x)}) \), \( m \) is the number of piezoelectric patches bonded on the upper and lower surfaces of the tapered cantilever.

To obtain the time varying displacement field, \( w(x, t) \), in Eq. (1) by the classical elastic beam model, the cantilever is simply separated into \( m/2 \) sections, as shown in Fig. 1, by considering each piezoelectric bonded area connected each other. Based on the Euler-Bernoulli beam theory, the vibration governing equation for the tapered cantilever can be expressed as:

\[
E_0 \frac{\partial^2}{\partial x^2} \left[ \frac{P(x,t)}{E(x)} \frac{\partial^2 W(x,t)}{\partial x^2} \right] + m(x) \frac{\partial^2 W(x,t)}{\partial t^2} = 0
\]

where, \( m_0 = \rho h b_0 l_0 = \frac{h b_0 l_0 l}{12} \), \( u = |x + (1 - \beta) \frac{a}{b_0} - \beta \frac{a}{b_0}| \), \( \nu = \frac{1}{\mu + 2\nu}, \)

\( a = b_1/b_0, b = h_1/h_0 \).

In order to obtain the natural frequencies and corresponding vibration shapes of the tapered cantilever, we use the modal method:

\[
w(x,t) = \sum_{n=1}^{\infty} W_n(x) q_n(t),
\]

Substitution of Eq. (6) into Eq. (5) leads to the following expression:

\[
\frac{d^4 W}{dx^4} + a_1(x) \frac{d^2 W}{dx^2} + a_2(x) \frac{d^2 W}{dx^2} + a_3(x) W = 0,
\]

in which, \( a_1(x) = 2 \left( \frac{1}{\mu} + \frac{3}{\mu + 2\nu} \right), \quad a_2(x) = 6 \left( \frac{1}{\mu} + \frac{1}{\mu + 2\nu} \right), \quad a_3(x) = - \frac{m_0 \omega^2}{E_0 b_0}
\).

Since there is no analytical solution for the Eq. (7), here the finite differential method is used to obtain the natural frequencies and corresponding vibration shapes of the tapered cantilever.

For the tapered cantilever, the boundary conditions are as below:

\[
W''(0) = 0, (EIW''|_0 = EI'(0)|W'(0) + El(0)|W(0)|m = 0
\]

\[
W'(l) = 0, W(l) = 0
\]

By expressing the first three order derivatives of the boundary points by Taylor Series, then the boundary conditions can be expressed as:

\[
\frac{2}{h^2} W_1 - \frac{5}{h^2} W_2 + \frac{4}{h^2} W_3 - \frac{1}{h^2} W_4 = 0,
\]

\[
- \frac{1}{h^2} W_1 + \frac{3}{h^2} W_2 - \frac{3}{h^2} W_3 + \frac{1}{h^2} W_4 = 0,
\]

\[
- \frac{1}{3h} W_{n-3} + \frac{3}{2h} W_{n-2} - \frac{3}{h} W_{n-1} + \frac{11}{6h} W_n = 0
\]

\[
W_n = 0
\]

in which, \( h = \frac{l}{n+1}, n \) is the number of discrete points on the tapered cantilever including the free end and fixed end points, \( W_{l-1} \) and \( W_{n-3-n} \) are the displacements of the vibration shape at corresponding discrete points on the tapered cantilever.

By expressing the first four order derivatives of the internal points by central differential formula, the finite differential equations of the internal points can be expressed as:
\[ Z_{i}W_{i-2} + Z_{i}W_{i-1} + Z_{i}W_{i} + Z_{i}W_{i+1} + Z_{i}W_{i+2} = 0 \quad (3 \leq i \leq n - 2) \]  

In which, \( Z_{1} = \frac{1}{\beta^{2}} - \frac{\delta_{1}(\alpha_{1})}{\beta^{2}C_{0}}, Z_{2} = -\frac{1}{\beta^{2}} + \frac{\delta_{2}(\alpha_{1})}{\beta^{2}C_{0}}, Z_{3} = \frac{6}{\beta^{4}} - \frac{2\delta_{1}(\alpha_{1})}{\beta^{2}C_{0}} + \delta_{2}(\alpha_{2}), Z_{4} = -\frac{4}{\beta^{2}} - \frac{\delta_{1}(\alpha_{1})}{\beta^{2}C_{0}}, Z_{5} = \frac{1}{\beta^{2}} - \frac{\delta_{1}(\alpha_{1})}{\beta^{2}C_{0}} \) and \( \alpha_{i} = (i - 1)h \). In order to obtain the value of the vibration shape at each discrete point, let \( W(0) = W_{1} = 1 \), then the formula of \( W_{2-n} \) can be derived based on Eqs. (9)–(12) and (13).

\[ W_{2} = U_{2}W_{4} + V_{2}, \]  

\[ W_{3} = U_{3}W_{4} + V_{3}, \]  

\[ W_{i+1} = U_{i+1}W_{i+2} + V_{i+1}, \quad 3 \leq i \leq n - 2, \]  

\[ W_{n} = V_{n}. \]  

The expressions of \( U_{i} \) and \( V_{i} \) can be seen in Appendix. Let \( V_{n} = 0 \), each natural frequency can be obtained by iteration method, and then the corresponding vibration shape can be achieved through back substituting natural frequency into Eqs. (16) and (17). So the generalized coordinates can also be obtained below:

\[ q_{n} = \frac{C}{Bm_{0}(\alpha_{n}^{2} - \omega^{2})} \sin \omega t, \]  

in which, \( B = \int_{0}^{l} W_{2}^{2}(x)dx, C = W_{n}(0)/\omega^{2} \).

Substitution of Eq. (18) into Eq. (6) and Eq. (6) into Eqs. (2)–(4) lead to the expressions of the electric power \( P_{e}(t) \). Further the RMS (root mean square) of the generated power from 0 to \( t/4 \) from the piezoelectric patches can be obtained as:

\[ P_{e}^{rms} = 2\sqrt{\frac{4}{T} \int_{0}^{t/4} |P_{e}(\tau)|^{2}d\tau}, \]  

where \( P_{e}(\tau) \) is the generated power of the piezoelectric coupled tapered cantilever harvester at time \( \tau (0 < \tau < t/4) \).

To estimate the RMS of the generated power, the period, \( t/4 \), can be separated into \( j \) time steps with a sufficiently short time interval \( \Delta t \). As a result, the expression in Eq. (19) can be rewritten in a discrete form below:

\[ P_{e}^{rms} = 4\sqrt{\frac{\Delta t}{T} \sum_{i=1}^{j} |P_{e}(t_{i})|^{2}} \]  

### 4. Results and discussions

Based on the finite differential model developed in Section 3, the generated charge, voltage and electric power from the piezoelectric patches can be obtained. In this section, we conduct simulations to study the effects of the taper ratios in both thickness and width of the cantilever, the width of piezoelectric patches, the width and thickness at the fixed end of the cantilever on the harvester power. The dimensions and materials properties of the piezoelectric coupled cantilever in the simulations are provided in Table 1, unless otherwise noted. The amplitude of the harmonic

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Table 1

<table>
<thead>
<tr>
<th>Natural frequencies of cantilever beam with different parameters ( \beta ).</th>
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<tbody>
<tr>
<td>N = 601</td>
</tr>
<tr>
<td>( \omega_{1} )</td>
</tr>
<tr>
<td>Calculating solution</td>
</tr>
<tr>
<td>Analytical solution [31]</td>
</tr>
<tr>
<td>Tolerance</td>
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</tbody>
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Note: \( E = 2.1e11, b_{0} = 0.04m, h_{0} = 0.003m, \alpha = b_{1}/b_{0}, \beta = h_{1}/h_{0} \).
excitation, Y, is set to be 1 N to assure a safe operation of the harvester without losing any generality. The piezoelectric patches and the host cantilever beam are made of PZT4 (lead zirconate titanate) and steel, respectively.

Firstly, to verify the effectiveness of the developed finite difference method, we set \( a = 1, b = 1 \) and \( \alpha = 1, \beta = 0 \) to study a pure uniform cantilever and a pure tapered cantilever, respectively. The corresponding natural frequencies of the cantilever are calculated by finite differential method taking the discrete point number of \( n = 601 \), and compared to the analytical solutions. The results (see in Table 2) show that the finite differential method has a good precision.

Fig. 2 shows the effect of the taper ratios in width and thickness of the cantilever on the RMS of electric power and the amplitude of first generalized coordinate of the cantilever harvester. The parameters in this simulation are set to be: \( n = 601, b_0 = 0.04 \text{ m}, h_0 = 0.003 \text{ m}, l = 0.6 \text{ m}, w_0 = 0.012 \text{ m}, Y = 1 \text{ N}, \) and \( \omega = 10 \text{ rad/s} \). Fig. 2a displays that the RMS nonlinearly decreases from 0.1376 W to 0.0263 W, 0.1376 W to 0.0077 W, and 0.1376 W to 0.002 W when the width ratio of \( b_1/b_0 \) and the thickness ratio of \( h_1/h_0 \) change from (0.2, 0.2) to (1.0, 0.2), (0.2, 0.2) to (0.2, 1.0), and (0.2, 0.2) to (1.0, 1.0), respectively. It also shows that the RMS, at \( b_1/b_0 = h_1/h_0 = 0.2 \), is 70 times of the RMS at \( b_1/b_0 = h_1/h_0 = 1.0 \), and 17.9 times of that at \( b_1/b_0 = 0.2 \) and \( h_1/h_0 = 0.2 \), respectively. Similarly, the amplitude of the first generalized coordinate decreases as shown in Fig. 2b from 0.0405 m to 0.0159 m, 0.0405 m to 0.0082 m, and 0.0405 m to 0.0038 m when \( b_1/b_0 \) and \( h_1/h_0 \) change from (0.2, 0.2) to (1.0, 0.2), (0.2, 0.2) to (0.2, 1.0), and (0.2, 0.2) to (1.0, 1.0), respectively. The findings on the decrease in RMS with increases in the ratios are mainly caused by the similar change of the first generalized coordinate \( q_1 \). A nonlinear increase in \( q_1 \) can lead to a nonlinear increase in the electric power as shown in Eqs. (6), (2)–(4). The results indicate that the taper ratio in thickness has a more obvious effect on the RMS than the taper ratio in width of the cantilever. More importantly, the results show that the tapered cantilever harvester has a much higher energy harvesting efficiency and is more economic than the counterpart uniform cantilever.

Fig. 3 illustrates the variation of RMS of the generated power and the amplitude of the sum of voltage on the piezoelectric patches versus the width of pzt4 patch with the following parameters: \( n = 601, b_0 = 0.04 \text{ m}, h_0 = 0.003 \text{ m}, l = 0.6 \text{ m}, h_1/h_0 = 1.0, b_1/b_0 = 0.2 \). It can be seen from the Fig. 3a that the RMS nonlinearly decreases from 0.1376 W to 0.1354 W when the pzt4 width changes from 0.01 m to 0.06 m. This phenomenon is mainly because the increase in the width of pzt4 patch would cause an increase in \( C_v \), which in turn causes a decrease in the voltage and electric power as shown in Eqs. (3) and (4). As we can see from Fig. 3b, the amplitude of the sum of voltage on the piezoelectric patches similarly decreases from 1074 V to 1050 V in this range. This finding indicates that a larger energy harvesting requires a
series of piezoelectric patches with a smaller width based on the theoretical model.

Fig. 4 demonstrates the effect of the width at the fixed end of the cantilever on the RMS and the amplitude of the first generalized coordinate with the parameters: \( n = 601 \), \( h_0 = 0.003 \text{ m} \), \( l = 0.6 \text{ m} \), and \( h_1/h_0 = b_1/b_0 = 0.2 \). Fig. 4a shows a nonlinear decrease in RMS from 0.1376 W to 0.055 W under the condition of the width at the fixed end of the cantilever changing from 0.04 m to 0.1 m. The findings on the decrease in RMS with an increase in the width at the fixed end of the cantilever are owing to the fact that an increase in \( b_0 \) would lead to an increase in the mass, \( m_0 = \rho b_0 h_0 \), and hence the decreases in the amplitude of generalized coordinates, \( q_1 \), the voltage on PZT4 patches, \( V_g \), and the electric power, \( P_e \), which can be seen from Eqs. (18), (3) and (4). For example, the amplitude of \( q_1 \) nonlinearly decreases from 0.036 m to 0.016 m in the same section shown in the Fig. 4b. This result suggests that a smaller \( b_0 \) for the tapered cantilever harvester will lead to more electric power harvesting from the same ambient excitation.

Fig. 5 displays the effect of the thickness at the fixed end of the cantilever on the RMS, the first generalized coordinate and the first natural frequency of the cantilever. The dimensions of the energy harvester in this simulation are set to be, \( n = 601 \), \( b_0 = 0.04 \text{ m} \), \( l = 0.6 \text{ m} \), and \( h_1/h_0 = b_1/b_0 = 0.2 \). From Fig. 5a it can be found that the RMS nonlinearly decreases from 0.1376 W to 0.0142 W with an increase in the thickness at the fixed end of the cantilever from 3 mm to 5 mm. This phenomenon is caused by the similarly variational tendency in the amplitude of the first generalized coordinate which is relevant to two factors. The main factor is the linearly increase in the first natural frequency of the tapered harvester from 77.75 rad/s to 129.25 rad/s in the same thickness section which can be seen from Fig. 5b; the second factor is the linearly increase in the mass of \( m_0 = \rho b_0 h_0 \). Two reasons above lead to a nonlinearly decrease in the amplitude of the first generalized coordinate from 0.0405 m to 0.0087 m shown as Fig. 5c. These phenomena in Fig. 5 can also be explained explicitly by Eqs. (18) and (2)–(4). The results reveal that a thicker cantilever harvester with the same taper ratio in thickness would lead to a decrease of RMS significantly.

5. Conclusion

A piezoelectric energy harvester made of a tapered cantilever surface bonded with piezoelectric patches is developed to harness energy from ambient vibrations. The current available models for
Appendix A.

\[
U_2 = \frac{1}{\Delta z} h_2, \quad V_2 = \frac{2}{\Delta h_2}, \quad U_3 = \frac{2}{\Delta h_2}, \quad V_3 = \frac{1}{\Delta h_2}, \quad \Delta_2 = \Delta_3 = \frac{3}{h_2}.
\]

\[
U_4 = -\frac{Z_3}{\Delta h}, \quad V_4 = -\frac{1}{\Delta h} \left[ (z_2 V_1 + z_3 V_2 + z_3 V_3) \right], \quad \Delta_4 = z_3 U_2 + z_3 U_3 + z_4 U_4.
\]

\[
U_5 = -\frac{Z_4}{\Delta h}, \quad V_5 = -\frac{1}{\Delta h} \left[ (z_4 V_2 + z_4 V_3 + (z_4 U_2 + z_4 U_3 + z_4 V_4) \right], \quad \Delta_5 = (z_4 U_2 + z_4 U_3 + z_4 U_4 + z_4 U_5).
\]

\[
U_{i+1} = U_i + \frac{1}{\Delta h} \left[ (z_1 U_{i-2} + z_2 U_{i-1}) V_{i-1} + \frac{1}{\Delta h} \left[ (z_1 U_{i-2} + z_2 U_{i-1} + z_3 U_{i-1}) \right] \right],
\]

\[
V_n = -\frac{1}{\Delta h} \left[ (z_1 U_{n-2} + z_2 U_{n-3} + z_2 U_{n-1}) V_{n-2} + \frac{1}{\Delta h} \left[ (z_1 U_{n-2} + z_2 U_{n-3} + z_2 U_{n-1}) \right] \right].
\]

References


