GO 60
Pure & Applied Algebraic Geometry
celebrating Giorgio Ottaviani's 60th birthday

June 21-25, 2021

Tentative schedule (CEST)

	Mon 6/21	**Tue 6/22**	**Wed 6/23**	**Thu 6/24**	**Fri 6/25**
11:00-11:50		Conca	Mourrain	Sturmfels	Schreyer
		Gather Town break	*GT break*	*GT break*	*GT break*
14:00-14:50	**14:30** opening session	Arrondo	Bernardi	Chiantini	Mezzetti
14:50-15:50	**15:00-15:50** Pardini	Sodomaco Cazzador	Poster session	Sorea Gałązka	Poster session
	GT break	*GT break*	*GT break*	*GT break*	*GT break*
16:20-17:10	Alzati	Miro-Roig	Ciliberto	Fania	Manivel
17:10-18:00	Vallès	Weyman	Abo	Draisma	Landsberg
	GT break	*GT break*	*GT break & birthday party*	*GT break*	*GT break*

Titles & abstracts

- **Hirotachi Abo -** ***The discriminant locus of a vector bundle***

  This talk is concerned with the classification of singular elements of the space of global sections of a vector bundle on a non-singular projective variety.
  A non-zero global section of a vector bundle of rank r on an n-dimensional non-singular projective variety defines a closed subscheme of the projective variety called the zero scheme of the global section. The zero scheme of a global section of a rank r vector bundle has codimension at most r (if it is non-empty). The global sections of the vector bundle whose zero scheme is non-singular of codimension r form a Zariski open set of the projective space of the global sections of the vector bundle. We call the complement of such an open set the discriminant locus of the vector bundle. The discriminant locus of a vector bundle generally is irreducible of codimension one. However, there are vector bundles whose discriminant loci are not irreducible or have higher codimensions. Thus, it is a tempting problem to classify such degenerate discriminant loci.

The main purpose of this talk is threefold; the first is to show that the ampleness of the vector bundle on the non-singular projective variety implies the irreducibility of its discriminant locus, the second is to discuss the criteria that a very ample vector bundle needs to satisfy, so that its discriminant locus has codimension one, and the third is to use these criteria to complete the classification of degenerate discriminant loci for $n = r$.
This is joint work (work in progress) with Robert Lazarsfeld and Gregory Smith.

- **Alberto Alzati - *Some results about Weak Lefschetz Property***

  The Weak Lefshetz Property deals with properties of multiplication maps within quotient algebras $A := \mathbb{C}[x_0, \ldots, x_m]/I$ where I is an Artinian homogeneous ideal generated by degree d polynomials. WLP holds if the multiplication (between homogeneous parts of A) by general linear element is of maximal rank. Although the matter is rather elementary, there are currently only a few results in this regard, lacking of a general strategy, also for low values of m and d and for regular sequences of r polynomials, for which it is conjectured that WLP holds. In the talk it will be proved that WLP holds when $r = 5, m = 4, d = 2$; a case not previously considered.

- **Enrique Arrondo - *Using algebraic geometry in the representation theory of finite groups***

  Given a finite group G, the center of its group algebra $K(G)$ if a finite commutative K-algebra, so it corresponds to a finite number of points. We use this idea to reconstruct all the representation theory of G in terms of those points. In particular, we will give a way of finding all the irreducible representations of G and deciding which fields K are suitable as ground fields for the representations. We will show, as an application when G is the symmetric group S_d, how to describe the different symmetries of functions in d variables. As a consequence, we will re-prove a theorem of Tocino stating that the hyperdeterminant of a d-dimensional matrix is zero for all but two types of symmetric matrices.

- **Alessandra Bernardi - *Decomposition of polynomials and minimal apolar schemes***

  I will present the relation among minimal apolar schemes of a given form and its so-called Generalized Additive Decomposition.

- **Elisa Cazzador - *Inverting catalecticants of ternary quartics***

  We study the reciprocal variety to the LSSM of catalecticant matrices associated with ternary quartics. With numerical tools, we obtain 85 to be its degree and 36 to be the ML-degree of the LSSM. We provide a geometric explanation to why equality between these two invariants is not reached, as opposed to the case of binary forms, by describing the intersection of the reciprocal variety and the orthogonal of the LSSM in the rank loci. Moreover, we prove that only the rank-1 locus, namely the Veronese surface $\nu_4(\mathbb{P}^2)$, contributes to the degree of the reciprocal variety.

- **Luca Chiantini-** ***Four times four***

  In the last years, methods for tensor analysis have involved a series of increasingly deep tools for the study of projective varieties. In return, the process suggested to algebraic geometers a focus on some special aspects of the theory. I will present an instance of this process, based on recent join works with Ottaviani, Vanniuwenhoven, Bocci, Angelini, Mazzon, on the construction of criteria for the minimality of Waring decompositions of forms. The investigation is strictly linked to the study of singularities of higher secant varieties. The general ideas and methods will be illustrated with special attention to the case of forms of degree four.

- **Ciro Ciliberto -** ***Extensions of canonical curves and double covers***

  A variety of dimension n is said to be extendable r times if it is the linear section of a variety of dimension $n+r$ which is not a cone. I will recall some general facts about extendability, with special regard for extensions of canonical curves to $K3$ surfaces and Fano 3-folds. Then I will focus on double covers and on their extendability properties. In particular I will consider $K3$ surfaces of genus 2, that are double covers of the plane branched over a general sextic. A first results is that the general curve in the linear system pull back of plane curves of degree $k > 6$ lies on a unique $K3$ surface, so it is only once extendable. A second result is that, by contrast, if $k < 7$ the general such curve is extendable to a higher dimensional variety. In fact in the cases $k = 4, 5, 6$, this gives the existence of singular index k Fano varieties of dimensions 8, 5, 3, genera 17, 26, 37, and indices 6, 3, 1 respectively. For $k = 6$ one recovers the Fano variety $\mathbb{P}(3, 1, 1, 1)$, one of two Fano threefolds with canonical Gorenstein singularities with the maximal genus 37, found by Prokhorov. A further result is that this latter variety is no further extendable. For $k = 4$ and 5 these Fano varieties have been identified by Totaro.

- **Aldo Conca -** ***Ideals associated to subspace arrangements***

  Given a subspace arrangement we may associate to it two ideals, the intersection of the linear ideals associated to each subspace or their product. The structure of the intersection ideal is mostly unknown. For example already for a finite collection of generic points in a projective space the degree of the generators of the intersection is not known. On the other hand the product ideal is better understood. An old theorem of Herzog and myself asserts that the product ideal has a linear resolution, or, which is the same, its regularity is given by the number of subspaces in the arrandment. In the talk we will discuss the structure of the resolution of the product ideal. We will see that such a resolution is supported on a polymatroid.
  In collaboration with Manolis Tsakiris of ShanghaiTech.

- **Jan Draisma -** ***The geometry of*** GL***-varieties***

  A GL-variety is an infinite-dimensional affine variety equipped with a suitable action of the infinite-dimensional general linear group. GL-varieties arise naturally in the study of

properties of polynomials (and more general tensors) that do not depend on their number of variables, a research theme that is attracting attention in diverse areas of mathematics. I will report on joint work with Arthur Bik, Rob Eggermont, and Andrew Snowden, on the structure of GL-varieties. The basic building blocks of GL-varieties are the affine spaces A^λ corresponding to a finite tuple λ of Schur functors. Indeed, one of our theorems says that any irreducible GL-variety X admits a dominant morphism $B \times A^\lambda \to X$ for some λ and some finite-dimensional variety B. If λ and B are taken minimal, then λ is unique and the dimension of B is the transcendence degree of the invariant function field $K(X)^{GL}$.

- **Maria Lucia Fania -** ***Ulrich bundles on 3-dimensional scrolls***

  I will report on a joint paper with M. Lelli-Chiesa and J. Pons Llopis, where we construct Ulrich bundles of low rank on 3-dimensional scrolls, with a special attention to 3-folds in P5 which are scrolls.

- **Maciej Gałązka -** ***Distinguishing secant from cactus varieties***

  Cactus varieties are defined using linear spans of arbitrary finite schemes of bounded length, while secant varieties use only isolated reduced points. In particular, any secant variety is always contained in the respective cactus variety, and, except in a few initial cases, the inclusion is strict. It is known that lots of natural criteria that try to test membership in secant varieties are actually only tests for membership in cactus varieties. In this talk, we present the first techniques to distinguish actual secant variety from the cactus variety in the case of the Veronese variety. We focus on the case of $\kappa_{14}(\nu_d(\mathbb{P}^n))$ the simplest that exhibits the difference between cactus and secant varieties. We show that for $d > 4$, the component of the cactus variety $\kappa_{14}(\nu_d(\mathbb{P}^6))$ other than the secant variety consists of degree d polynomials divisible by a $(d-3)$-th power of a linear form. We generalize this description to an arbitrary number of variables. We present an algorithm for deciding whether a point in the cactus variety $\kappa_{14}(\nu_d(\mathbb{P}^n))$ belongs to the respective secant variety for $d > 5, n \neq 6$. Our intermediate results give also a partial answer to analogous problems for other cactus varieties to any Veronese variety.
  The talk is about a joint project with Tomasz Mandziuk and Filip Rupniewski.

- **J.M. Landsberg -** ***Tensors of minimal border rank***

  In his paper Symplectic bundles on the plane, secant varieties and Lüroth quartics revisited GO gave a determinantal description of Strassen's equations for secant varieties of Segre varieties that led us to a vast generalization of these equations (Young flattenings). There had not been much progress regarding equations of secant varieties after that until very recently we used the border apolarity of Buczynska-Buczynski to obtain new equations.
  I will report on work in progress with Chia-Yu Chang, Arpan Pal and Joachim Jelisiejew on the geometry of these new equations and how they compare to the old.

- **Laurent Manivel -** ***Complete quadrics and Gaussian models***

  I will explain how the maximum likelihood degree (ML-degree) for linear concentration models, as well as the algebraic degree of semidefinite programming (SDP), can be understood in terms of Schubert calculus on the very classical varieties of complete quadrics. This allows to prove a conjecture by Sturmfels and Uhler on the polynomiality of the ML-degree, and a conjecture by Nie, Ranestad and Sturmfels providing an explicit formula for the degree of SDP. Joint work with M. Michalek, L. Monin, T. Seynnaeve and M. Vodicka.

- **Emilia Mezzetti -** ***Congruences of lines and families of matrices of constant rank***

  I will report on recent joint work with A. Boralevi and M.L. Fania, about quadric surfaces contained in the PfaffIan hypersurface in $\mathbb{P}^{14}$. I will then explain the connections with congruences of lines in the 5-dimensional projective space.

- **Rosa Maria Miro-Roig -** ***The Tea Theorem***

  The goal of this talk is to show the ubiquity of the Weak Lefschetz Property (WLP) and to prove the "Tea Theorem". More precisely, I will establish a close relationship between a priori two unrelated problems: (1) the existence of Togliatti systems (i.e. homogeneous Artinian ideals $I \subset k[x_0, \ldots, x_n]$ generated by forms of degree d which fail the WLP in degree $d-1$; and (2) the existence of (smooth) projective varieties $X \subset \mathbb{P}^N$ satisfying at least one Laplace equation of order $d-1 \geqslant 2$. These are two longstanding problems which lie at the crossroads between Commutative Algebra, Algebraic Geometry, Differential Geometry and Combinatorics. In the monomial case, I will classify some relevant examples, I will establish minimal and maximal bounds, depending on n and $d \geqslant 2$, for the number of generators of Togliatti systems. Finally, I will related Galois coverings with cyclic group $\mathbb{Z}/d$ to the so called GT-systems and study the minimality of GT-systems in terms of the number of monomials that appear in the expand of the determinant of a 3-line circulant matrix.
  All I will say is based in joint work with either L. Colarte, P. Di Poi, E. Evo, E. Mezzetti, M. Michałek G. Ottaviani, or M. Salat.

- **Bernard Mourrain -** ***Tensors, Eigenvectors and Simultaneous DiaGOnalisation***

  Tensor decomposition is a problem which can be much difficult than matrix rank decomposition, but it has many fascinating facets both from the algebraic, geometric and application point of view. In this talk, we will explore the relationship between this difficult problem and standard linear algebra operations such has eigenvector computation, simultaneous diagonalisation of matrix pencils. We will describe some algebraic approaches based on flat extension properties and simultaneous diagonalisation for computing tensor decomposition. Some links between varieties of commuting matrices and the Hilbert scheme of points will be discussed.

- **Rita Pardini -** ***Deformations of semi-smooth varieties***

  A variety X is semi-smooth if locally in the étale topology its singularities are either double crossing points ($xy = 0$) or pinch points ($x^2 - y^2z = 0$). Alternatively, X is semi-smooth if it can be obtained from a smooth variety X' by gluing it along a smooth divisor D' via an involution g of D'. We describe explicitly in terms of the triple (X', D', g) the two sheaves on X that control its deformation theory, that is, the tangent sheaf T_X and the sheaf $T^1_X := \mathrm{ext}^1(\Omega_X, \mathcal{O}_X)$. As an application, we discuss the smoothability of the semi-smooth Godeaux surfaces ($K^2 = 1, p_g = q = 0$).
  This is joint work with Barbara Fantechi and Marco Franciosi.

- **Frank-Olaf Schreyer -** ***Godeaux surfaces***

  In this talk I report on joined work with Isabel Stenger. We describe the construction of an 8-dimensional locally complete family of simply connected numerical Godeaux surfaces, building on an homological algebra approach. We also describe how the families of Reid and Miyaoka with torsion $\mathbb{Z}/3\mathbb{Z}$ and $\mathbb{Z}/5\mathbb{Z}$ arise in our homological setting.

- **Luca Sodomaco -** ***Algebraic degree of optimization over a variety with an application to p-norm distance degree***

  We study an optimization problem constrained on a real algebraic variety X and whose parametric objective function f_u is gradient solvable with respect to the parametric data u. This class of problems includes Euclidean distance optimization as well as maximum likelihood optimization. For these particular optimizations, a prominent role is played by the ED and ML correspondence, respectively. To our generalized optimization problem, we attach an optimization correspondence and show that it is equidimensional. This leads to the notion of algebraic degree of optimization on X. We apply these results to p-norm optimization, where p is a positive integer, and we define the p-norm distance degree of X. When $p = 2$, we recover the ED degree introduced by Draisma, Horobeţ, Ottaviani, Sturmfels, and Thomas. Finally, we derive a formula for the p-norm distance degree of X as a weighted sum of the polar classes of X under suitable transversality conditions.
  This is joint work with Kaie Kubjas and Olga Kuznetsova.

- **Miruna-Ştefana Sorea -** ***Signatures of paths and the shuffle algebra***

  Our work is motivated by the theory of rough paths in stochastic analysis, where information from a path is usually encoded in a sequence of tensors with real entries, called the path signature. Using tools from representation theory and applied algebra, we prove an intriguing combinatorial identity in the shuffle algebra. It has a close connection to de BruijnÕs formula.
  This talk is based on joint work with Laura Colmenarejo and Joscha Diehl.

- **Bernd Sturmfels - *Algebraic Statistics with a View towards Physics***

  We discuss the algebraic geometry of maximum likelihood estimation from the perspective of scattering amplitudes in particle physics. A guiding example is the CHY model, where the underlying very affine variety is the moduli space $M_{0,n}$ of n-pointed rational curves. The scattering potential plays the role of the log-likelihood function, and its critical points are solutions to rational function equations. Their number coincides with the Euler characteristic. Soft limit degenerations are combined with certified numerical methods for concrete computations.

- **Jean Vallès - *Free divisors in a pencil of plane curves***

- **Jerzy Weyman - *Finite free resolutions and root systems***

  I will discuss the connection between structure of perfect ideals of codimension 3 and Gorenstein ideals of codimension 4 with root systems and Schubert varieties in homogeneous spaces. This will be a friendlier version of discussion of this topic.